Lecture 13: Differential Amplifiers (Part 2)

(Quiz)

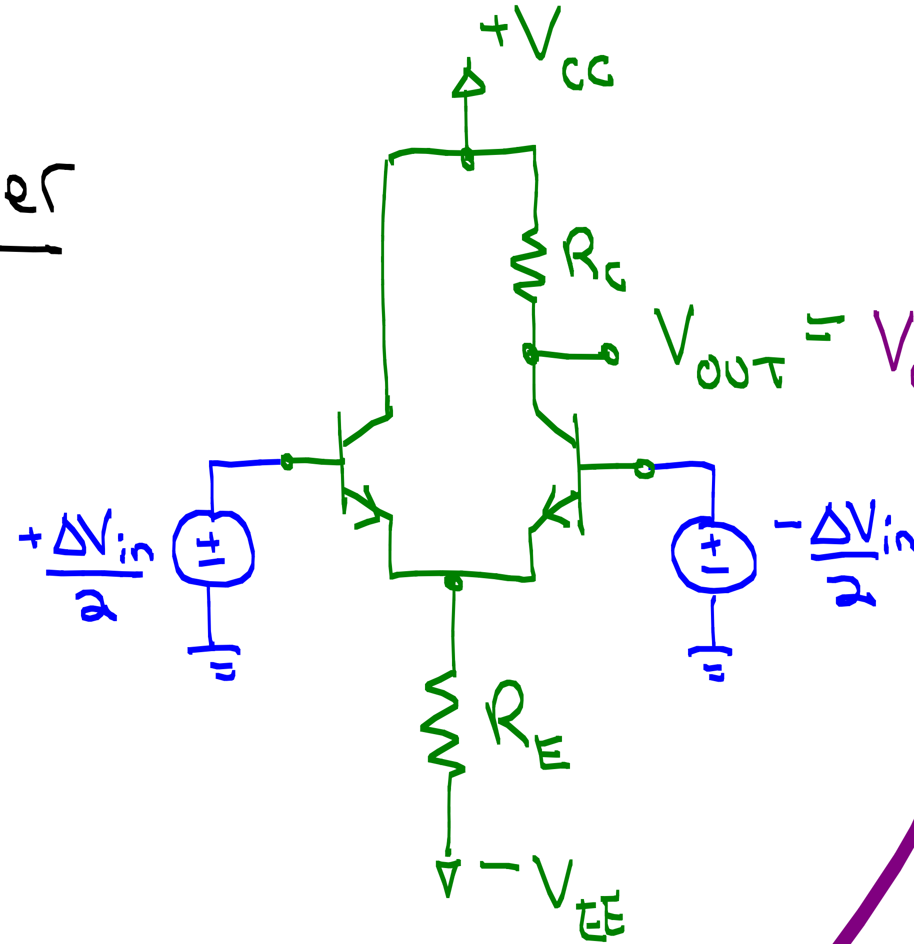
0. Review
1. Common mode rejection ratio
2. Current Source Biasing
3. BJT current source

Textbook reading: 15-5 Common Mode Gain

- PreLab 5b (Multisim) due at lab session
- HW 6 due this Fri (Nov 08)
- Exam #2 next Tue (Nov 12)
  - → HW4, 5, 6
  - → Q24, 5, 6 ← sample on website
  - → Sample exam on course website

