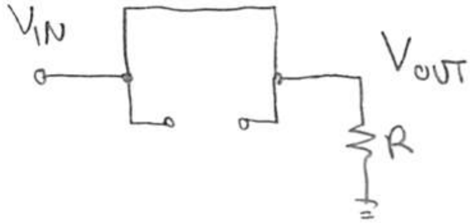
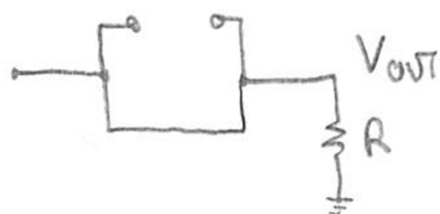
ECE 363
HW7 Solns

Total = 100 pts

① ⓐ $f = 0$ : Inductor ⇒ short
Capacitor ⇒ open

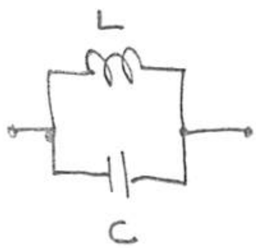



$f \to \infty$ : Inductor ⇒ open
capacitor ⇒ short




ⓑ Due to short, $V_{OUT} = V_{IN} \rightarrow \boxed{\dfrac{V_{OUT}}{V_{IN}} = 1}$

ⓒ $Z = j\omega L \,//\, \dfrac{1}{j\omega C}$




$$= \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{-\omega^2 LC + 1} = \underline{\frac{j\omega L}{1-\omega^2 LC}}$$

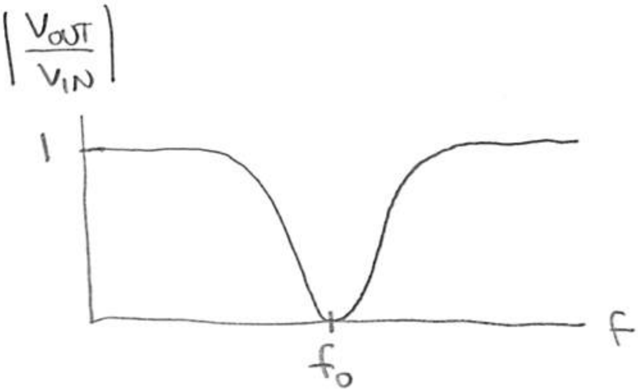
Looks like OPEN at $f = f_o$.

$|Z| \to \infty$ when denominator $= 0$

$$\rightarrow 1 - \omega_o^2 LC = 0$$

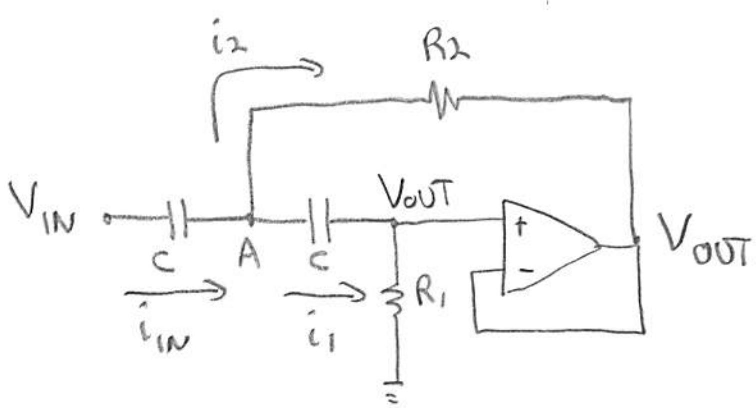
$$\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{f_o = \frac{1}{2\pi\sqrt{LC}}}$$

ⓓ




Notch filter
↑
or band stop

(2)




(a) KCL at node A:

$$i_{IN} = i_1 + i_2$$

$$\frac{V_{IN}-V_A}{Z} = \frac{V_A - V_{OUT}}{Z} + \frac{V_A - V_{OUT}}{R_2} \quad V_A?$$

$$Z = \frac{1}{j\omega C}$$

$$i_1 = \frac{V_A - V_{OUT}}{Z} = \frac{V_{OUT} - 0}{R_1}$$

$$\frac{R_1}{Z} V_A = \left(1 + \frac{R_1}{Z}\right) V_{OUT}$$

$$V_A = \left(1 + \frac{Z}{R_1}\right) V_{OUT} \quad (2)$$

$$V_A - V_{OUT} = \frac{Z}{R_1} V_{OUT} \quad (3)$$

$$V_{IN} - V_A = V_A - V_{OUT} + \frac{Z}{R_2}(V_A - V_{OUT})$$

$$V_{IN} - V_A = \left(1 + \frac{Z}{R_2}\right)(V_A - V_{OUT}) \quad (1)$$

Plug (2) and (3) into (1):

