Lecture 12: Differential Amplifier (Part 1)

0. Review
1. Intro
2. Long-Tail Pair
3. Input Impedance
4. Output Impedance

Textbook reading:

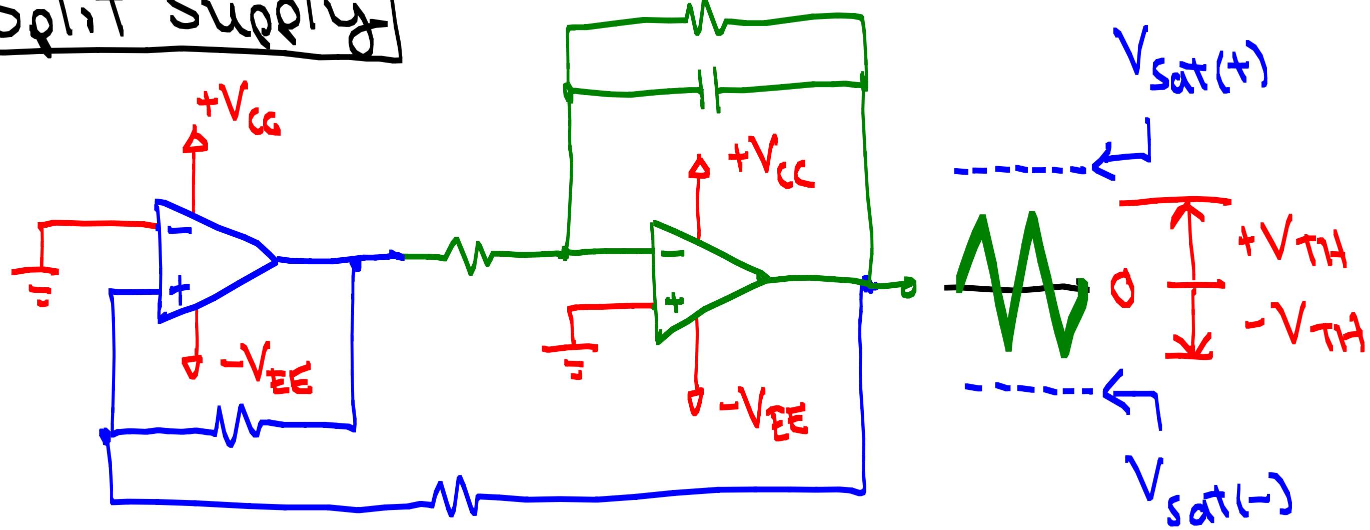
15-1 The differential amplifier
15-2 DC analysis
15-3 AC analysis

- HW 5 due Fri (Nov 01)
- PreLab 5 due today
  - → Team Project (2-3 people)
  - → Choose from 3 topics
  - → Determine responsibilities (who does what circuit)
  - → Formulate design requirements (e.g. servo pulse freq)
  - "Rough" circuit schematics (<u>no</u> component values)
- Lab 3 report due Nov 4 (Mon)
- Exam #2 (Nov 12) !

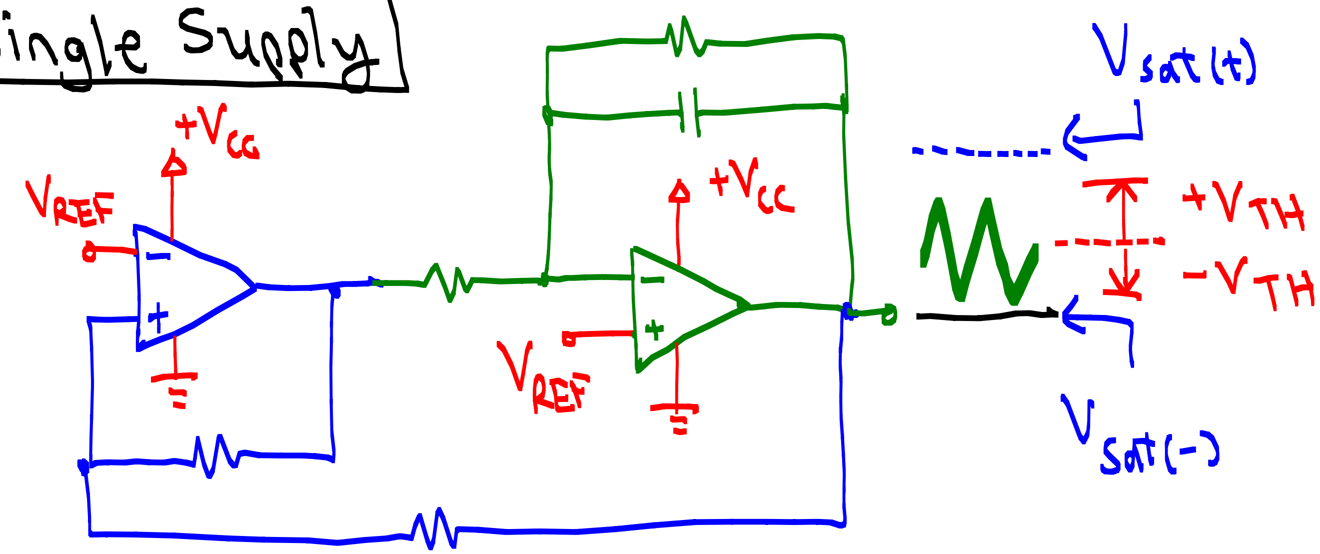
0. Review

- Triangle Wave Generator using op amps

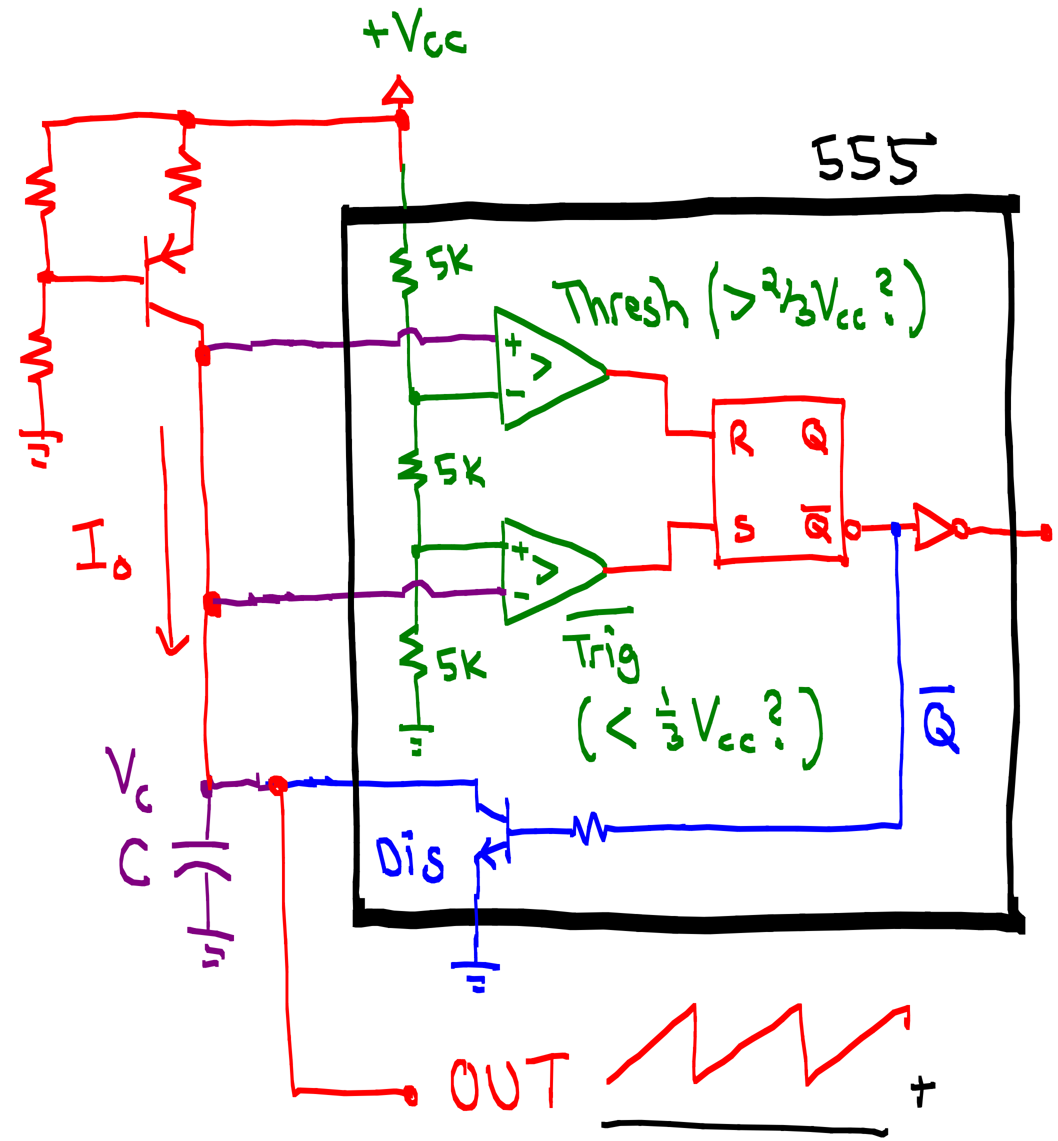
Split Supply



Single Supply



- Sawtooth Generator using 555 timer:

