Homework 5
ECE 363 (F19)
10 problems for 100 pts
Due Nov 01

A. RELAXATION OSCILLATOR

1) Show that the relaxation oscillator frequency is given by: $f = \frac{1}{2RC\,ln\left(\frac{1+B}{1-B}\right)}$

An easy way to do this is to consider a capacitor with an initial voltage $V_C = -V_{TH}$. As shown in Fig. 1, it is charged by a resistor R connected to $+V_{SAT}$. We can figure out how long it takes the capacitor to go from $-V_{TH}$ to $+V_{TH}$ (assuming $V_{TH} < V_{SAT}$, which is indeed true).

   a) Derive the first-order differential equation for V_C.

   Hint #1: The current through R is the same as the current through C. Write the expression for each current and set them equal to each other.

   Hint #2: You should get: $\frac{dV_C}{dt} + \frac{1}{RC}V_C = \frac{1}{RC}V_{SAT}$

   b) Let the initial condition be $V_C(0) = -V_{TH}$ and the steady state solution be $V_C(\infty) = +V_{SAT}$. Solve the differential equation for $V_C(t)$. Hint: You should get $V_c(t) = V_{SAT} - (V_{TH} + V_{SAT})e^{-t/RC}$.

   c) Derive the time value when V_C reaches $+V_{TH} = V_{SAT}\,R1/(R1 + R2)$. Hint: You should get $t = RC\,ln\left(\frac{1+B}{1-B}\right)$, where $B = R_1/(R_1+R_2)$.

   d) Derive the frequency of the square wave output from the relaxation oscillator.

2) Design a relaxation oscillator with a frequency of 1 kHz. Use standard 5% resistor values and 10% capacitor values.

   a) A reasonable choice of the feedback ratio is B = 0.5. R1 and R2 are typically in the 100 kohm range.

   b) Choose R and C. Note: R is typically around 100 kohm, and C is typically 0.1 uF or less.

   NOTE: Using resistors in the 100 kohm range ensures the max op amp output current is well below 1 mA.

   c) Based on your choice of components, show that your frequency is within 5% of the desired value.

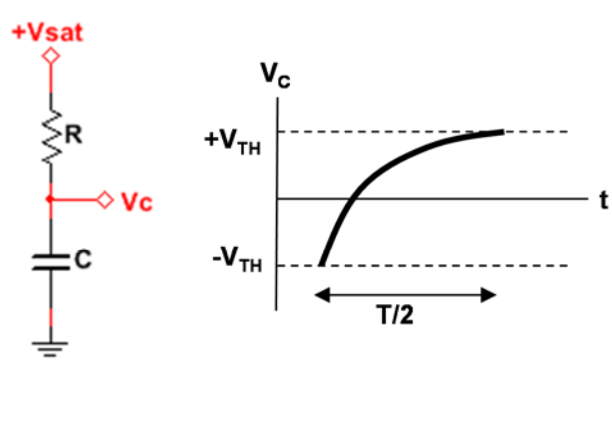


Fig. 1

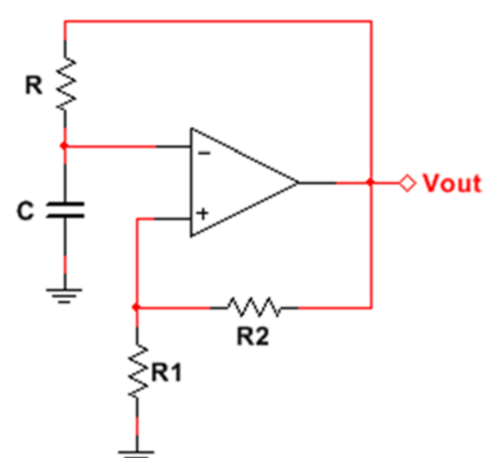


Fig. 2

B. SCHMITT TRIGGER

3) A non-inverting Schmitt trigger is basically a voltage comparator with hysteresis.

- The output goes HIGH ($V_{OUT} = +V_{SAT}$) when the input rises above the UTP.
- The output goes LOW ($V_{OUT} = -V_{SAT}$) when the input dips below the LTP.