$$V_{IN} - \left(1 + \frac{Z}{R_1}\right) V_{OUT} = \left(1 + \frac{Z}{R_2}\right) \frac{Z}{R_1} V_{OUT}$$

$$V_{IN} = \left[\left(1 + \frac{Z}{R_2}\right)\frac{Z}{R_1} + \left(1 + \frac{Z}{R_1}\right)\right] V_{OUT}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{\frac{Z}{R_1} + \frac{Z^2}{R_1 R_2} + 1 + \frac{Z}{R_1}} = \frac{1}{1 + 2\frac{Z}{R_1} + \frac{Z^2}{R_1 R_2}}$$

$$= \frac{1}{1 + 2\frac{1}{j\omega R_1 C} + \frac{1}{-\omega^2 R_1 R_2 C^2}} = \frac{-\omega^2 R_1 R_2 C^2}{-\omega^2 R_1 R_2 C^2 \left(1 - j\frac{2}{\omega R_1 C}\right) + 1}$$

$$= \boxed{\frac{-\omega^2 R_1 R_2 C^2}{1 + j2\omega R_2 C - \omega^2 R_1 R_2 C^2}}$$

(b) $-\omega^2 \underbrace{R_1 R_2 C^2}_{\frac{1}{\omega_p^2}} = -\omega^2/\omega_p^2 \rightarrow \boxed{\omega_p = \frac{1}{\sqrt{R_1 R_2}\, C}}$

Also, $2\cancel{\omega} R_2 C = \frac{\cancel{\omega}}{\omega_p Q} \rightarrow Q = \frac{1}{\omega_p 2 R_2 C} = \frac{\sqrt{R_1 R_2}\,\cancel{C}}{2 R_2 \cancel{C}}$

$$\boxed{Q = \frac{1}{2}\sqrt{\frac{R_1}{R_2}}}$$

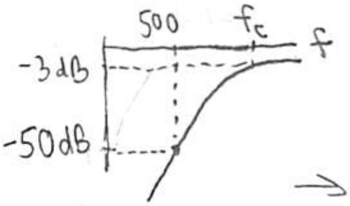
(c) When $Q = \frac{1}{\sqrt{2}}$ and $f_c = \frac{1}{2\pi}\omega_p$

$$\rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{-f^2/f_p^2}{1 + j\frac{f}{f_p}\sqrt{2} - f^2/f_p^2}$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{f^2/f_p^2}{\sqrt{(1 - f^2/f_p^2)^2 + \left(\frac{f}{f_p}\sqrt{2}\right)^2}} = \frac{(f/f_p)^2}{\sqrt{1 \cancel{- 2(f/f_p)^2} + (f/f_p)^4 \cancel{+ 2(f/f_p)^2}}}$$

$$\boxed{\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{(f/f_p)^2}{\sqrt{1 + (f/f_p)^4}}}$$

3) (a) $20\log_{10}|A| = -50\,dB \Rightarrow |A| = 10^{-50/20} = .00316 = \dfrac{(500/f_c)^2}{\sqrt{1+(500/f_c)^4}}$




$$\rightarrow 10^{-5} = \frac{(500/f_c)^4}{1+(500/f_c)^4} \Rightarrow 1+\left(\frac{500}{f_c}\right)^4 = 10^5\left(500/f_c\right)^4$$

$$1 = (10^5-1)\left(500/f_c\right)^4$$

$$1 = (10^5-1)^{1/4}\,\frac{500}{f_c}$$

$$\rightarrow \boxed{f_c = 8891.4\,Hz}$$

$$f_c = \frac{1}{2\pi\sqrt{R_1R_2}\,C},\quad Q = \frac{1}{2}\sqrt{\frac{R_1}{R_2}} = \frac{1}{\sqrt{2}} \rightarrow \frac{R_1}{R_2} = 2$$

$$\rightarrow f_c = \frac{1}{2\pi\sqrt{2R_2^2}\;C} = \frac{1}{2\pi\sqrt{2}\,R_2C} = 8891.4\,Hz$$