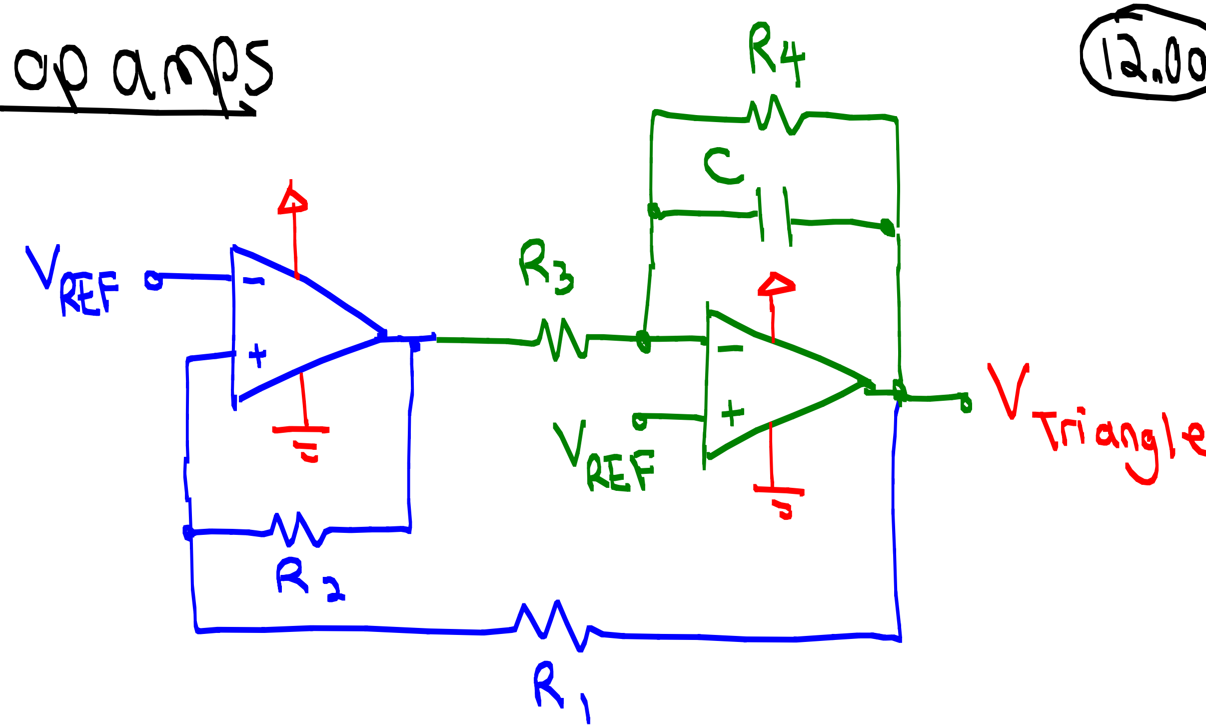


• Design example — Triangle wave w/ single-supply op amps

$f = 1\ \text{KHz}$, $6\ V_{pp}$

$V_{cc} = 9V$	$+V_{sat} = V_{cc} - 1$
$-V_{EE} = 0V$	$-V_{sat} = 0$

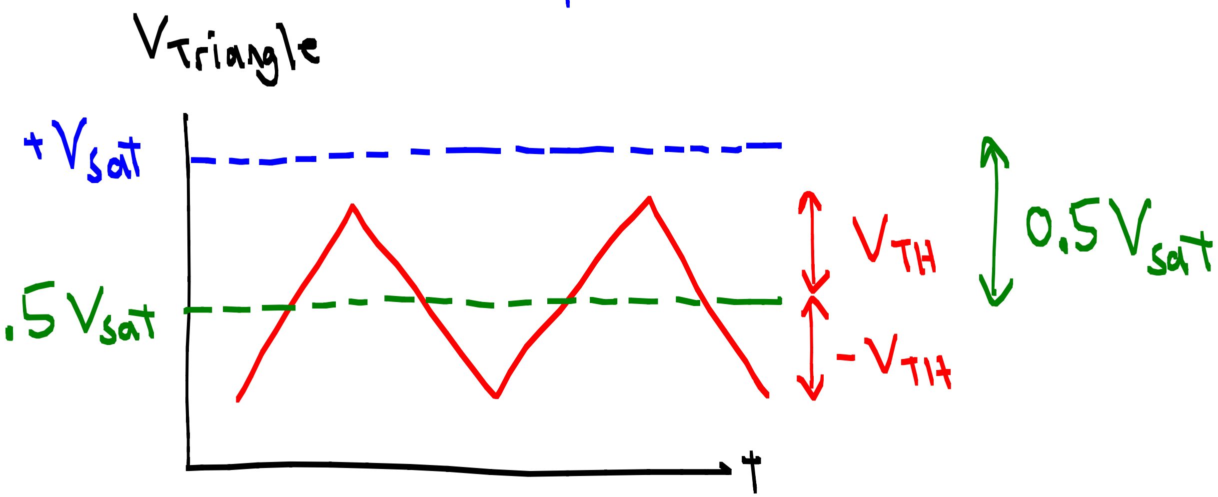
single supply op amp (e.g. LM358)



① $V_{TH} = \boxed{\dfrac{R_1}{R_2}} \times \boxed{0.5\,V_{sat}} = 3V$

$\dfrac{9-1}{2} = 4V$

$V_{REF} = 0.5\,V_{sat}$



$\rightarrow \dfrac{R_1}{R_2} = 0.75$

Typically, $R_2 \sim 100k$ range

Choose $\boxed{\begin{array}{l} R_1 = 75K \\ R_2 = 100K \end{array}}$

② $f = \dfrac{R_2}{4R_1R_3C} = \dfrac{100K}{4 \times 75K \times R_3C} = 1000\,Hz \Rightarrow R_3C = 3.333 \times 10^{-4}\,s$

Typically $C < 0.1\mu F$ and $R_3 \sim 100K$:

R_3	C	$C_{5\%}$	ΔC
100K	3333 pF	3300 pF	33 pF
110 K	3030	3000	30 ←
120 K	2778	2700	78
130 K	2564	2400	164

$\dfrac{100K}{4 \times 75K \times (110K)(3000pF)}$

$= 1010.1\,Hz$

(within 1%)

③ Choose $R_4 > 10R_3$ (or higher)

$\rightarrow R_4 = 1.5M$ (limit to few $M\Omega$ or less)

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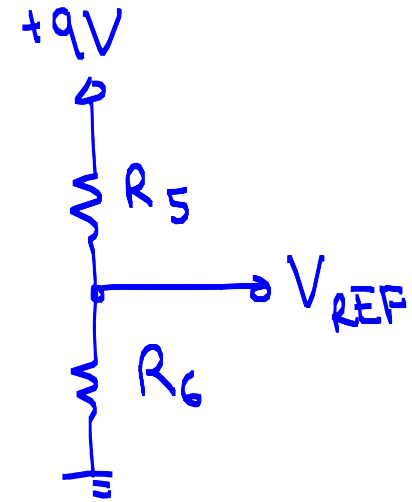
④ $V_{REF} = 0.5\left(V_{sat(+)} - V_{sat(-)}\right)$