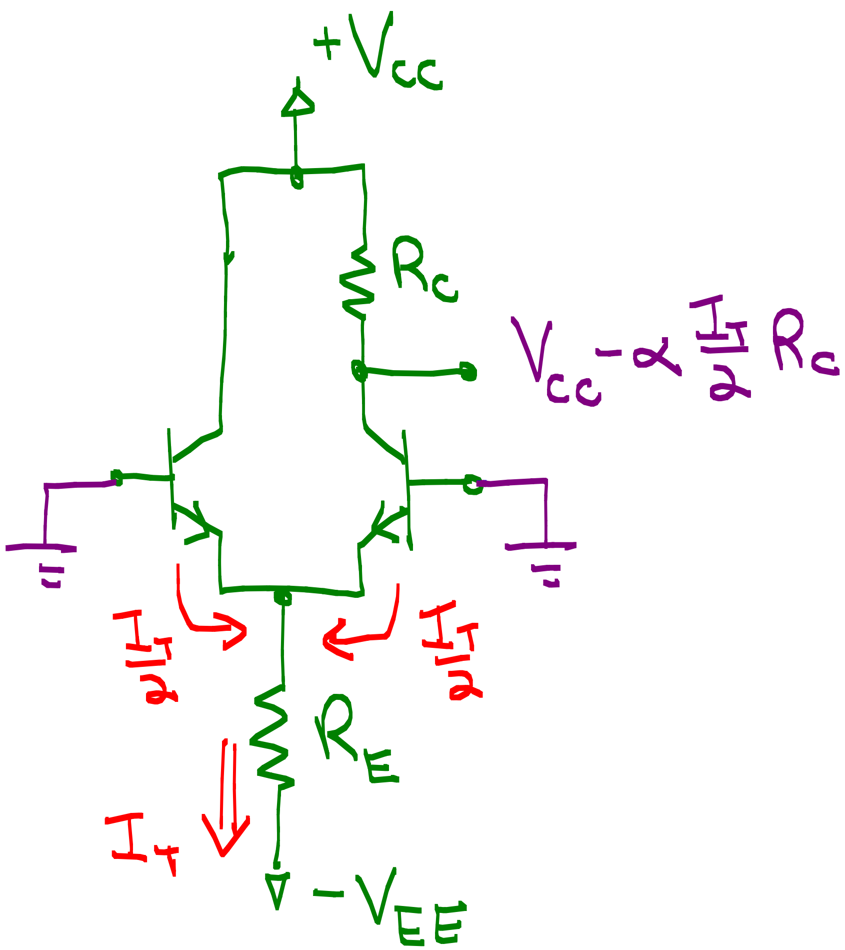
0. Review

• Differential Amplifier

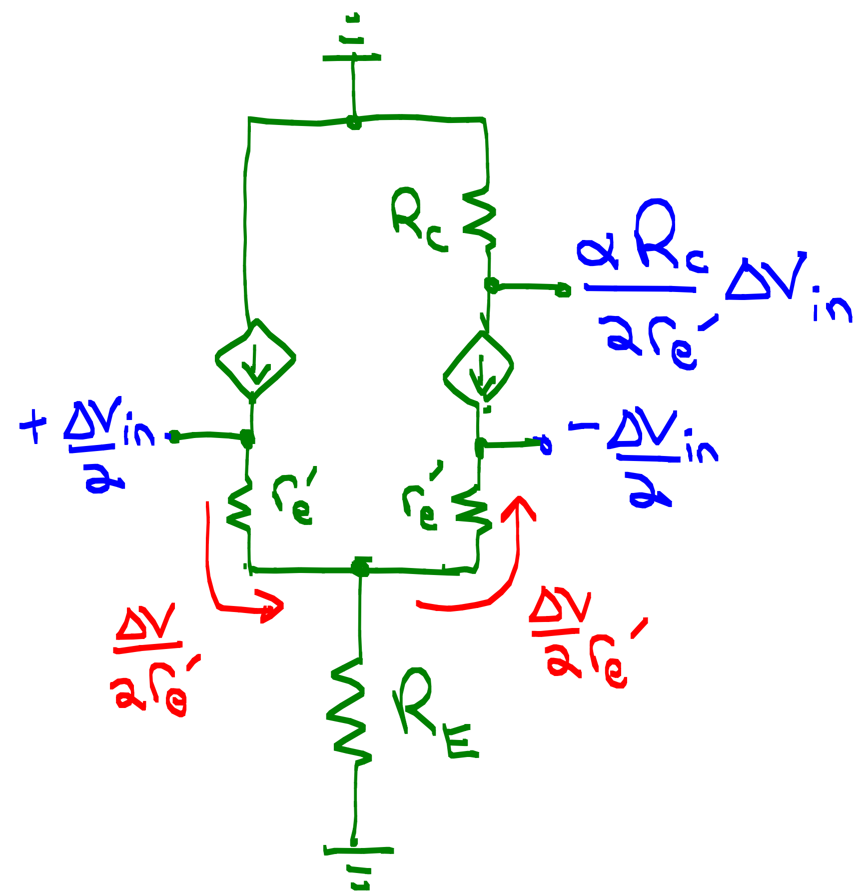


$V_{OUT} = V_{CQ} + \overbrace{\Delta V_{OUT}}^{A_d \Delta V_{IN}}$

Quiescent



Signal



Also...

$Z_{IN} = 2(\beta+1) r_e'$

$Z_{OUT} = R_c$

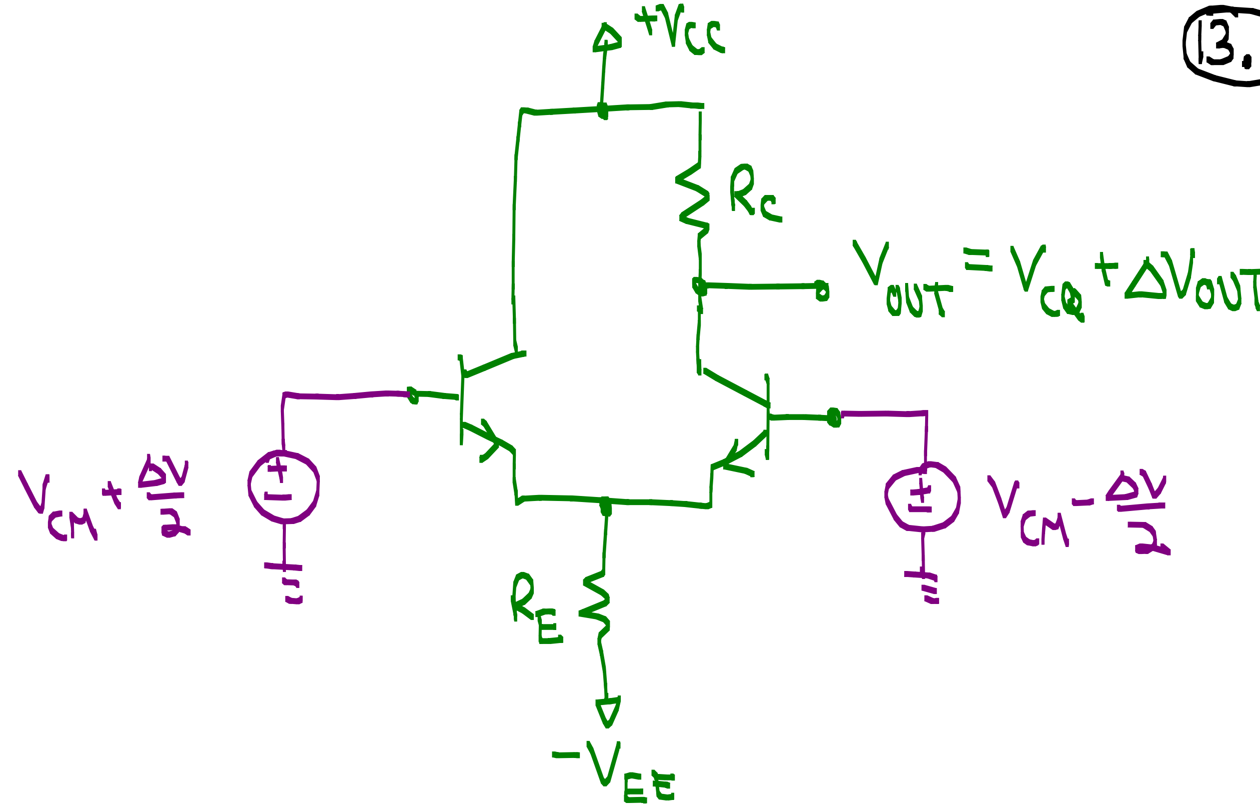
1. Common Mode Rejection Ratio (CMRR)