$$R_2C = \frac{1}{2\pi\sqrt{2}\,(8891.4)} = 1.266\times10^{-5}\,s$$

$$\text{Let } \boxed{C = 1\,nF} \rightarrow R_2 = 12657\,\Omega$$

Want $R_1, R_2 \gtrsim k\Omega$ to reduce power dissipation } Choose $\boxed{R_2 = 13K}$ $\boxed{R_1 = 26K}$




$$\Rightarrow \text{Actual } f_c = \frac{1}{2\pi\sqrt{26\times10^3\times13\times10^3}\times10^{-9}}$$

$$= 8656.9\,Hz$$

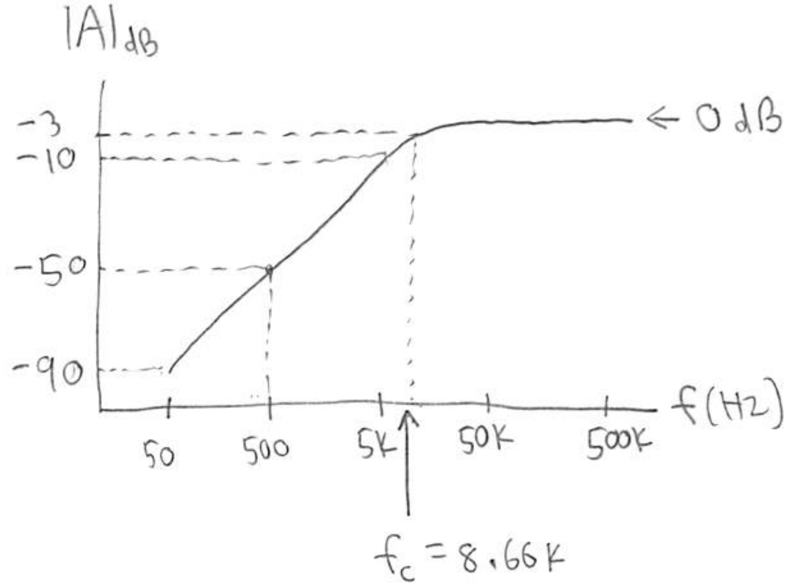
within 1 dB!

$$|A| = \frac{(500/8656.9)^2}{\sqrt{1+(500/8656.9)^4}} = .003336 \rightarrow \boxed{-49.54\,dB}$$

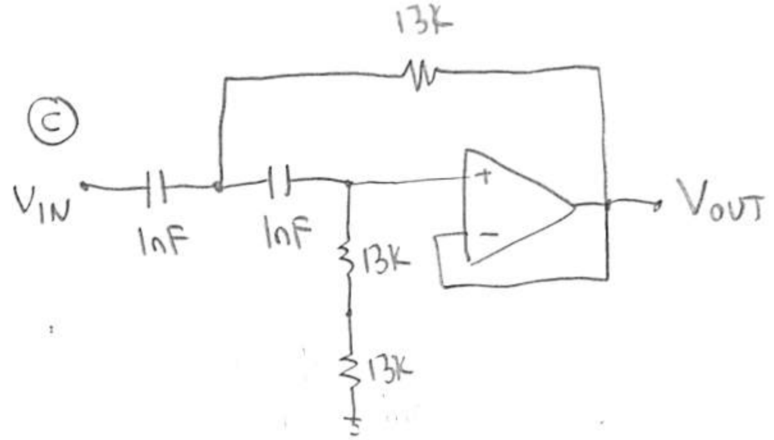
(b) $f_c = 8656.9\,Hz$

$|A| \sim -50\,dB$ at $500\,Hz$

$2^{nd}$ order filter rolloff is $-40\,dB/decade$




(c)




(4) (a) $Q_1 = 0.54 = \frac{1}{2}\sqrt{\frac{C_2}{C_1}} \rightarrow (1.08)^2 = \frac{C_2}{C_1} \rightarrow C_2 = 1.1664\,C_1$

Try $\boxed{C_1 = 1000\,pF \quad C_2 = 1200\,pF}$

$$f_c = \frac{1}{2\pi R\sqrt{C_1 C_2}} = 20 \times 10^3$$

$$R = \frac{1}{2\pi\sqrt{1000 \times 10^{-12} \times 1200 \times 10^{-12}} \times 20 \times 10^3} = 7264.4\,\Omega \leftarrow \text{Try } \boxed{7.5K}$$

$$\rightarrow \text{Actual } f_c = \frac{1}{2\pi(7.5 \times 10^3)\sqrt{1000 \times 1200 \times 10^{-24}}} = 19371.7\,Hz \quad ✓$$