$= 0.5(8 - 0) = \underline{\underline{4V}}$



$\rightarrow 9 \cdot \dfrac{R_6}{R_5 + R_6} = 4V$

$\dfrac{9}{4} R_6 = R_5 + R_6$

$\dfrac{5}{4} R_6 = 1.25 R_6 = R_5$

Typically, $R_6 \sim 100K$:

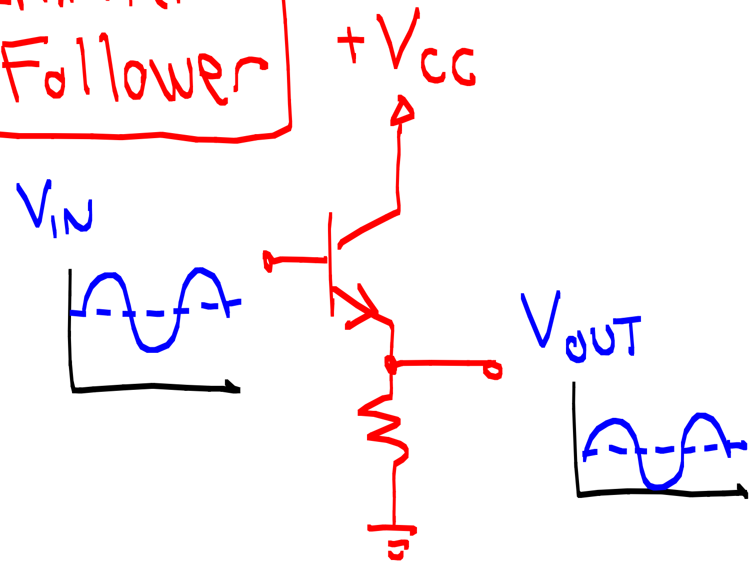
R_6	R_5
100K	125K
110K	137.5K
$\boxed{120K}$	$\boxed{150K}$