- In many situations, V_{IN1} and V_{IN2} have a large offset V_{CM}.

$$V_{IN1} = \boxed{V_{CM}} + \frac{\Delta V}{2}$$

$$V_{IN2} = \boxed{V_{CM}} - \frac{\Delta V}{2}$$

Common Mode Input



- Ideally, ΔV_{OUT} only depends on $\Delta V_{IN} = V_{IN1} - V_{IN2} = \Delta V$

- In reality, ΔV_{OUT} is affected by V_{CM}. $\Rightarrow$

$$\Delta V_{OUT} = \boxed{A_d} \Delta V_{IN} + \boxed{A_{CM}} V_{CM}$$

A_d: Differential Gain

A_{CM}: Common Mode Gain (Ideally zero)

$\Delta V_{IN} = V_{IN1} - V_{IN2}$

$V_{CM} = \frac{V_{IN1} + V_{IN2}}{2}$

Example

$V_{IN1} = 3.01V$, $V_{IN2} = 2.99V$

$A_d = 100$, $A_{CM} = .01$

What is ΔV_{OUT}?

$$\Rightarrow \Delta V_{OUT} = A_d \Delta V_{IN} + A_{CM} V_{CM}$$

$$V_{CM} = \frac{V_{IN1} + V_{IN2}}{2} = 3V$$

$$\Rightarrow \Delta V_{OUT} = \underbrace{100 \times .02}_{\text{Ideal result} = 2V} + \underbrace{.01 \times 3}_{\text{Error term}} = \boxed{2.03V}$$

- A good differential amplifier has very small A_{cM} (e.g. 0.001)

→ Called common mode rejection

- Extremely important metric of a differential amplifier is the common mode rejection ratio (CMRR)

$$\Rightarrow \boxed{CMRR = 20\log_{10}\left|\frac{A_d}{A_{cM}}\right|}$$

Example $A_d = 120$, $A_{cM} = .005$

$$CMRR = 20\log_{10}\left(\frac{120}{.005}\right)$$

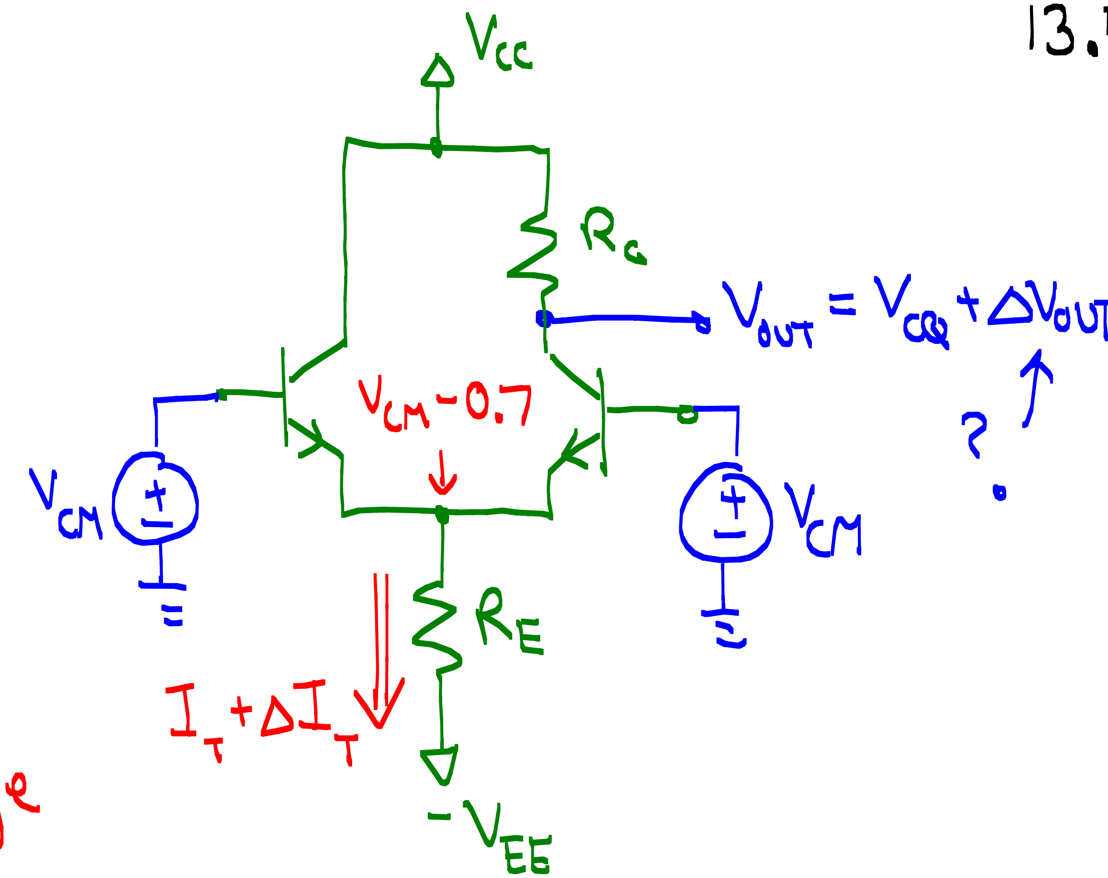
$$= 20\log_{10}(24000)$$

$$= \boxed{87.6\text{ dB}}$$

★ Good differential amplifier has CMRR > 80 dB

80 dB = $20\log_{10}(10{,}000)$

- Common mode gain for long-tail pair



★ V_{CM} causes a change in tail current! :(

$$I_T' = \frac{(V_{CM} - 0.7) - (-V_{EE})}{R_E}$$

$$= \frac{V_{EE} - 0.7}{R_E} + \boxed{\frac{V_{CM}}{R_E}} = I_T + \boxed{\Delta I_T}$$