$$100 \times \frac{19371.7 - 20000}{20000} = \boxed{-3.1\%}$$

(b) $Q_2 = 1.31$

$(2.62)^2 = \frac{C_2}{C_1}$

$C_2 = 6.8644\,C_1$

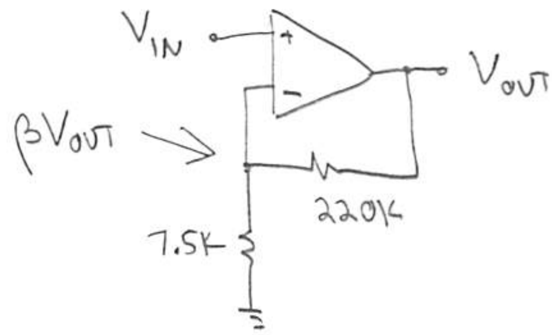
Try $\boxed{C_2 = 6800\,pF,\ C_1 = 1000\,pF}$

$$R = \frac{1}{2\pi\sqrt{6800 \times 1000 \times 10^{-12}} \times 20 \times 10^3} = 3051.6 \rightarrow \text{Try } \boxed{3K}$$

$$\text{Actual } f_c: \frac{1}{2\pi\sqrt{6800 \times 1000} \times 10^{-12} \times 3000} = \boxed{20.344\,kHz}$$

$\Delta = 1.7\%$

(5)




(a) $\beta V_{OUT} = \frac{7.5K}{7.5K + 220K} V_{OUT}$

$\rightarrow \boxed{\beta = 0.03297}$

(b) $A_o = 106\,dB \rightarrow A_o = 10^{106dB/20dB} = 1.995 \times 10^5$

$\Rightarrow \boxed{A_o\beta = 6577}$

(c) $G = \frac{A}{1 + A\beta} = \frac{1.995 \times 10^5}{1 + 6577} = \boxed{30.328}$

(d) $Z_{IN} = (1 + A\beta) R_i = (1 + 6577)(200K) = \boxed{1.316\,G\Omega}$

(e) $Z_{OUT} = \frac{R_o}{1 + A\beta} = \frac{25}{1 + 6577} = 3.8 \times 10^{-3}\,\Omega = \boxed{3.8\,m\Omega}$

(6)

(a) want $G = 100 = \frac{A_o}{1 + A_o\beta} \rightarrow 1 + A_o\beta = \frac{A_o}{100}$

$\beta = \frac{\frac{A_o}{100} - 1}{A_o} = \frac{1}{100} - \frac{1}{A_o}$

Since $A_o = 66\,dB = 10^{66dB/20dB}$

$= 1995.3 \longrightarrow \beta = .01 - \frac{1}{1995.3} = \boxed{0.00950}$

(b) $\frac{\Delta G}{G} = \frac{G_{MAX} - G_{MIN}}{G_o} \times 100\%$.

NOTE: Only $A_o$ varies while $\beta$ is <u>constant</u>!

Max: $A_{o,MAX} = 10^{69/20} = 2818.4 \Rightarrow G_{MAX} = \frac{2818.4}{1+(2818.4)(.0095)} = \underline{\underline{101.47}}$

Min: $A_{o,MIN} = 10^{63/20} = 1412.5 \Rightarrow G_{MIN} = \frac{1412.5}{1+(1412.5)(.0095)} = \underline{\underline{97.96}}$

$$\Rightarrow \frac{\Delta G}{G} = \frac{101.47 - 97.96}{100} \times 100\% = \boxed{3.5\%}$$

Does NOT satisfy design spec!

(c) Want $G_1 \times G_2 = G^2 = 100 \rightarrow G = 10 = \frac{A_o}{1+A_o\beta}$

$$1 + A_o\beta = 0.1A_o$$

$$\beta = 0.1 - \frac{1}{A_o} = 0.1 - \frac{1}{1995.3}$$

$$\boxed{\beta = 0.09950}$$

(d) $G_{MAX}^2 = \left(\frac{A_{MAX}}{1+A_{MAX}\beta}\right)^2 = \left(\frac{2818.4}{1+2818.4 \times .0995}\right)^2 = 100.29$

$G_{MIN}^2 = \left(\frac{A_{MIN}}{1+A_{MIN}\beta}\right)^2 = \left(\frac{1412.5}{1+1412.5 \times .0995}\right)^2 = 99.59$

$$\Rightarrow \frac{\Delta G}{G} = \frac{100.29 - 99.59}{100} \times 100\% = \boxed{0.7\%}$$