☺

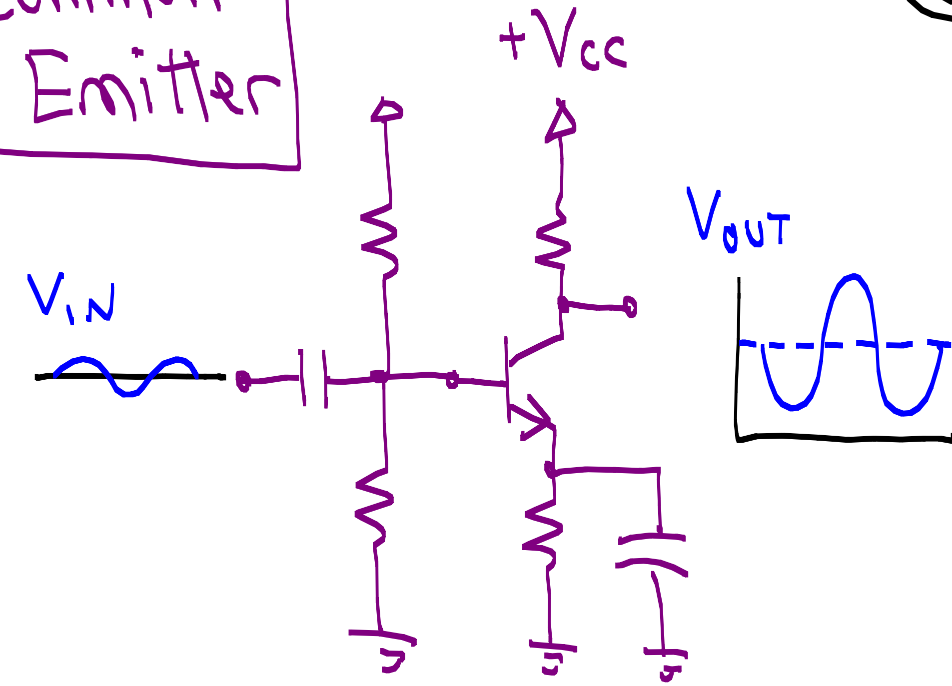
1. Intro

- Many amplifiers have a single input V_{IN}

Emitter Follower

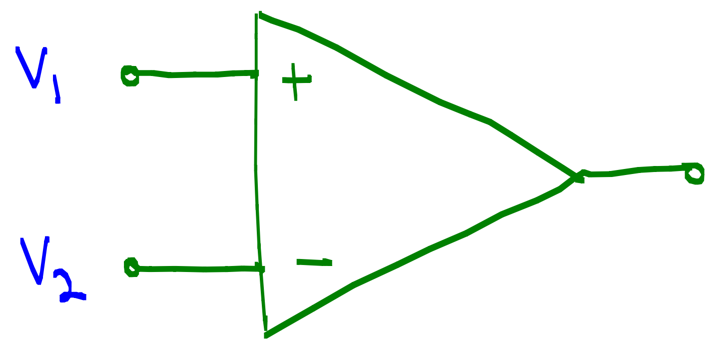


Common Emitter



- A differential amplifier has 2 inputs

→ Want to amplify the DIFFERENCE between V_1 and V_2



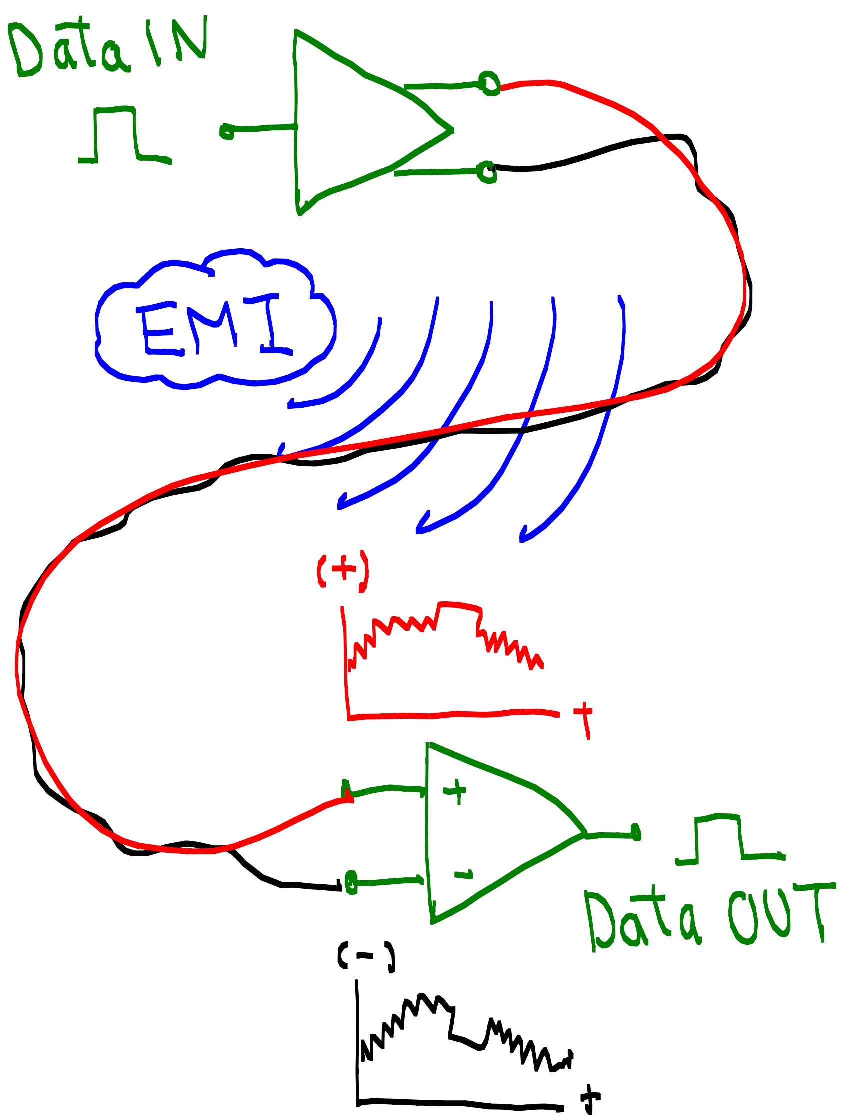
Diff Amplifier

$$V_{OUT} = A_d(V_1 - V_2)$$

A_d = Differential Gain

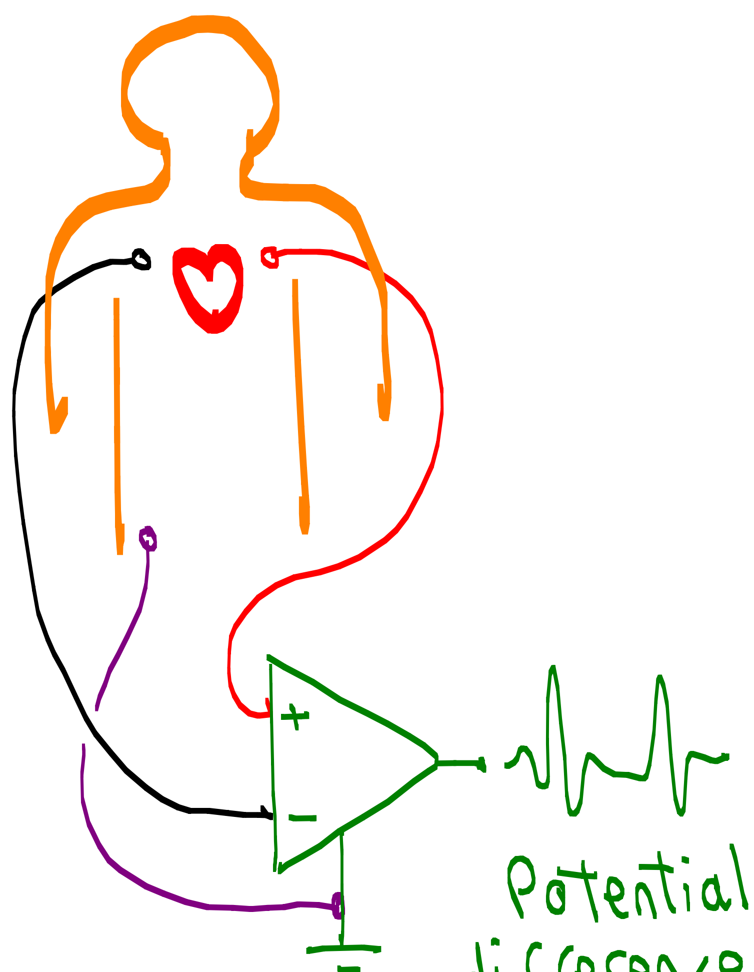
Application #1

* Signal transmission over long cables



Application #2

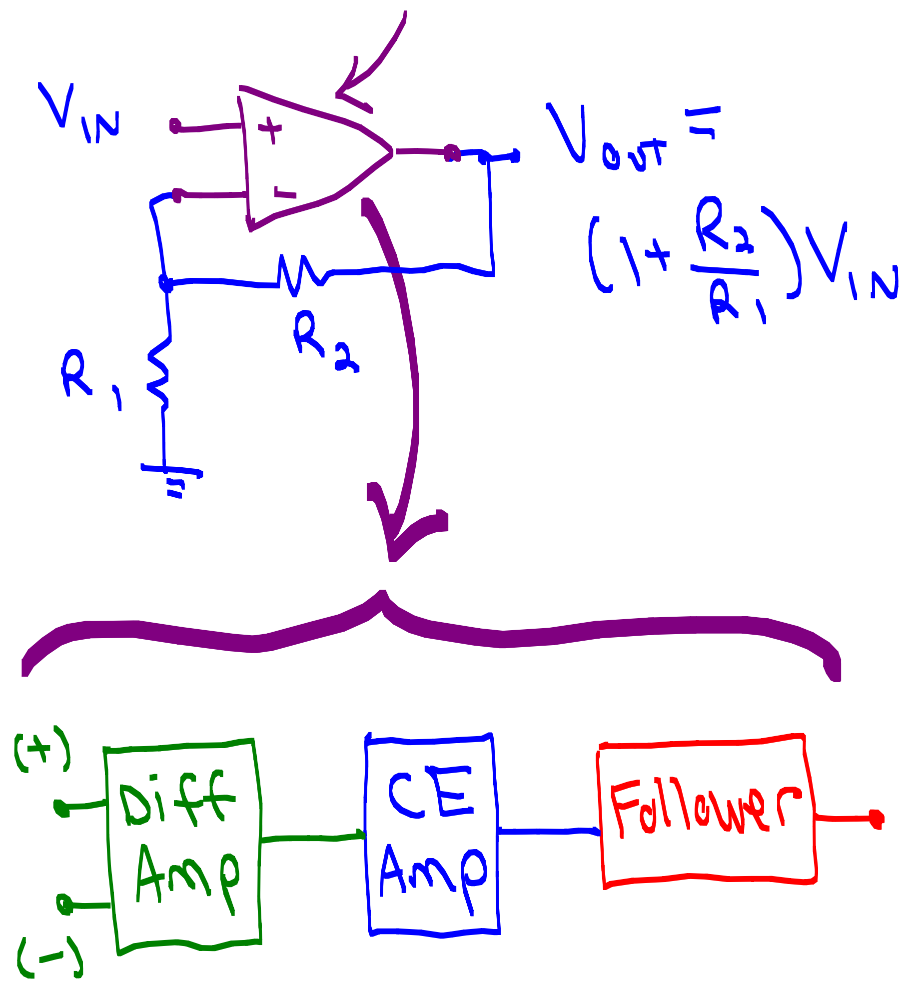
Electrocardiogram (ECG) signals



Potential difference between two points on body

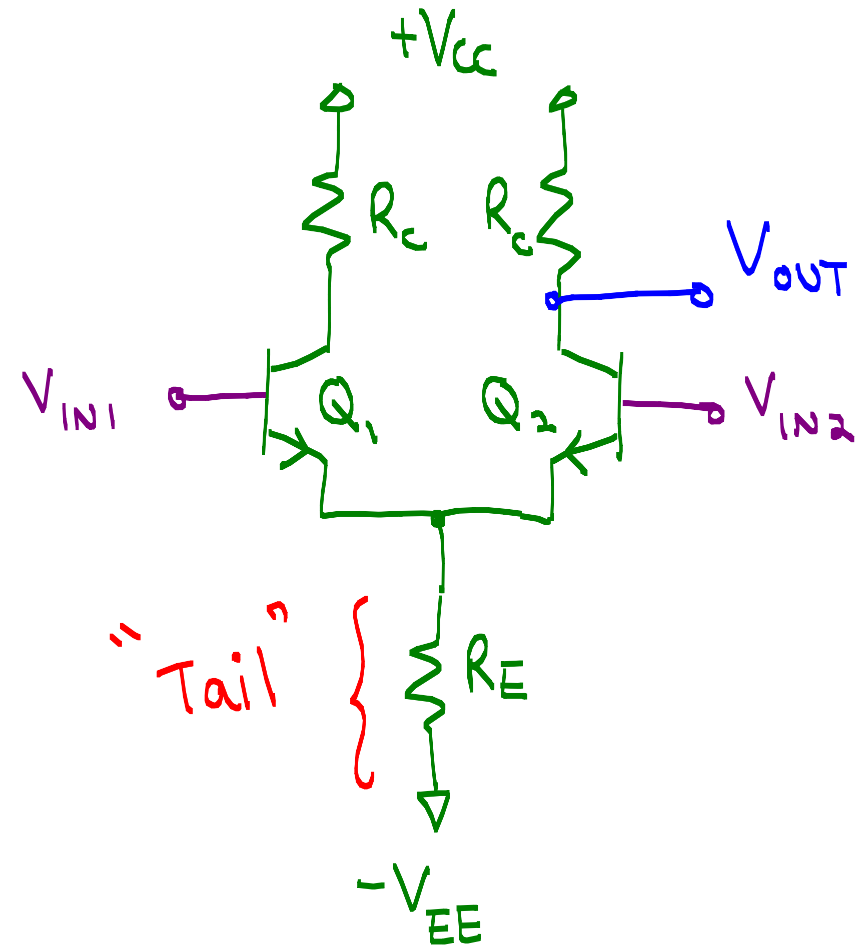
Application #3

Input stage for an <u>op amp</u>



2. BJT Long Tail Pair

- Classic configuration for differential amplifier



① Two inputs V_{IN1} and V_{IN2}

② Note the symmetry

→ Ideally, Q_1 and Q_2 are identical ← "matched pair"