causes change in V_{OUT}! :(

$$\text{So, } I_{EQ}' = \frac{I_T}{2} + \boxed{\frac{\Delta I_T}{2}}$$

$$\rightarrow V_o' = \underbrace{V_{CC} - \alpha \frac{I_T}{2} R_C}_{\text{Quiescent}} + \boxed{-\alpha \frac{\Delta I_T}{2} R_C}$$

$$\hookrightarrow \Delta V_{OUT} = -\frac{\alpha V_{CM}}{2 R_E} R_C \Rightarrow \frac{\Delta V_{OUT}}{V_{CM}} = \boxed{A_{CM} = -\frac{\alpha R_C}{2 R_E}}$$

(-) sign!

- **CMRR for long-tail pair**

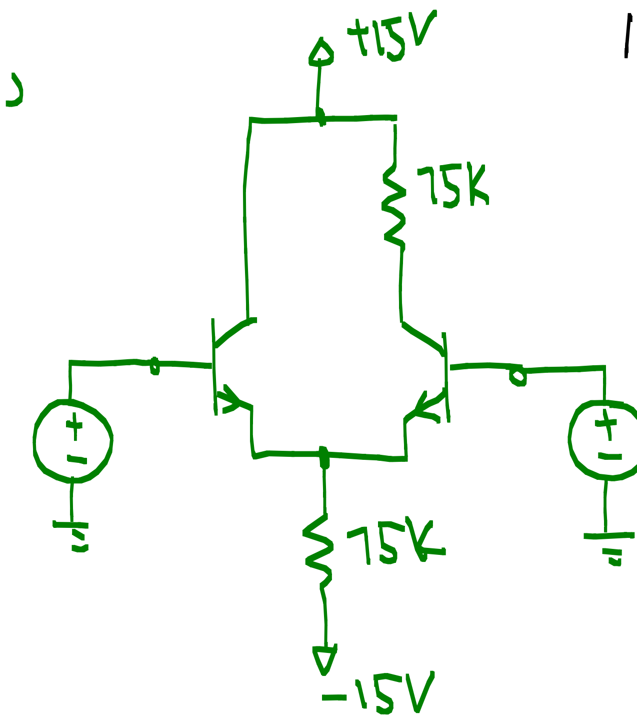
$$CMRR = 20\log_{10}\left|\frac{A_d}{A_{CM}}\right|$$

$$= 20\log_{10}\left|\frac{\alpha R_C/2r_e'}{-\alpha R_C/2R_E}\right|$$

$$= \boxed{20\log_{10}\left(R_E/r_e'\right)}$$

↑ Only depends on R_E and r_e'

Example Compute A_d, A_{CM}, and CMRR



- $I_T = \frac{15-0.7}{75K} = .19\,mA$

$r_e' = \frac{.026V}{.095mA} = .274K$

- $A_d = \frac{75k}{2 \times .274k} = 137$, $A_{CM} = \frac{-75k}{2(75k)} = \underline{\underline{-0.5}}$!

$\Rightarrow CMRR = 20\log_{10}\left(\frac{75k}{.274k}\right)$ or $20\log_{10}\left|\frac{137}{-0.5}\right|$

$= 20\log_{10}(174) = \boxed{48.8\ dB}$

(not very good)

:(

2. Current Source Biasing

Q: How to improve CMRR? ← $20\log_{10}(R_E/r_e')$

Increase R_E? Tiny I_T → Q_1, Q_2 OFF ☹

☆ Recall that:

