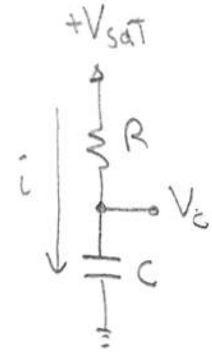
ECE 363
HW 5 solns

Total = 100 pts



① ⓐ $i = \dfrac{V_{sat} - V_c}{R} = C\dfrac{dV_c}{dt}$

$$\frac{1}{RC}V_{sat} - \frac{1}{RC}V_c = \frac{dV_c}{dt}$$

$$\Rightarrow \boxed{\frac{dV_c}{dt} + \frac{1}{RC}V_c = \frac{1}{RC}V_{sat}}$$

Method #1

ⓑ Total soln = Homogeneous + Particular

- Homogeneous: $\dfrac{dV_c}{dt} + \dfrac{1}{RC}V_c = 0$ Try $V_c = Ae^{xt}$

$$\Rightarrow xAe^{xt} + \frac{1}{RC}Ae^{xt} = 0$$

$$\left(x + \frac{1}{RC}\right)Ae^{xt} = 0$$

$$\Rightarrow x = -1/RC$$

$$V_{c,H} = Ae^{-t/RC}$$

- Particular: (steady state) $\dfrac{dV_c}{dt} + \dfrac{1}{RC}V_c = \underbrace{\dfrac{1}{RC}V_{sat}}_{\text{constant}}$ Try $V_c = B \equiv$ constant

$$\Rightarrow 0 + \frac{1}{RC}B = \frac{1}{RC}V_{sat}$$

$$\Rightarrow B = V_{sat}$$

So, $V_c(t) = Ae^{-t/RC} + V_{sat}$

- Initial condition: $V_c(0) = -V_{TH} = A + V_{sat} \Rightarrow A = -(V_{TH} + V_{sat})$

$$\Rightarrow \boxed{V_c(t) = V_{sat} - (V_{TH} + V_{sat})e^{-t/RC}}$$

$\boxed{\text{METHOD \#2}}$ Another way is to use an integrating factor

$$\frac{dV_c}{dt} + \left(\frac{1}{RC}\right) V_c = \left(\frac{1}{RC} V_{sat}\right)$$

$$\frac{dV_c}{dt} + \alpha V_c = \beta$$

Integrating factor is $\mu(t) = e^{\int \alpha \, dt}$

$$V_c(t) = \int_0^t \frac{\mu(t')}{\mu(t)} \beta \, dt' + \underbrace{V_c(0)}_{-V_{TH}} \frac{\mu(0)}{\mu(t)}$$

So, $\dfrac{\mu(t')}{\mu(t)} = \dfrac{e^{\frac{1}{RC}t'}}{e^{\frac{1}{RC}t}} = e^{-t/RC} e^{t'/RC}$

$$\rightarrow V_c(t) = e^{-t/RC} \int_0^t e^{t'/RC} \left(\frac{1}{RC} V_{sat}\right) dt' + (-V_{TH}) e^{-t/RC}$$

$$= \frac{1}{RC} V_{sat} e^{-t/RC} \int_0^t e^{t'/RC} dt' - V_{TH} e^{-t/RC}$$

$$= \cancel{\frac{1}{RC}} V_{sat} e^{-t/RC} \Big[\underbrace{RC e^{t'/RC} \Big|_0^t}_{\cancel{RC}(e^{t/RC} - 1)} \Big] - V_{TH} e^{-t/RC}$$

$$= V_{sat} (1 - e^{-t/RC}) - V_{TH} e^{-t/RC}$$

$$\boxed{V_c(t) = V_{sat} - (V_{TH} + V_{sat}) e^{-t/RC}}$$

METHOD #3

$$\frac{dV_c}{dt} + \frac{1}{RC} V_c(t) = \underbrace{\frac{1}{RC} V_{sat}}_{\text{constant}}$$