③ Output is single-ended ← most common

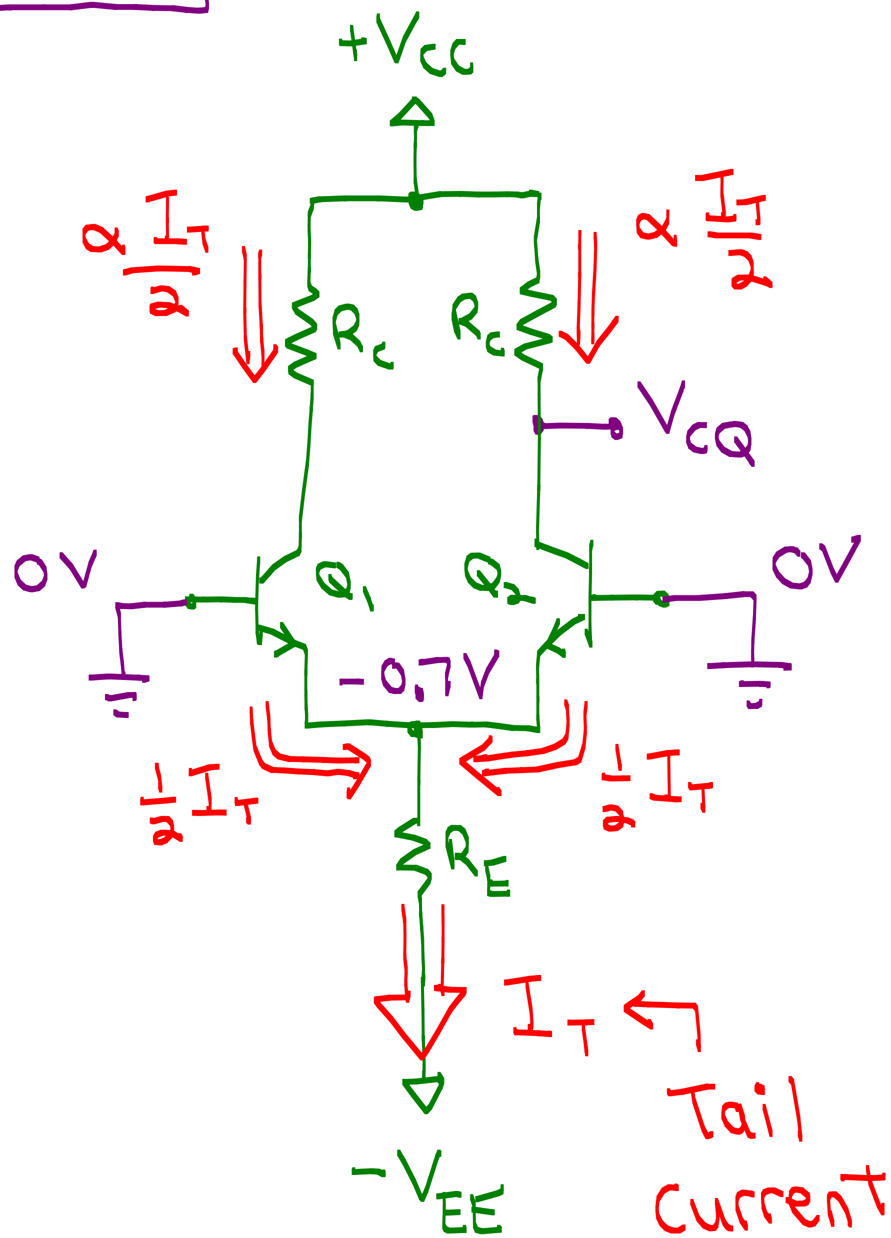
differential output is possible

④ R_E is the "tail"

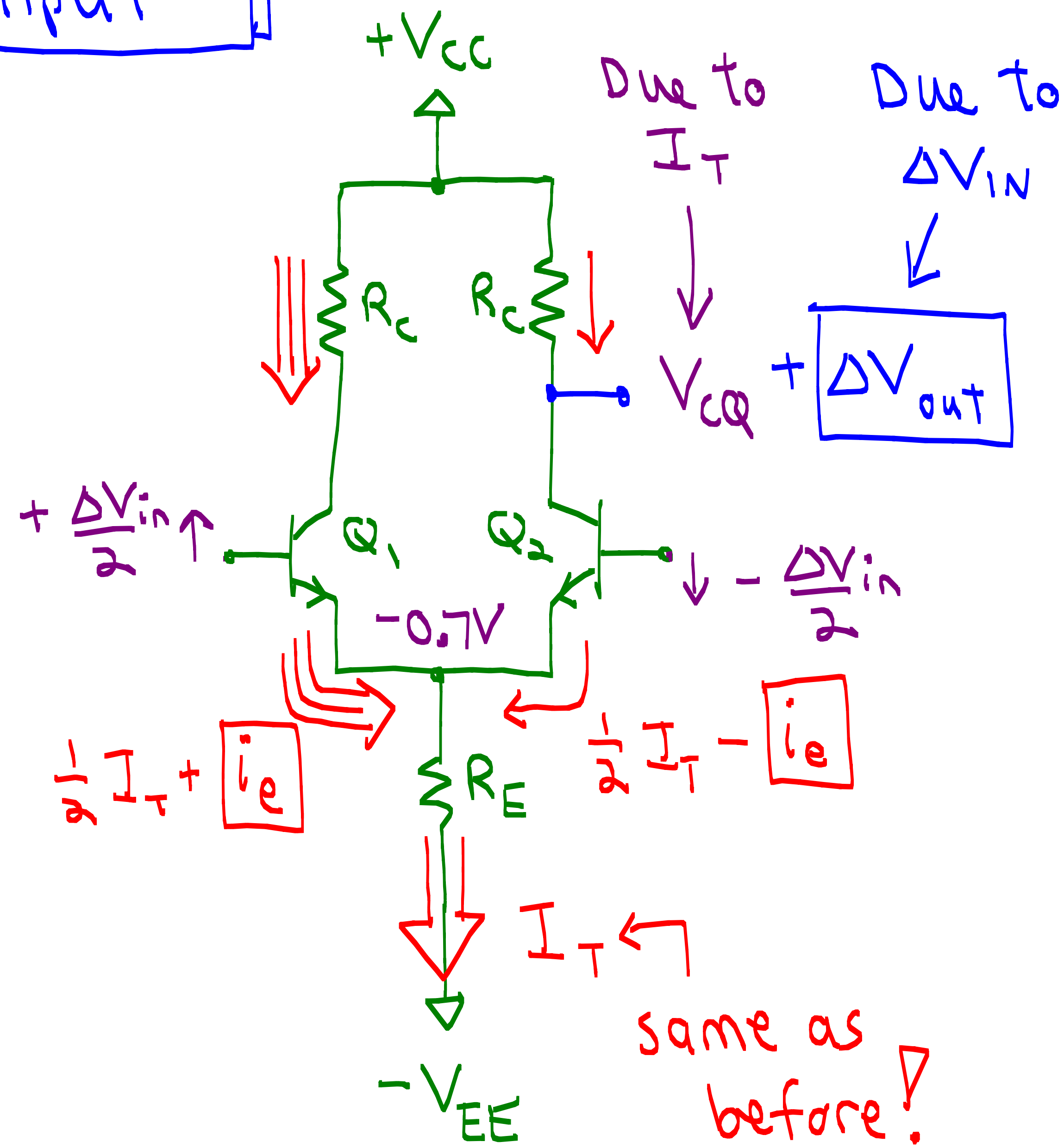
usually a pretty large resistor

- How does this work?