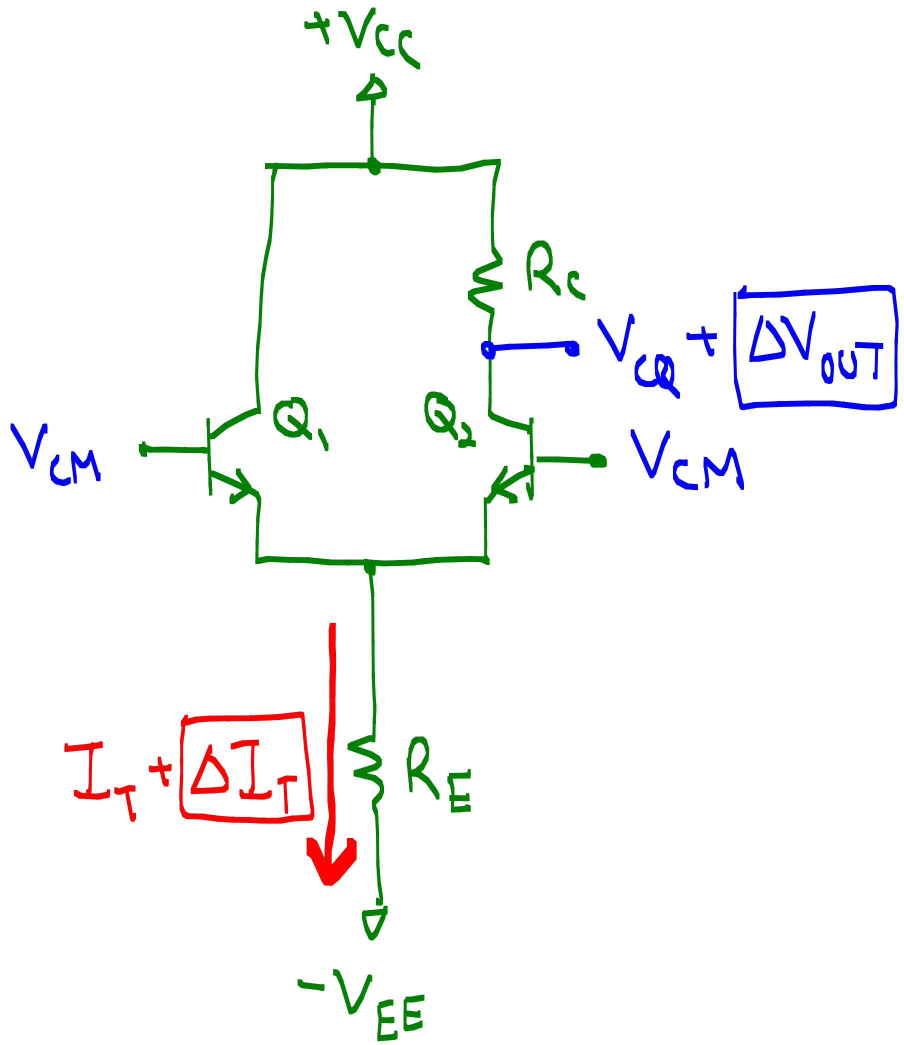
V_{CM}

↓

$I_T + \boxed{\Delta I_T}$

↓

$\boxed{\Delta V_{OUT}}$!



☆ Key is to make I_T insensitive to V_{CM}.

⇒ use constant current source for I_T!

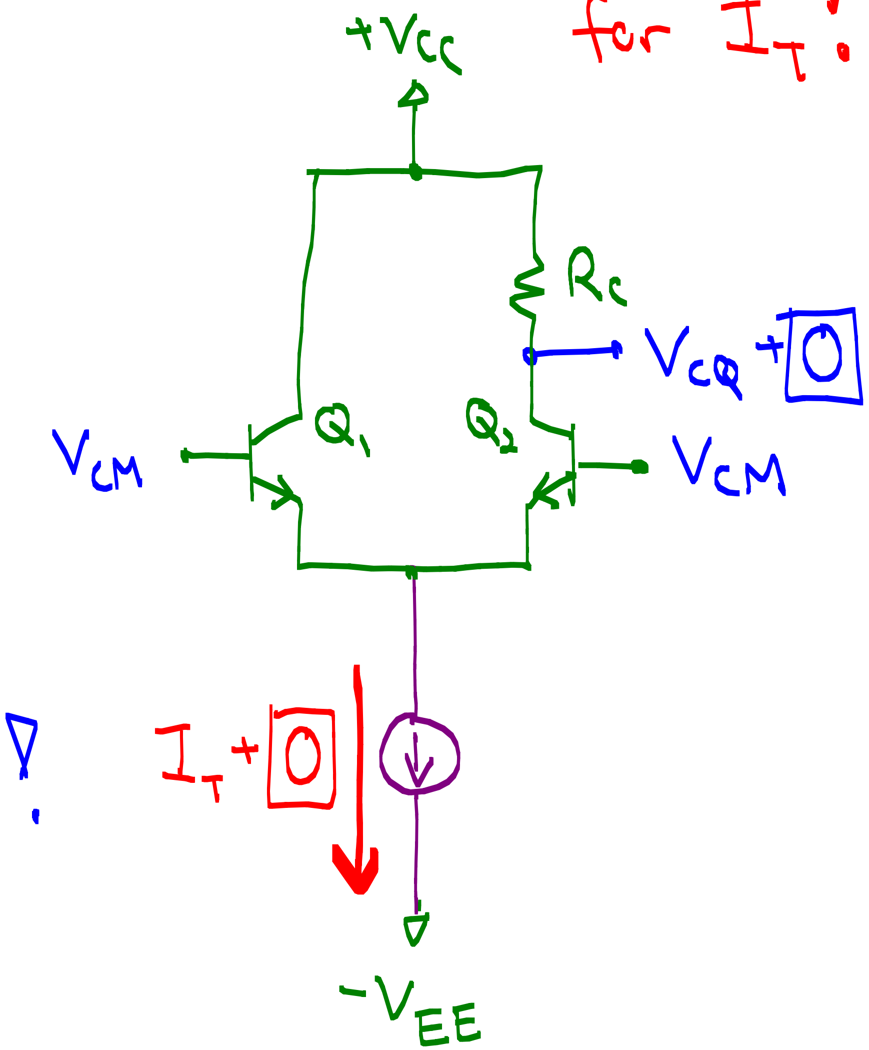
V_{CM}

↓

$I_T + \boxed{0}$

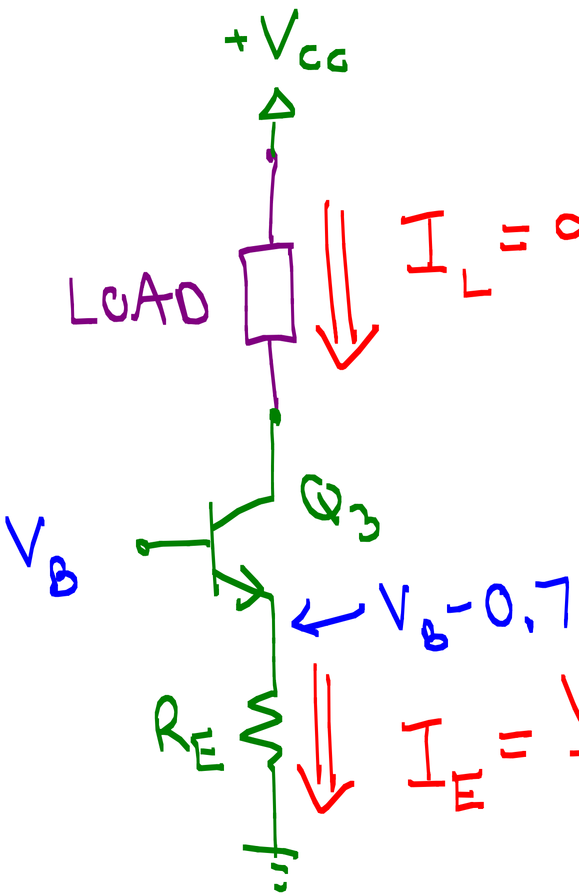
↓

Zero ΔV_{OUT}!