This makes it easy to directly integrate:

$$RC \frac{dV_c}{dt} = V_{sat} - V_c(t)$$

$$\frac{dV_c}{V_{sat} - V_c} = \frac{1}{RC} dt$$

$$\int_{V_c(0)}^{V_c(t)} \frac{dV_c}{V_{sat} - V_c} = \frac{1}{RC} \int_0^t dt = t/RC$$

using: $d\left[\ln(a-y)\right] = \frac{-dy}{a-y}$

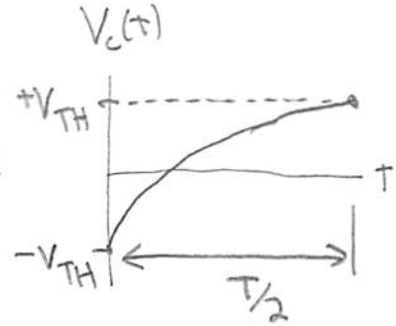
$$\rightarrow -\int_{V_c(0)}^{V_c(t)} \frac{dV_c}{V_{sat} - V_c} = \ln(V_{sat} - V_c)\Big|_{V_c(0)}^{V_c(t)} = -t/RC$$

$$= \left[\ln\left(V_{sat} - V_c(t)\right) - \ln\left(V_{sat} + V_{TH}\right)\right] = -t/RC$$

$$= \ln\left(\frac{V_{sat} - V_c(t)}{V_{sat} + V_{TH}}\right) = -t/RC$$

$$\rightarrow e^{-t/RC} = \frac{V_{sat} - V_c(t)}{V_{sat} + V_{TH}} \Rightarrow \boxed{V_c(t) = V_{sat} - (V_{sat} + V_{TH})\, e^{-t/RC}}$$

(c) At $t = \frac{T}{2}$, $V_c = +V_{TH}$



$$\rightarrow \quad V_{TH} = V_{sat} - (V_{TH} + V_{sat})e^{-T/2RC}$$

$$(V_{sat} + V_{TH})e^{-T/2RC} = V_{sat} - V_{TH}$$

$$\frac{V_{sat} + V_{TH}}{V_{sat} - V_{TH}} = e^{T/2RC}$$

using $V_{TH} = V_{sat}\frac{R_1}{R_1+R_2}$ $\rightarrow$ $\frac{V_{sat} + V_{TH}}{V_{sat} - V_{TH}} = \frac{V_{sat}\left(1 + \frac{R_1}{R_1+R_2}\right)}{V_{sat}\left(1 - \frac{R_1}{R_1+R_2}\right)}$

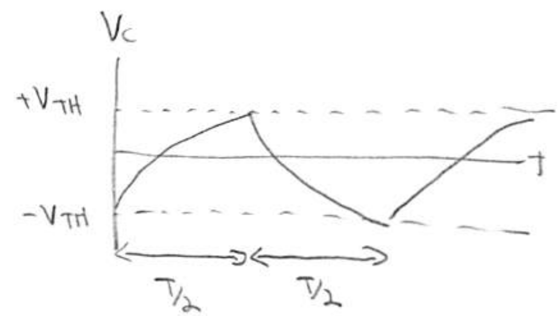
$$= \frac{1+B}{1-B}, \quad B = \frac{R_1}{R_1+R_2}$$

$$\rightarrow \quad \frac{1+B}{1-B} = e^{T/2RC} \rightarrow \frac{T}{2RC} = \ln\frac{1+B}{1-B}$$