Including some hysteresis (meaning UTP ≠ LTP) produces a much less jittery output V_{OUT} when the input contains noise fluctuations.

Consider the non-inverting Schmitt trigger shown in Fig. 3. Assume $V_{CC} = 15V$, $+V_{SAT} = V_{CC}-1$ and $-V_{SAT} = -V_{CC}+1$.

a) Compute the UTP and LTP levels.

b) The input voltage V_{IN} has some fluctuations, as shown in Fig. 3. Sketch both V_{IN} and V_{OUT}. Label important features, such as the UTP and LTP on the V_{IN} plot and +/-V_{SAT} on the V_{OUT} plot.

4) Consider the non-inverting Schmitt trigger shown in Fig. 4. The reference voltage $V_{REF} = 5V$ is connected to the (-) input of the op amp. Assume $V_{CC} = 15V$, $+V_{SAT} = V_{CC}-1$ and $-V_{SAT} = -V_{EE}+1$.

a) Derive general expressions for the UTP and LTP levels (e.g. in terms of V_{REF}, R1, and R2).

Hint #1: Use a similar procedure as the derivations in the Lecture 11 notes.

Hint #2: You should get UTP = (1 + R1/R2) V_{REF} + V_{SAT}R1/R2 and LTP = (1 + R1/R2) V_{REF} – V_{SAT}R1/R2.

b) The input voltage V_{IN} has some fluctuations, as shown in Fig. 4. Sketch both V_{IN} and V_{OUT}. Label important values, such as the UTP and LTP on the V_{IN} plot and +/-V_{SAT} on the V_{OUT} plot.

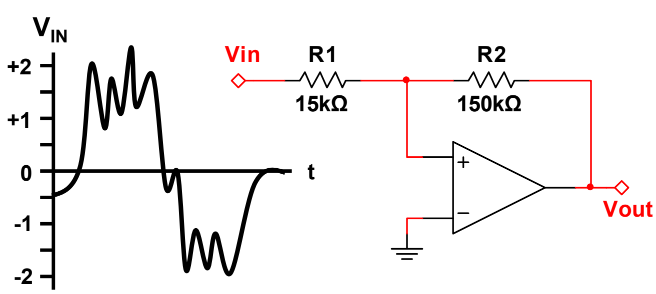


Fig. 3

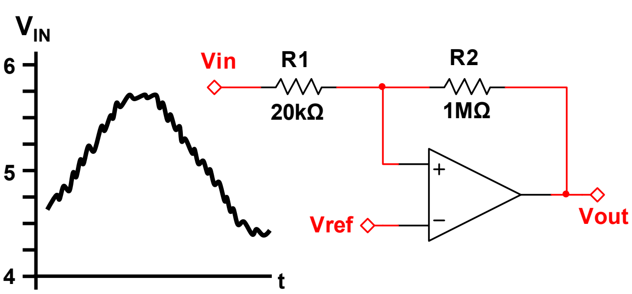


Fig. 4

C. TRIANGLE WAVE GENERATOR

5) As we've seen in Lecture 11, a good way to generate a triangle wave is to use a closed loop circuit formed with a non-inverting Schmitt trigger and an op-amp integrator (see Fig. 5). The square wave (Schmitt trigger output) goes from $+V_{SAT}$ to $-V_{SAT}$ while the triangle wave goes between $-V_{TH}$ to $+V_{TH}$. However, we did not discuss the frequency of the triangle wave. Show that the triangle wave frequency is given by $f = {}^{R_2}/_{4R_1R_3C}$

Hint #1: The integrator output is $V_{OUT} = -\left({}^{1}/_{R_3C}\right)\int V_{IN}dt$, where V_{IN} is the square wave from the Schmitt trigger.

Hint #2: It helps to sketch one cycle of the square wave and integrator output, as shown in Fig. 5. During a duration T/2, the integrator output changes from $+V_{TH}$ to $-V_{TH}$ while the square wave is held at $+V_{SAT}$. You know that $V_{TH} = V_{SAT}R_1/R_2$, so you can therefore solve for T.