3. BJT Current Source

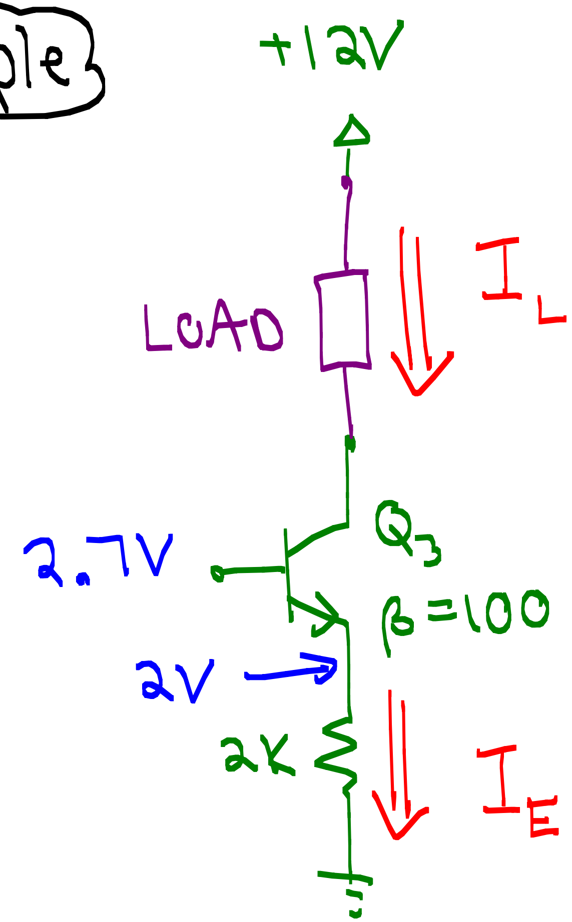
Fortunately, a good current source can be made from a BJT



$$I_L = \alpha \frac{V_B - 0.7}{R_E}$$

$$I_E = \frac{V_B - 0.7}{R_E}$$

Example



$I_L = 0.99\,mA$ ← **Does not depend on V_{cc}!**

$I_E = 1\,mA$

✱ I_L depends on V_B and R_E!

Q: How to set the Q_3 base voltage?

A common method is to use some diodes as a voltage reference.