$$\boxed{\frac{T}{2} = RC\ln\frac{1+B}{1-B}}$$

(d) $T = 2RC\ln\left(\frac{1+B}{1-B}\right)$

$$\rightarrow \quad \boxed{f = \frac{1}{2RC\ln\left(\frac{1+B}{1-B}\right)}}$$

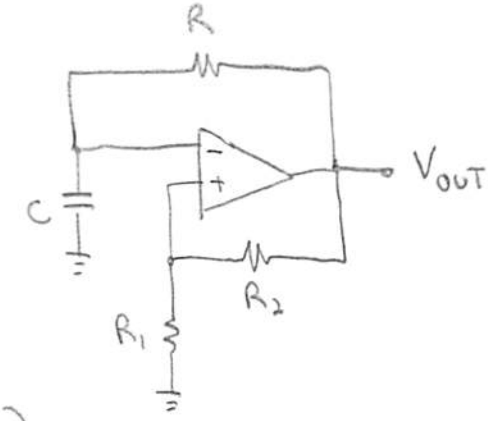


②

ⓐ Choose $\beta = 0.5 = \frac{R_1}{R_1 + R_2}$

$\Rightarrow R_1 = R_2$

Let $\boxed{R_1 = R_2 = 100K}$



ⓑ $T = 10^{-3} s = 2RC \ln\left(\frac{1+0.5}{1-0.5}\right)$

$RC = 4.551 \times 10^{-4} s$

C	Ideal R	Actual R
6.8nF	66.9K	68K
8.2nF	55.5K	56K
4.7nF	96.8K	100K
3.9nF	116.7K	120K

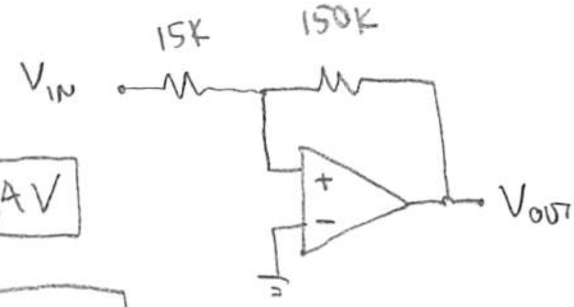
← Choose $\boxed{R = 56K \quad C = 8.2nF}$

$f = \frac{1}{2(56 \times 10^3)(8.2 \times 10^{-9}) \ln\left(\frac{1.5}{.5}\right)}$

$= \boxed{991.1 Hz}$

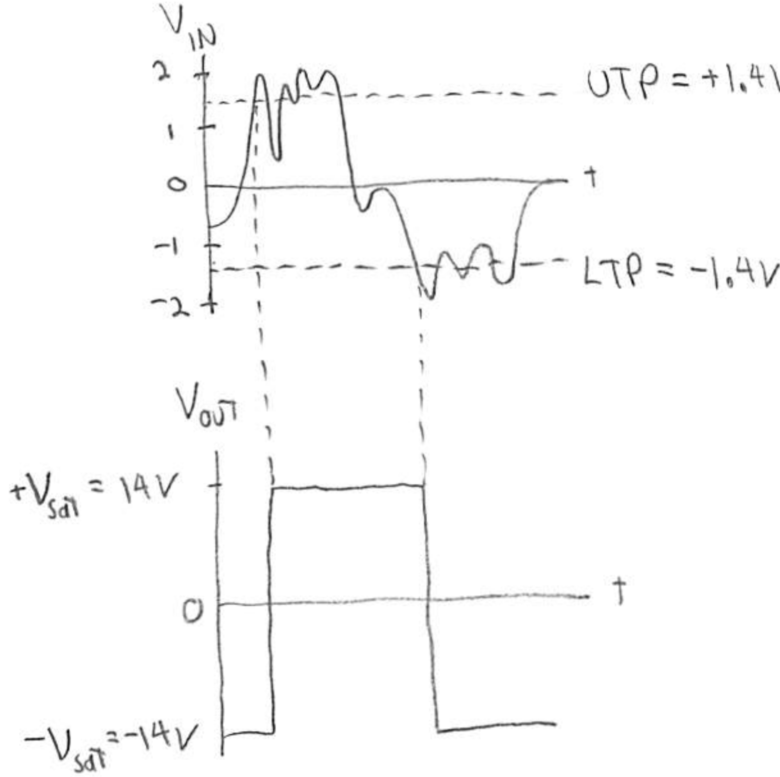
ⓒ $\% \text{ Error} = \frac{991.1 - 1000}{1000} \times 100\% = \boxed{-0.9\%}$

3 a



$$UTP = \frac{R_1}{R_2} V_{sat} = \frac{15K}{150K}(14) = \boxed{1.4V}$$

$$LTP = -\frac{R_1}{R_2} V_{sat} = -\frac{15K}{150K}(14) = \boxed{-1.4V}$$



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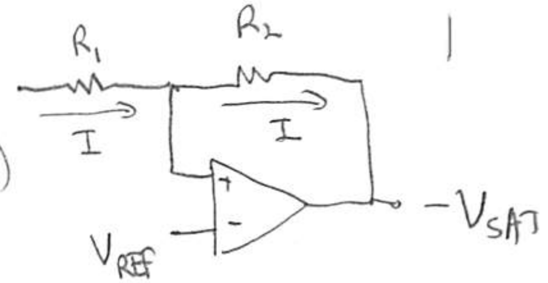
(a)