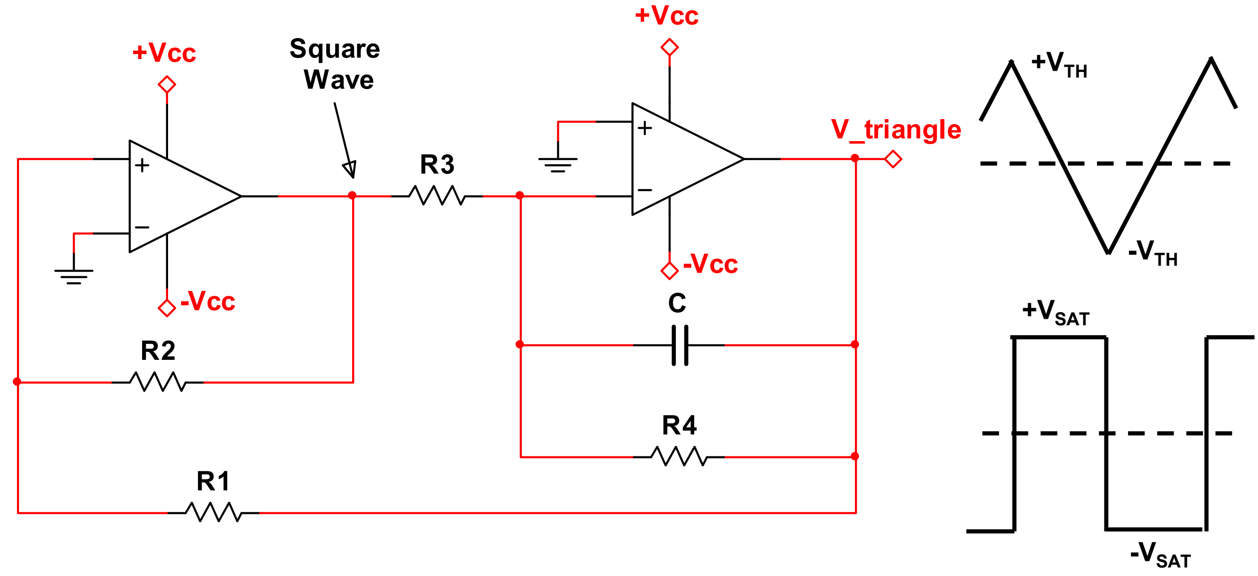


Fig. 5

6) Design a triangle wave generator to output a 20V peak-to-peak waveform with a 500 Hz frequency. Your op-amps are powered by +/- 15 V. Use standard 5% resistors and 10% capacitors.

a) Choose R_1 and R_2. Hint: R_2 is typically 100 kohm or higher. This ensures the max op amp current is pretty low.

b) Choose R_3, C, and R4. C is typically between 1 nF and 100 nF while R3 is typically between 1 kohm and 100 kohm.

c) Most integrators have a large resistor in parallel with the capacitor (see Fig. 6). The purpose is to suppress output drift over long periods of time. This resistor R_4 is typically chosen to be at least 10 times higher than R_3. What value should you use for your 500 Hz triangle wave generator?

d) Show that your resulting frequency is within 5% of the desired value.