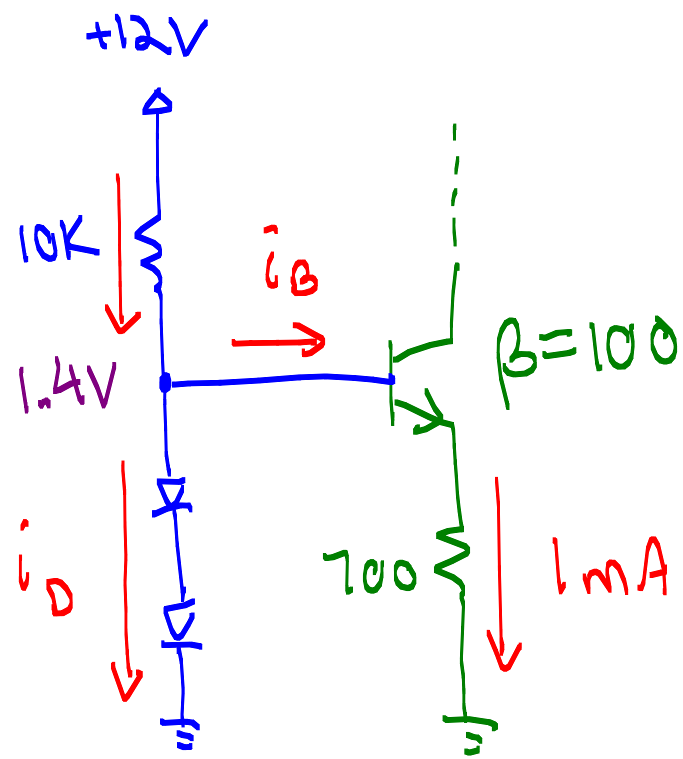
* Q_1 is biased with some diodes

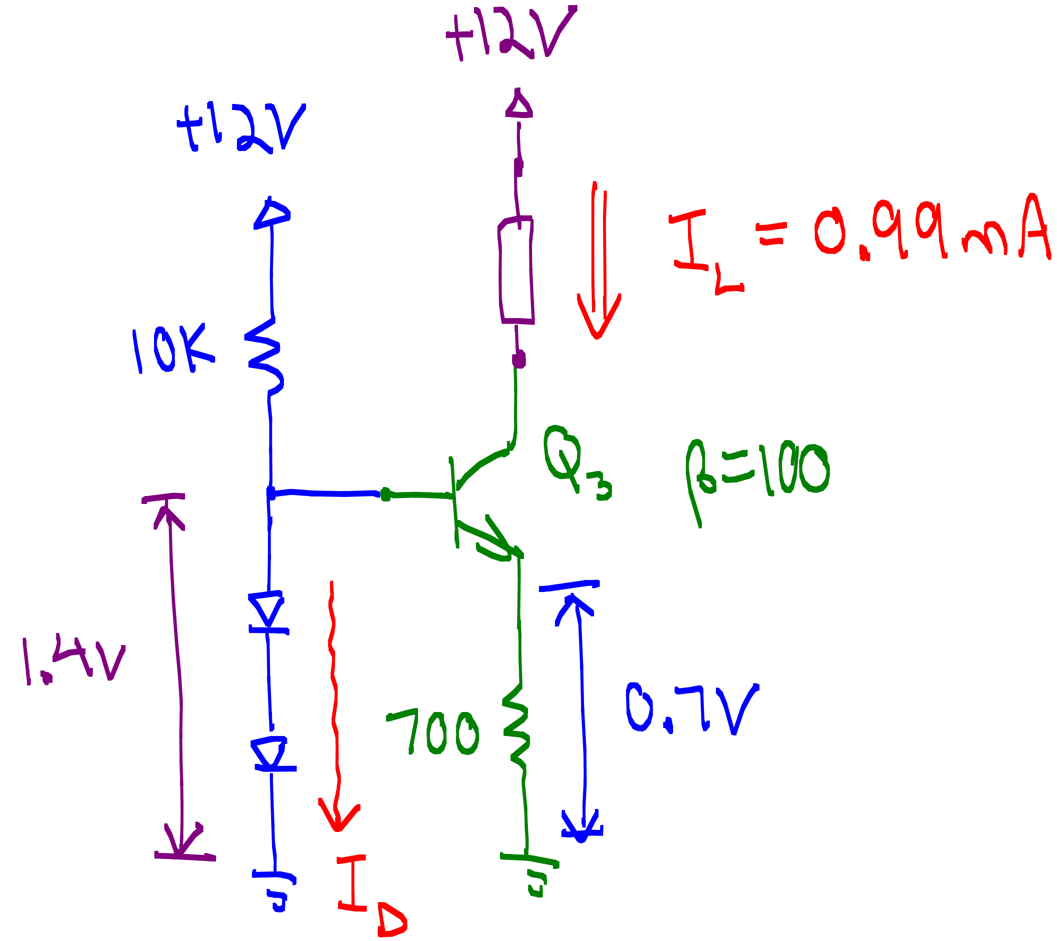
→ Make sure $I_D \sim 1\,mA$!

$$i_D = \frac{12-1.4}{10K} - \frac{1mA}{100+1}$$

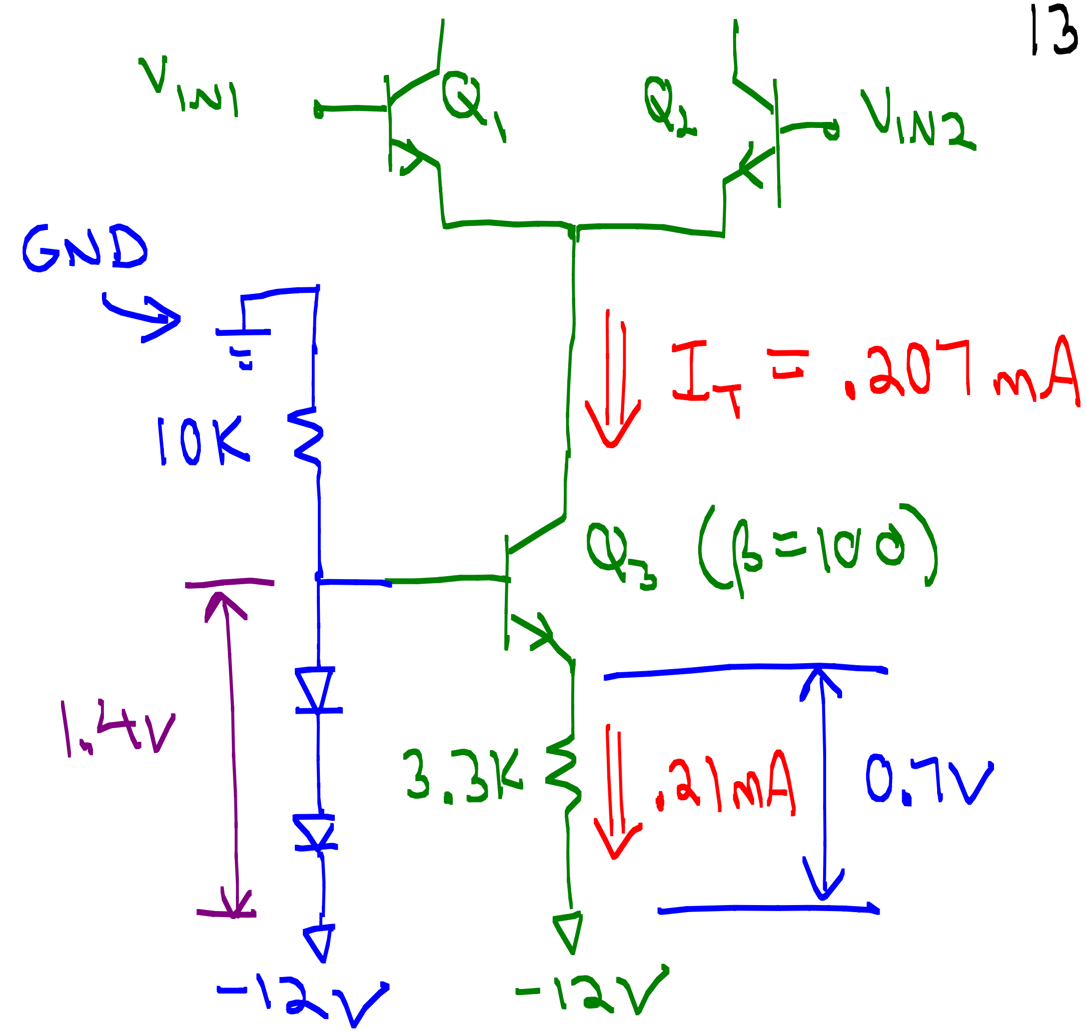
$$= 1.06\,mA - .01\,mA$$

$$= 1.05\,mA \checkmark$$





- For BJT diff pair, the current source often operates from negative voltage.