UTP:

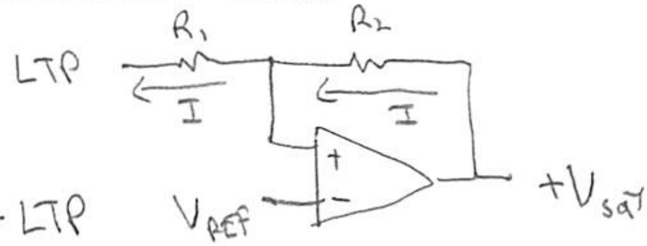


$$I = \frac{UTP - V_{REF}}{R_1} = \frac{V_{REF} - (-V_{sat})}{R_2}$$

$$UTP - V_{REF} = \frac{R_1}{R_2}V_{REF} + \frac{R_1}{R_2}V_{sat}$$

$$\rightarrow \boxed{UTP = \left(1 + \frac{R_1}{R_2}\right)V_{REF} + \frac{R_1}{R_2}V_{sat}}$$

LTP:



$$I = \frac{V_{sat} - V_{REF}}{R_2} = \frac{V_{REF} - LTP}{R_1}$$

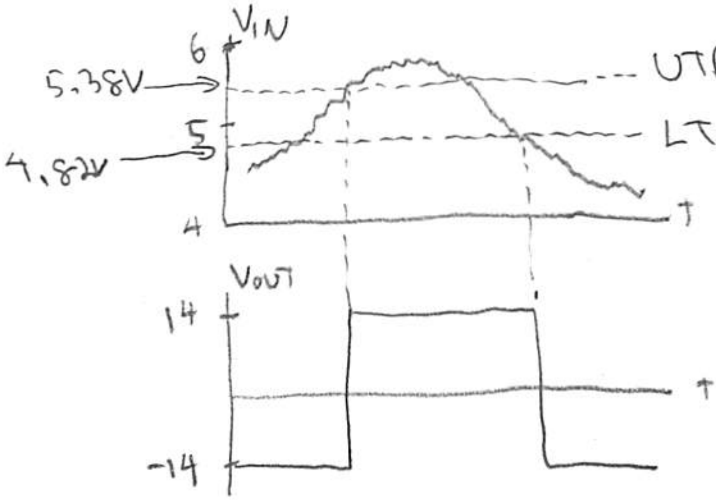
$$\frac{R_1}{R_2}V_{sat} - \frac{R_1}{R_2}V_{REF} = V_{REF} - LTP$$

$$\rightarrow \boxed{LTP = \left(1 + \frac{R_1}{R_2}\right)V_{REF} - \frac{R_1}{R_2}V_{sat}}$$

(b)

$$UTP = \left(1 + \frac{20K}{1000K}\right)5 + \frac{20K}{1000K}14 = \boxed{5.38V}$$

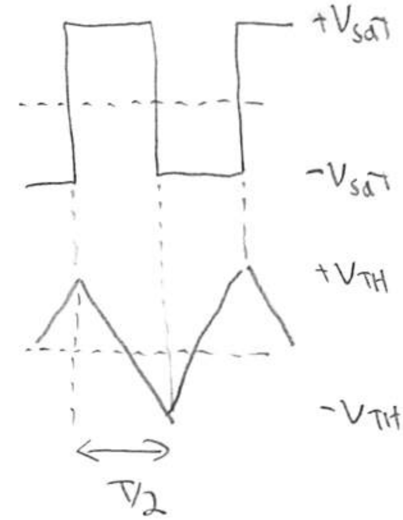
$$LTP = \left(1 + \frac{20K}{1000K}\right)5 - \frac{20K}{1000K}14 = \boxed{4.82V}$$



(5)