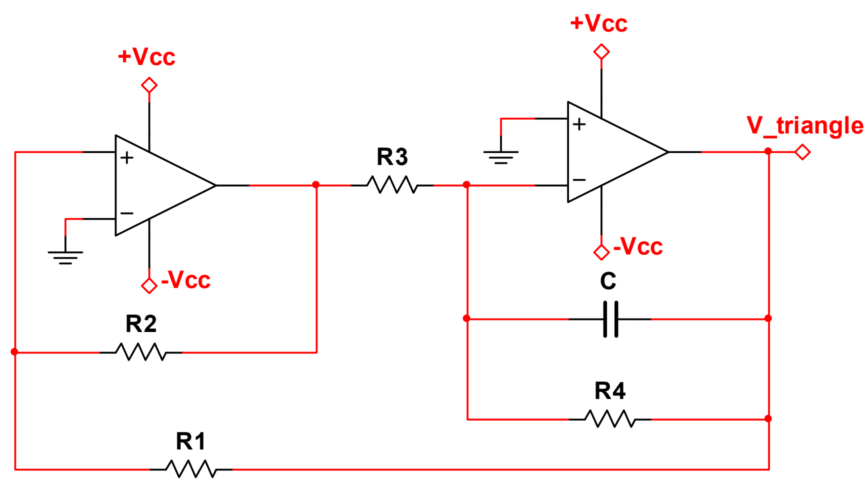


Fig. 6

7) A PWM signal can also be made with a "sawtooth" waveform, which is similar to a triangle wave but has steep falling edges. A common method to generate a sawtooth waveform is to charge a capacitor with a constant current source. The capacitor is discharged once the voltage has reached a threshold value. As shown in the figure below, we can use a 555

timer chip to do this! The discharge transistor turns on when the capacitor voltage reaches $2V_{CC}/3$ and turns off once the capacitor voltage drops to $V_{CC}/3$ (NOTE: Buma corrected the lecture notes to show this). Show that the sawtooth frequency is given by $f = 3\, I_O/(V_{CC}C)$

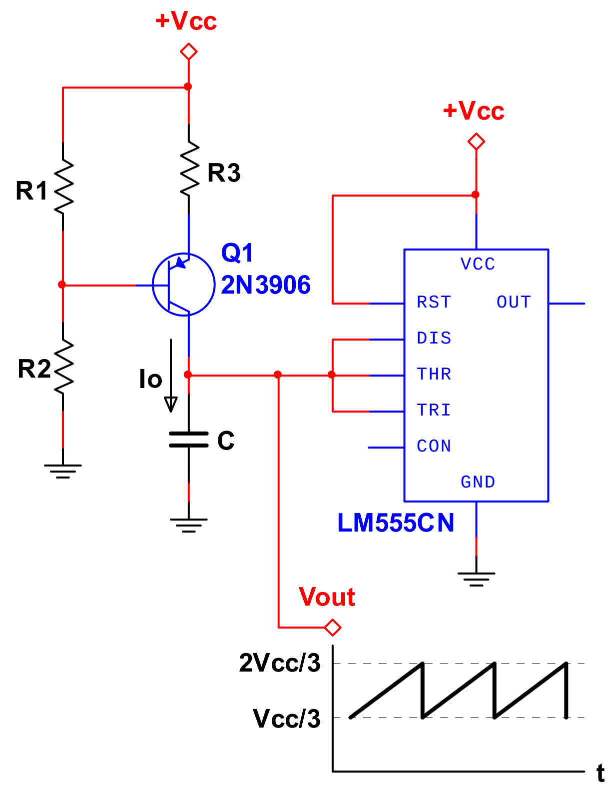
8) Design a sawtooth oscillator using a 555 timer to operate at 10 kHz (+/- 5% is fine). Let Vcc = 9V. Some comments:

- The available capacitor values are 100 pF, 1 nF, 10nF, and 100 nF.
- I_0 is typically between 0.1 to 1 mA.
- Assume "quick" analysis parameters for Q1.
- Typically, the voltage across R3 is about 1V.
- Make R1 and R2 a "firm" divider. This means that $R_1//R_2 \approx (\beta+1)R_3/10$.

a) Choose values for your capacitor C and current source value Io. Keep in mind the comments above!

b) Choose your resistor R3.

c) Choose your divider resistors R1 and R2.



D. Pulse Width Modulation

9) Analyze the pulse width modulator shown in Fig. 7. The comparator is just an op amp (no hysteresis). Assume $+/-V_{SAT} = +/- (V_{CC} - 1)$.

a) Compute the amplitude of the triangle wave.

b) Compute the frequency of the triangle wave.

c) Compute the duty cycle of the PWM output.

d) Sketch the triangle wave, V_{REF}, and output V_{PWM} over a 2 ms interval. Label important features!