7) a) $G = \dfrac{A}{1+A\beta}$ $\left\{ \begin{array}{l} G_{MAX} = \dfrac{A_{MAX}}{1+A_{MAX}\beta} = \dfrac{10A_{MIN}}{1+10A_{MIN}\beta} \\ G_{MIN} = \dfrac{A_{MIN}}{1+A_{MIN}\beta} \end{array} \right.$

we know $0.99\,G_o < G < 1.01\,G_o$

$$G_{MAX} = 1.01 \times 30 = 30.3 = \frac{10A_{MIN}}{1+10A_{MIN}\beta} \quad (1)$$

$$G_{MIN} = 0.99 \times 30 = 29.7 = \frac{A_{MIN}}{1+A_{MIN}\beta} \quad (2)$$

we have two equations and two unknowns ($A_{MIN}$ and $\beta$)

using (2): $A_{MIN}\beta = \dfrac{A_{MIN}}{29.7} - 1$

Plug into (1): $30.3 = \dfrac{10A_{MIN}}{1+10\left[\dfrac{A_{MIN}}{29.7} - 1\right]} = \dfrac{10A_{MIN}}{10\dfrac{A_{MIN}}{29.7} - 9}$

$$\frac{10}{29.7}A_{MIN} - 9 = \frac{10}{30.3}A_{MIN}$$

$$A_{MIN} = \frac{9}{\dfrac{10}{29.7} - \dfrac{10}{30.3}} = 1349.9 \sim \boxed{1350}$$

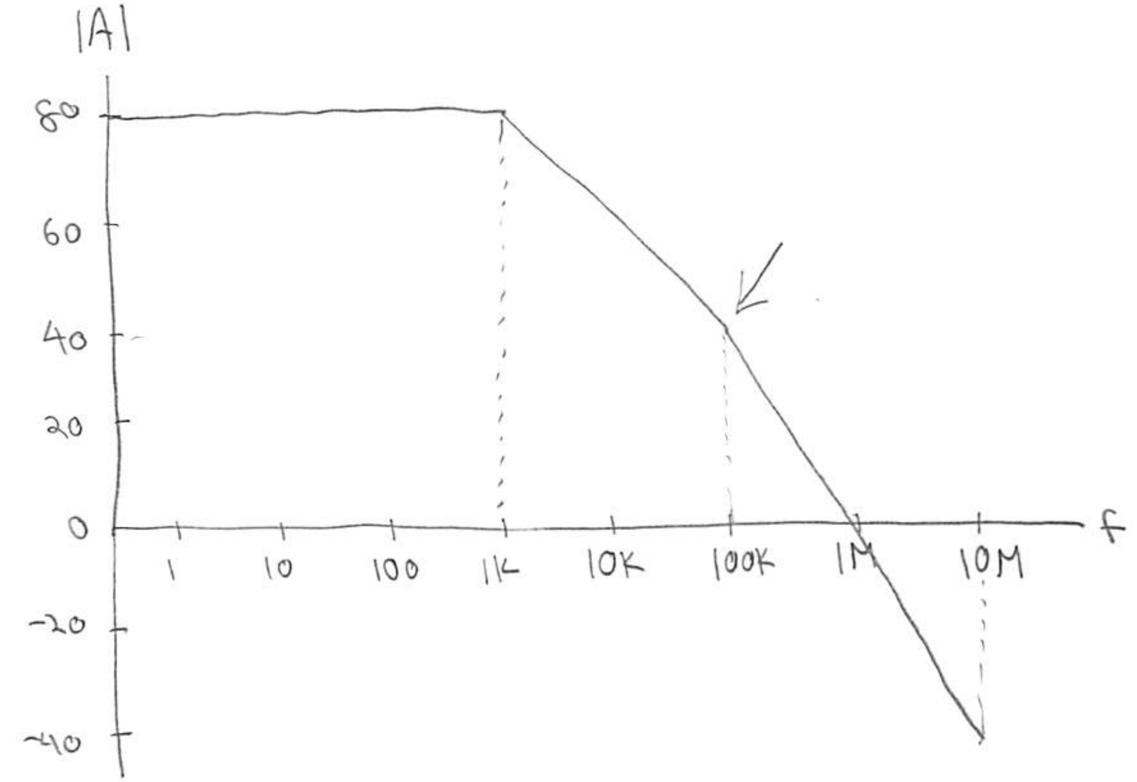
b) using (2): $A_{MIN}\beta = \dfrac{A_{MIN}}{29.7} - 1 \rightarrow \beta = \dfrac{\dfrac{1350}{29.7} - 1}{1350} = \boxed{.03293}$