Quiescent



Differential Input



$\boxed{\text{Quiescent}}$ $(V_{IN1} = V_{IN2} = 0)$

- Assuming $Q_1 + Q_2$ are ON $\quad V_E = V_B - 0.7 = 0 - 0.7 = -0.7V$
- Tail current is therefore $\quad I_T = \dfrac{(-0.7) - (-V_{EE})}{R_E} = \boxed{\dfrac{V_{EE} - 0.7}{R_E}}$
- Due to symmetry, both Q_1 and Q_2 have

$$I_{EQ} = \frac{1}{2} I_T$$

$$I_{CQ} = \alpha I_{EQ} = \alpha \frac{1}{2} I_T$$

- Quiescent output voltage is $V_{OUT} = V_{CQ} = V_{CC} - I_{CQ} R_C$

DC value without ΔV_{IN}! $\rightarrow$ $\boxed{V_{CQ} = V_{CC} - \alpha \frac{1}{2} I_T R_C}$

$\boxed{\text{Differential Input}}$ $\left(V_{IN1} = +\frac{\Delta V_{in}}{2},\ V_{IN2} = -\Delta V_{in}/2\right)$

- Q_1 and Q_2 act like a "see saw"

$$V_{BE1} = \overbrace{0.7}^{\text{Quiescent}} + \overbrace{\boxed{\frac{\Delta V_{in}}{2}}}^{\text{small signal}} \qquad V_{BE2} = 0.7 \overbrace{\boxed{-\frac{\Delta V_{in}}{2}}}^{\text{small signal}}$$

$$i_{E1} = \tfrac{1}{2} I_T + \boxed{i_e} \qquad i_{E2} = \tfrac{1}{2} I_T \boxed{-i_e}$$

$$i_{C1} = \alpha \tfrac{1}{2} I_T + \boxed{\alpha i_e} \qquad i_{C2} = \alpha \tfrac{1}{2} I_T \boxed{-\alpha i_e}$$

- Output voltage is therefore:

$$V_{OUT} = V_{CC} - i_{C2} R_C = \underbrace{V_{CC} - \alpha \tfrac{1}{2} I_T R_C}_{\text{Quiescent}} + \underbrace{\boxed{\alpha i_e R_C}}_{\Delta V_{OUT}}$$

- What is i_e?

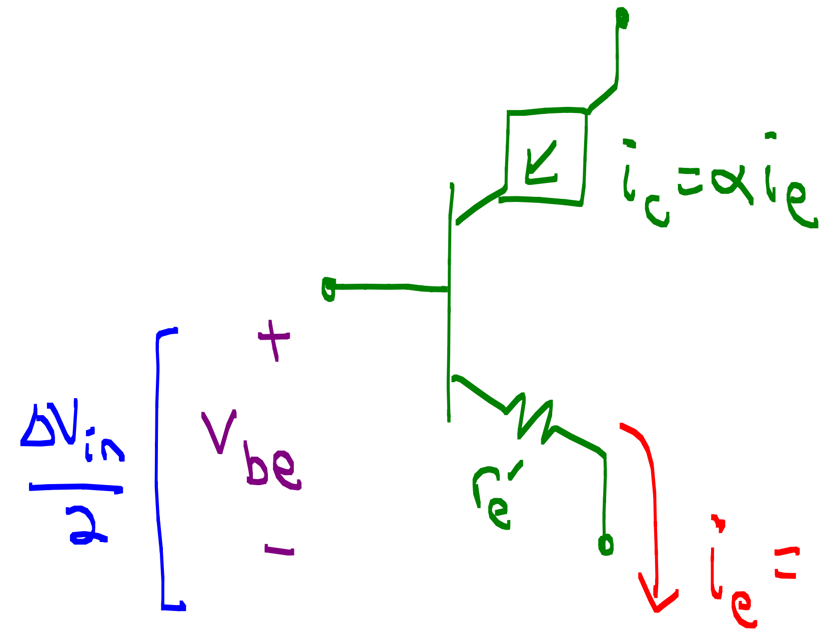
Recall the T-Model for BJT small signal behavior

v_{be} = small change in V_{BE}

i_e = small change in i_E

r_e' = AC emitter resistance

$$= \frac{.026V}{I_{EQ}}$$



$$i_e = \frac{v_{be}}{r_e'}$$

So, $i_e = \dfrac{\Delta V_{in}/2}{r_e'}$

- Therefore, $A_d = \dfrac{\Delta V_{out}}{\Delta V_{in}} = \dfrac{\alpha \, i_e R_c}{\Delta V_{in}} = \dfrac{\alpha \dfrac{\Delta V_{in}/2}{r_e'} R_c}{\Delta V_{in}} = \boxed{\dfrac{\alpha R_c}{2 r_e'}}$

Differential Gain!

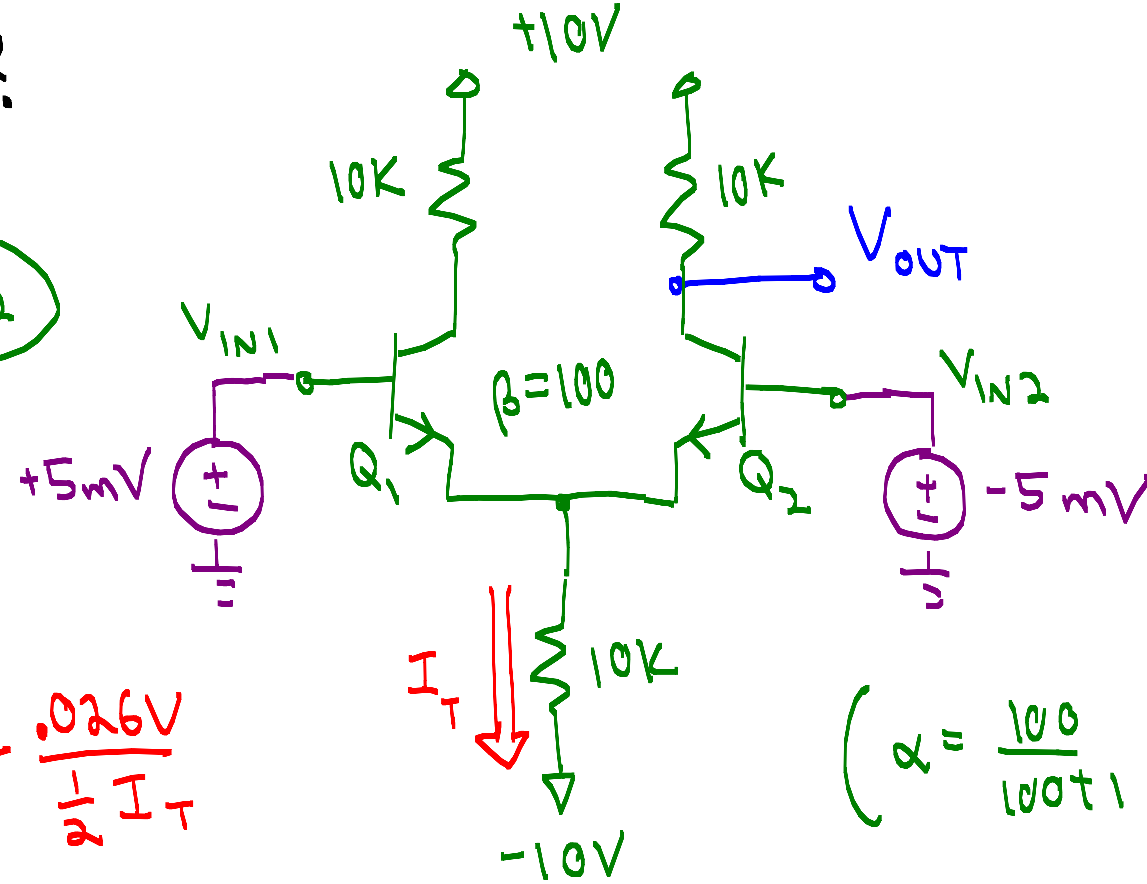
D. Analysis Example: What is V_{OUT}?

$$V_{OUT} = \boxed{V_{CQ}} + \boxed{\Delta V_{out}}$$

$V_{CQ} = V_{CC} - \alpha \frac{1}{2} I_T R_C$

$\Delta V_{out} = \boxed{A_d} \Delta V_{in}$, with $\Delta V_{in} = V_{IN1} - V_{IN2}$

$A_d = \dfrac{\alpha R_C}{2 \boxed{r_e'}}$, with $r_e' = \dfrac{.026V}{\frac{1}{2} I_T}$



$$\left(\alpha = \frac{100}{100+1} = 0.99\right)$$

✡ We need I_T!