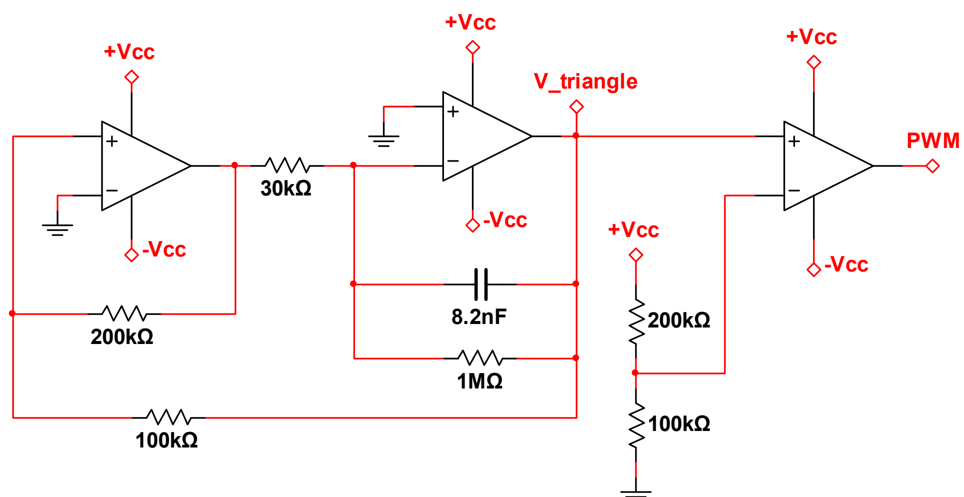


Fig. 7

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10) Suppose we want to make a night light that turns on when ambient light is low. Ambient light is detected with a photocell, which is basically a light-sensitive resistor. A photocell has high resistance in darkness, and low resistance in bright light. Photocells are notorious for having a wide resistance tolerance (see data sheet).

Consider the voltage comparator circuit shown in Fig. 10. The photocell and resistor R1 form a voltage divider. A reference voltage is produced with a voltage divider made from R2 and a 100 kohm resistor. All voltage dividers and the comparator are powered by V_{CC} = +5V. The design constraints are below:

- R1 and R2 must be chosen from: 1 kohm, 10 kohm, 100 kohm, or 1 Mohm
- High speed operation is NOT necessary from the comparator (this affects your choice of R).

    (a) Based on the photocell's data sheet, what is the appropriate value of R1 to ensure Vsig has widely different values between bright (> 10 lux) and dark conditions?

    (b) Compute the appropriate value for R2.

    (c) Would you use R = 220 ohm or 10 kohm for the comparator output? Explain.

    (d) Sketch your circuit, and **explain your reasoning** for how the comparator's (+) and (-) inputs are connected to the Vsig and Vref voltage dividers.

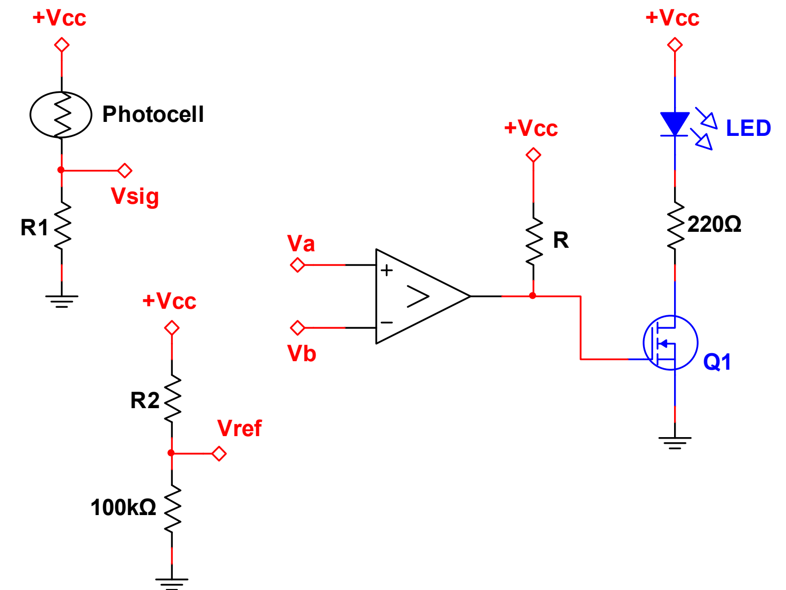

Fig. 10: Night light circuit for Problem 8. The photocell is a light sensitive resistor, where more light causes LOWER resistance.