Integrator output is

$$V_{OUT} = -\frac{1}{R_3 C}\int V_{IN}\,dT$$



$$V_{OUT}\left(\frac{T}{2}\right) - V_{OUT}(0) = -\frac{1}{R_3 C}\int_0^{T/2} V_{sat}\,dT$$

$$-V_{TH} - V_{TH} = -\frac{1}{R_3 C} V_{sat}\frac{T}{2}$$

$$-2V_{TH} = -\frac{1}{2}\frac{V_{sat}}{R_3 C}T$$

$$\frac{1}{T} = \frac{1}{4R_3 C}\frac{V_{sat}}{V_{TH}}$$

Since $V_{TH} = \frac{R_1}{R_2}V_{sat} \Longrightarrow \frac{1}{T} = \boxed{f = \frac{R_2}{4R_1 R_3 C}}$

(6)

(a) $V_{TH} = 10V = \frac{R_1}{R_2}V_{sat} = \frac{R_1}{R_2}14 \rightarrow \frac{R_1}{R_2} = \frac{10}{14} = 0.7143$

choose $\boxed{R_1 = 75K \quad R_2 = 100K} \rightarrow \frac{R_1}{R_2} = 0.7$

(b) $500\,Hz = \frac{100K}{4(75K)R_3 C} \rightarrow R_3 C = 6.67\times10^{-4}s$

C	Ideal R_3	Actual R_3
6.8nF	98.1K	100K
8.2nF	81.3K	82K
10 nF	66.7K	68K

Choose $\boxed{R_3 = 82K \quad C = 8.2nF}$

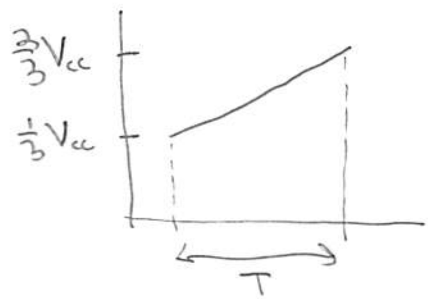
(c) $R_4 > 10R_3$

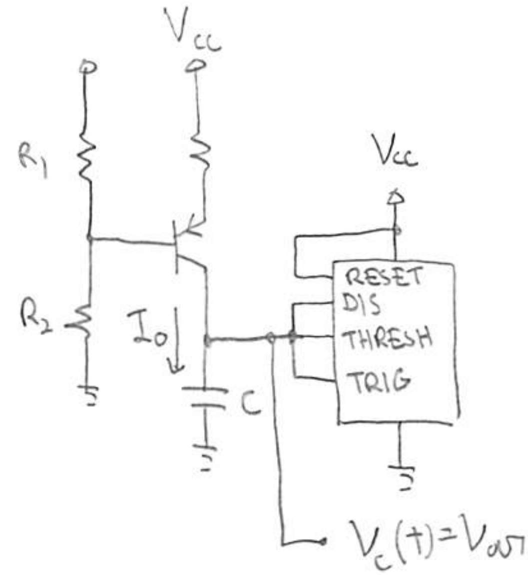
$> 820K$ Choose $\boxed{R_4 = 1M}$

(d) $f = \frac{100K}{4(75K)(82\times10^3)(8.2\times10^{-9})} = \boxed{495.7Hz}$

Error= $\boxed{-0.9\%}$

7





$$i = C\frac{dV_c}{dt}$$

$$V_c(t) = \frac{1}{C}\int i\,dt$$

$$V_c(T) - V_c(0) = \frac{1}{C}\int_0^T I_o\,dt = \frac{1}{C}I_o T$$

$$\frac{2}{3}V_{cc} - \frac{1}{3}V_{cc} = \frac{1}{C}I_o T \rightarrow \frac{1}{3}V_{cc} = \frac{1}{C}I_o T$$