$$I_T = \frac{(-0.7) - (-10V)}{10K} = \frac{9.3V}{10K} = 0.93mA$$

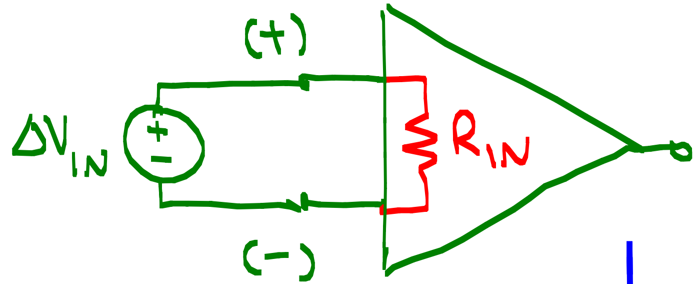
$\Rightarrow$ $V_{CQ} = 10 - 0.99 \frac{1}{2} (.93mA)(10K) = \underline{\underline{5.4V}}$ ← Quiescent

$r_e' = \dfrac{.026}{0.5 \times .93mA} = \underline{\underline{.056K}}$, $A_d = 88.4$

$A_d \Delta V_{IN} = 88.4 \times 10mV = \underline{\underline{884mV}}$ ⇒ $\boxed{V_{OUT} = 6.284V}$

$10mV = V_{IN1} - V_{IN2}$

3. Input Impedance

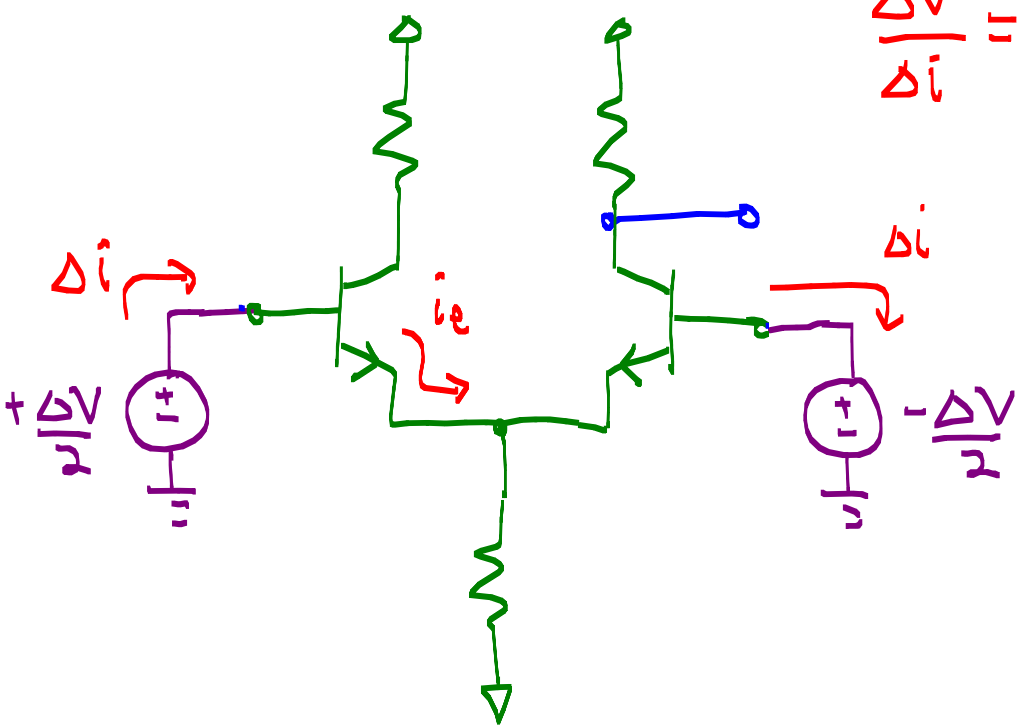


Derivation 1

$$R_{in} = \frac{\Delta V_{IN}}{\Delta I_{IN}} = \frac{+\frac{\Delta V}{2} - \left(-\frac{\Delta V}{2}\right)}{\Delta i} = \frac{\Delta V}{\Delta i}$$

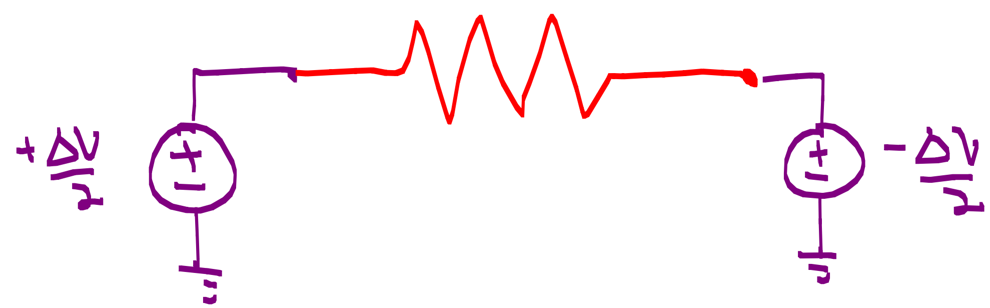
Since Δi is a <u>base</u> current, $\Delta i = \dfrac{i_e}{\beta+1} = \dfrac{\Delta V / 2r_e'}{\beta+1}$

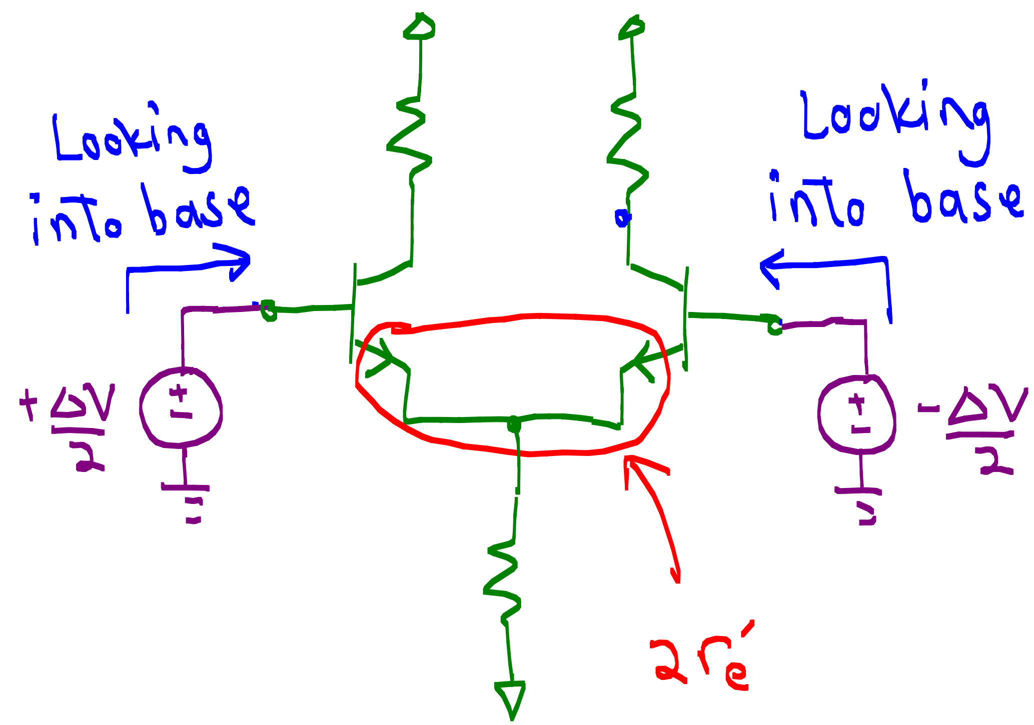
$$\frac{\Delta V}{\Delta i} = \boxed{R_{in} = 2(\beta+1) r_e'}$$