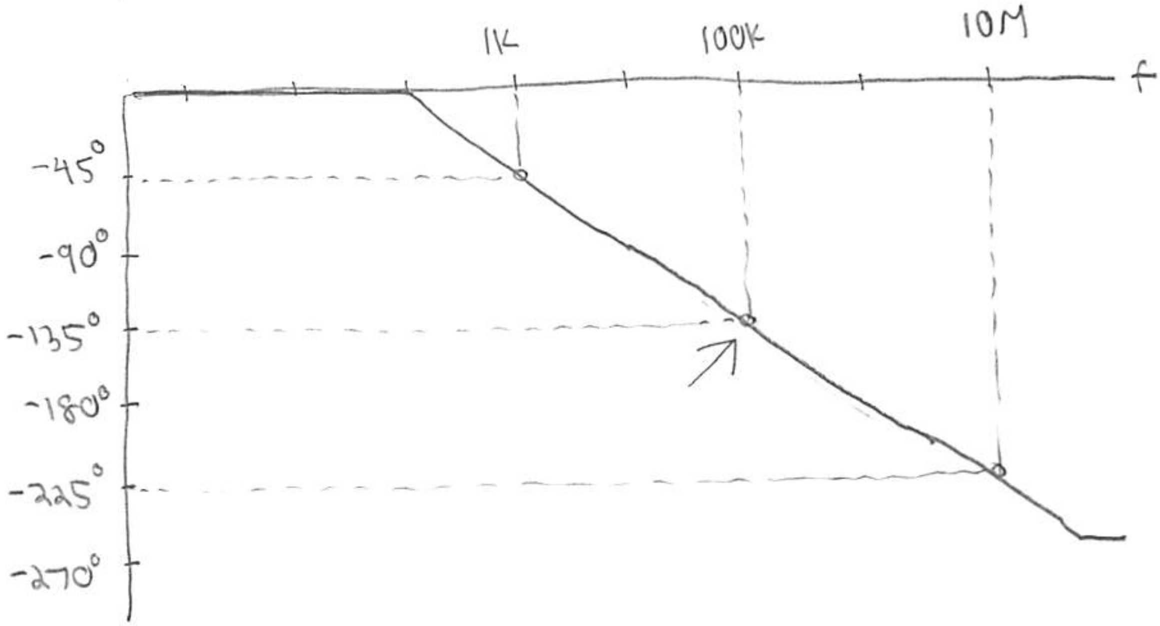
c) $G_o = 30 = \dfrac{A_o}{1+A_o\beta} \rightarrow 1 + A_o\beta = \dfrac{A_o}{30} \rightarrow 1 = A_o\left(\dfrac{1}{30} - \beta\right)$

$$A_o = \frac{1}{\frac{1}{30} - .03293} = \boxed{2479}$$

8 (a)

$20\log_{10} 10^4 = 80\,dB$



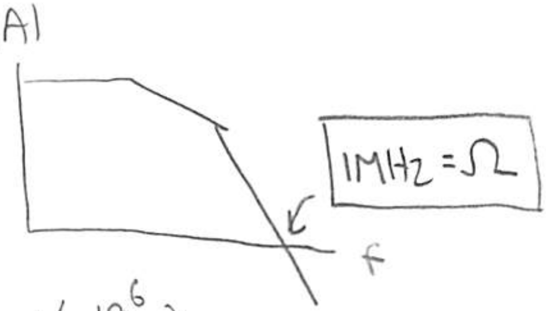

(b) $45^\circ$ phase margin $= \angle A - (-180^\circ) \Rightarrow \angle A = -135^\circ \leftarrow f = f_{p2} = 100\,KHz$

From Bode plot of $|A|$: $|A| = 40\,dB$ @ $100\,KHz$

$= 1/\beta \rightarrow 1/\beta = 10^{40/20} = 100$

$\Rightarrow \boxed{\beta = .01}$

ⓒ $\beta = 1 \rightarrow 20 \log_{10}(1) = 0 \text{ dB}$




$$\measuredangle A = -\tan^{-1}\left(\frac{10^6}{10^3}\right) - \tan^{-1}\left(\frac{10^6}{10^5}\right) - \tan^{-1}\left(\frac{10^6}{10^7}\right)$$

$$= -89.94^\circ - 84.29^\circ - 5.71^\circ = \boxed{-179.94^\circ}$$

Very close to $-180^\circ$!

ⓓ NO, phase margin is .06° when $\beta = 1$!