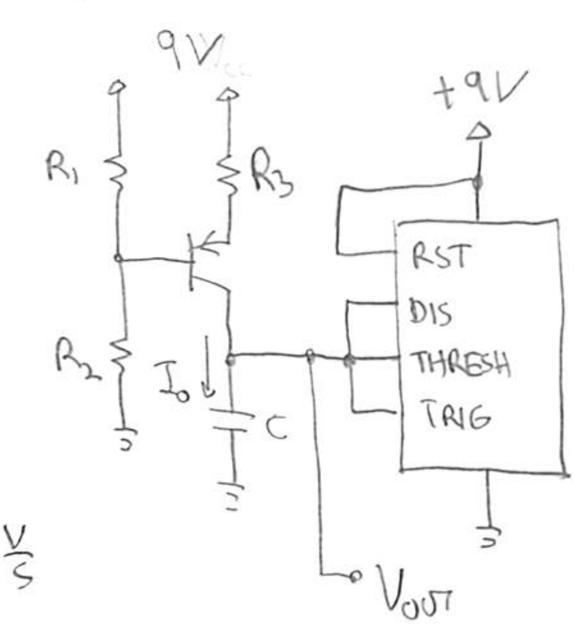
$$\Rightarrow T = \frac{CV_{cc}}{3I_o}$$

$$\Rightarrow \boxed{f = \frac{1}{T} = \frac{3I_o}{CV_{cc}}}$$

(8)

(a) Want $f = 10\,\text{KHz}$

use $f = \dfrac{3 I_o}{V_{cc} C}$



$10^4\,\text{Hz} = \dfrac{3 I_o}{9 \cdot C} \rightarrow \dfrac{I_o}{C} = 3\times10^4 \ \frac{V}{s}$

Let $C = 1\,nF \rightarrow I_o = 30\,\mu A$ (kind of small)

$\boxed{C = 10\,nF} \rightarrow \boxed{I_o = 0.3\,mA}$ (ok!)

(b)

$I_o = \alpha \dfrac{9 - V_E}{R_3} \sim \alpha \dfrac{1V}{R_3}$

So, $R_3 = 0.99 \times \dfrac{1V}{.3mA} = \boxed{3.3K}$

This is a standard 5% value!

(c) Want $V_B = 9 - 1 - 0.7 = 7.3V$

$9 \times \dfrac{R_2}{R_1 + R_2} = 7.3 \rightarrow \dfrac{9}{7.3} = \dfrac{R_1 + R_2}{R_2} = 1 + \dfrac{R_1}{R_2}$

$\rightarrow \dfrac{R_1}{R_2} = 0.233$

For firm divider, $R_1 \| R_2 \sim \dfrac{\beta+1}{10} R_3 = \dfrac{101}{10} \times 3.3K = 33.33K$

$R_1 \| R_2 = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{\frac{R_1}{R_2} R_2}{\frac{R_1}{R_2} + 1} = \dfrac{0.233\, R_2}{0.233 + 1} = 0.189\, R_2 = 33.33K$

$\Rightarrow R_2 = 176.4K$ Choose $\boxed{R_2 = 180K,\ R_1 = 43K}$

CHECK f!

$V_B = 9 \times \dfrac{180}{180 + 43}$

$= 7.265V$

$V_E = 7.965V$

$I_o = .99 \dfrac{9 - 7.965}{3.3K}$

$= 0.311\,mA$

$f = 10366.7\,\text{Hz}$

(3.7% error)

CHECK

(9)

(a) Triangle wave amplitude $= V_{TH} = \frac{R_1}{R_2} V_{sat} = \frac{100K}{200K} 14V = \boxed{7V}$

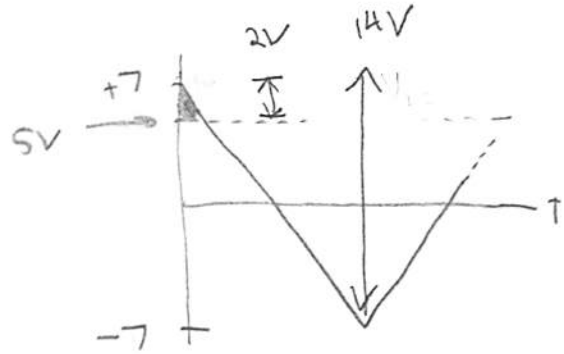
$(14V_{pp})$

(b) Frequency: $f = \frac{R_2}{4R_1R_3C} = \frac{200K}{4(100K)(30\times10^3)(8.2\times10^{-9})}$