Derivation 2

$$\overbrace{(\beta+1)\,2r_e'}^{R_{IN}}$$





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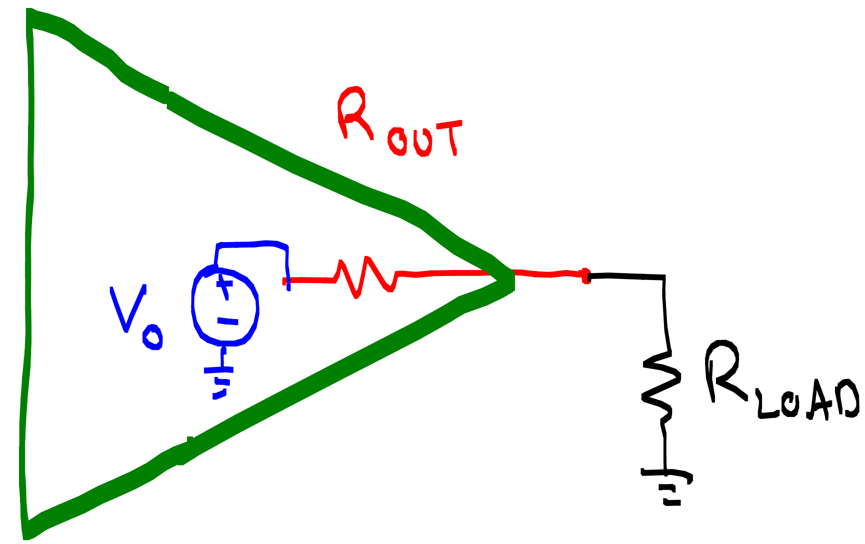
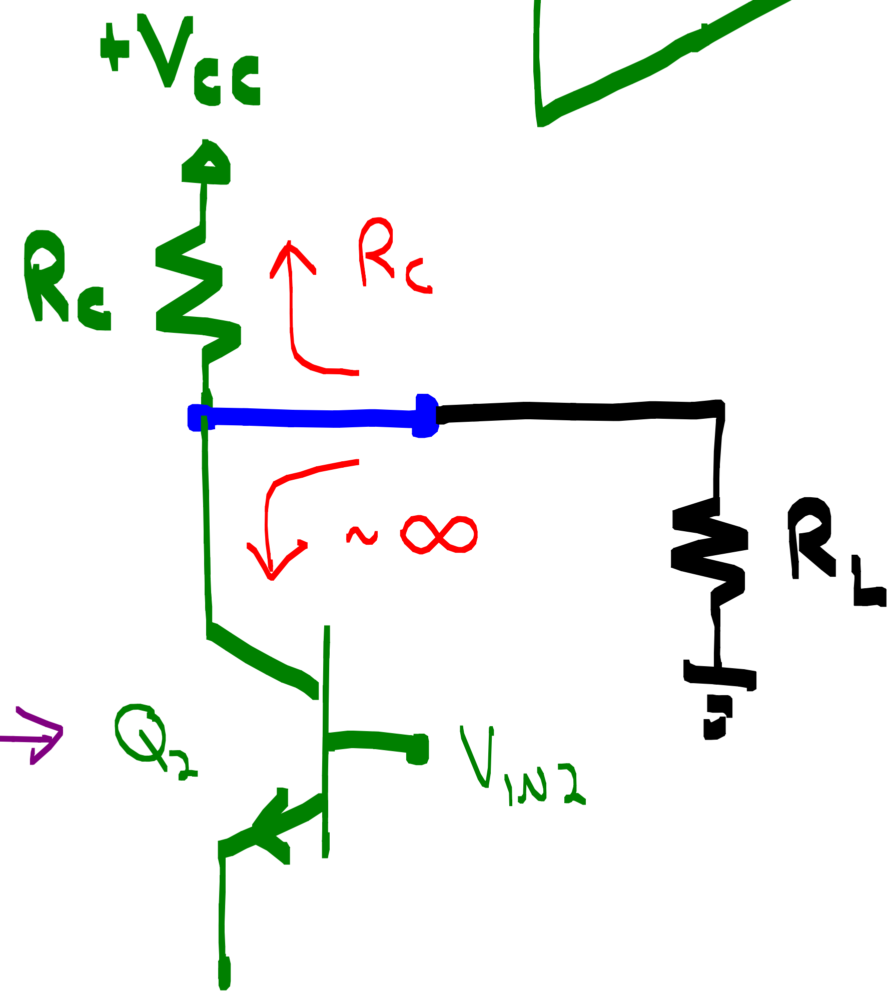
4. Output Impedance

- A voltage amplifier ideally has $R_{OUT} \approx 0\Omega$





BJT collector looks like a current source

~ Infinite source impedance

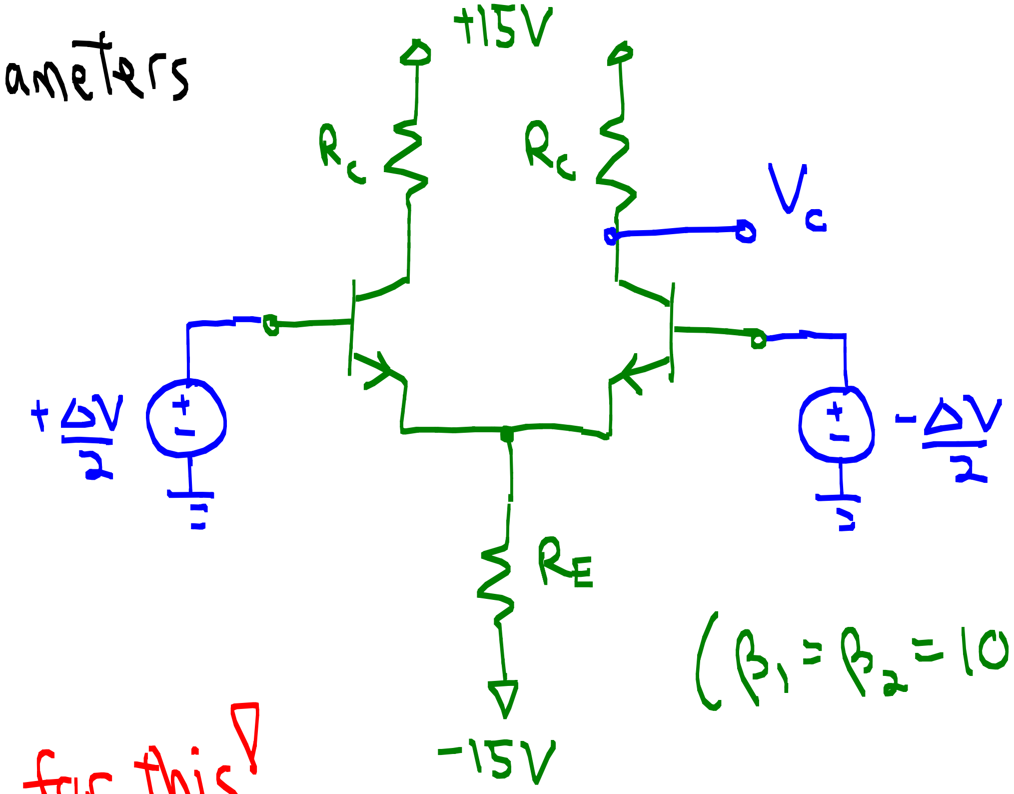
$$\boxed{R_{OUT} = R_C}$$

Typically > 10k

Not good :(

3. Design Example ← use "default" parameters

Gain ≥ 50, $R_{IN} \geq 10\,k\Omega$, $V_{CC} = V_{EE} = 15\,V$



$(\beta_1 = \beta_2 = 100)$

where to start?

Formulas: $A_d = \dfrac{\alpha R_C}{2 r_e'}$, $\boxed{R_{IN}} = 2(\beta+1)\boxed{r_e'}$

can solve for this!

① $R_{IN} = 2(100+1) r_e' \geq 10000\,\Omega$

$r_e' \geq 49.5\,\Omega$

Since $r_e' = \dfrac{.026V}{I_E} \geq 49.5\,\Omega \Rightarrow \underline{\underline{I_E \leq .53\,mA}}$ ⇨ $I_T \leq \underline{\underline{1.06\,mA}}$

② using $I_T = \frac{15-0.7}{R_E} < 1.06\,mA$

$R_E > 13.49\,K$

Choose $\boxed{R_E = 15\,K}$

③ Actual $I_T = \frac{15-0.7}{15K} = 0.95\,mA \Rightarrow r_e' = \frac{.026V}{\frac{1}{2}(0.95mA)} = \underline{\underline{.0547K}}$

$R_{IN} = 2(100+1)(.0547K) = \boxed{11.05\,K}$ ✓

④ $A_d = \frac{\alpha R_c}{2r_e'} = \frac{0.99\,R_c}{2 \times .0547K} \geq 50$

$R_c \geq 5.53K \Rightarrow \boxed{\text{Choose } 5.6\,K} \Rightarrow$ Actual gain is $A_d = \frac{0.99 \times 5.6K}{2 \times .0547K} = \boxed{50.7}$ ✓