$= \boxed{2032.5 \text{ Hz}}$

(c) $V_{REF} = 15 \times \frac{100K}{100K + 200K} = 5V$

PWM is HIGH when Triangle > V_{REF}

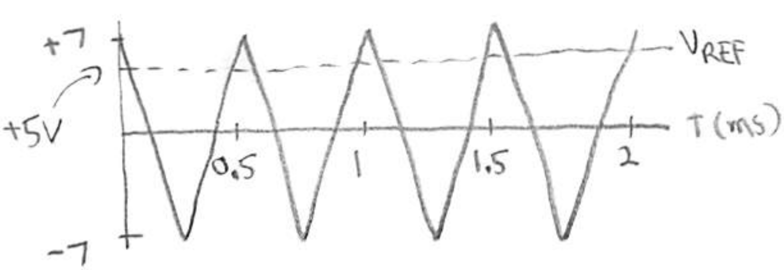


→ % above $= \frac{\text{Voltage Above } V_{REF}}{\text{Total peak-to-peak}} = \frac{7-5}{14} = \frac{2}{14}$

$= 0.143$

$\boxed{\text{Duty cycle} = 14.3\%}$

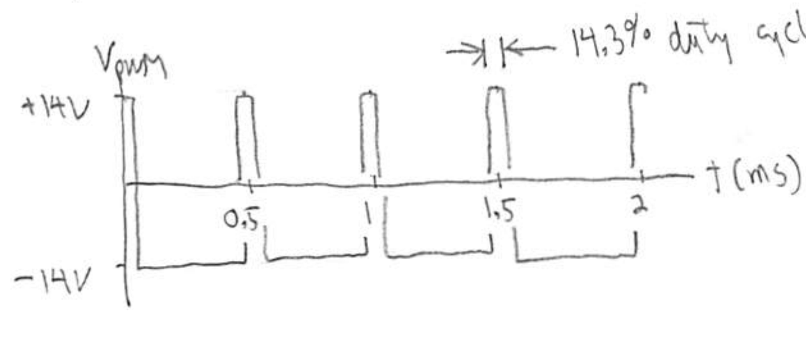
(d)



One period $= \frac{1}{2\text{kHz}}$

$= \underline{\underline{0.5 \text{ ms}}}$

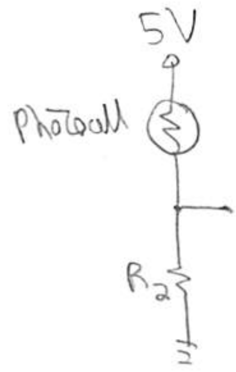
$\frac{2\text{ms}}{.5\text{ms}} = \underline{\underline{4 \text{ cycles}}}$



(10)

(a) Photocell data sheet: Dark resistance > 5M

Illuminated resistance: 10K To 30K

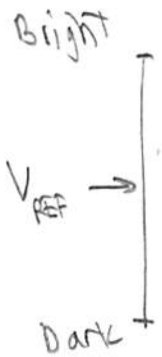


$$V_A = 5 \frac{R_2}{R_2 + R_{PC}}$$

	R_2	V_A Bright	V_A Dark	
	1K	.16 – .46V	.001V	X All Too low
	10K	1.25 – 2.5V	.010V	X Not enough swing
Both are OK	100K	3.85 – 4.6V	0.1V	
	1M	4.85 – 4.95V	0.83V	

$\boxed{\text{Choose 1M}}$ for lower power dissipation

(b)



Bright: 4.85 – 4.95V

Dark: < 0.83V

Make $V_{REF} = \frac{\text{Bright} + \text{Dark}}{2}$

$= 2.84V$

$$5 \frac{100K}{100K + R_3} = 2.84V \rightarrow \frac{5V}{2.84V} \times 100K = 100K + R_3$$

$R_3 = 76K$

OK $V_{REF} = 2.5V \longrightarrow$ Choose $\boxed{R_3 = 100K}$

(c) Do NOT need high speed operation, so $\boxed{10K}$ is fine.

(d) A table helps:

Condition	V_A	V_B	Comparator output ($V_B > V_A$?)	LED
Bright	HIGH	MED	LOW	OFF
Dark	LOW	MED	HIGH	ON