- Assuming diodes have enough current,

$$I_E = \frac{1.4-0.7}{3.3K} = 0.21\ mA$$

$$I_T = .99 \times .21 = 0.207\ mA$$

Check diodes:

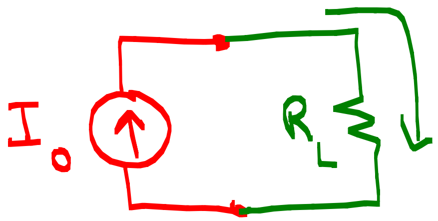
$$i_D = \frac{0-(-12+1.4)}{10K} - \frac{.21\ mA}{100+1}$$

$$= 1.06\ mA - .002\ mA = 1.058\ mA$$

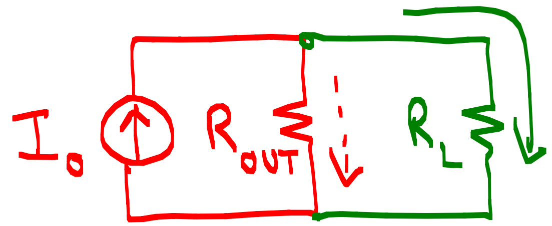
✓

• But what is the CMRR?

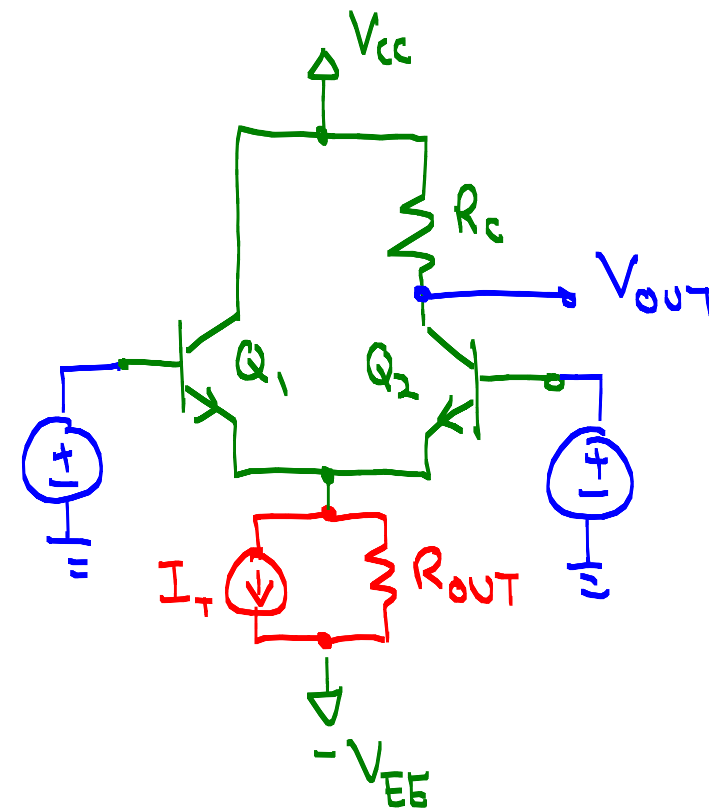
Ideal Current Source	Real Current Source



$I_L = I_o$



$I_L \neq I_o$



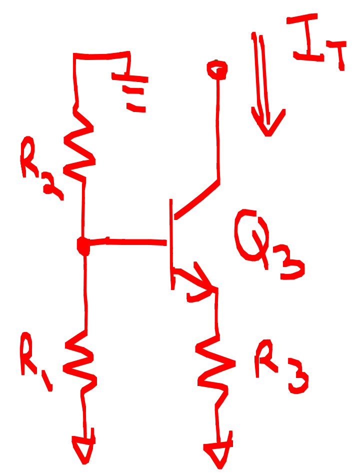
→ Output impedance $R_{OUT} < \infty$

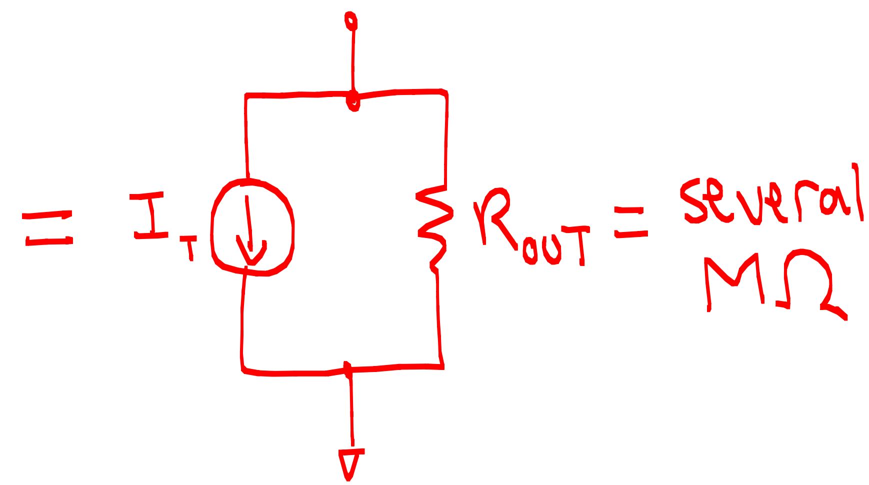
So,

$$CMRR = 20\log_{10}\left(\frac{R_{OUT}}{r_e'}\right)$$

• For our simple current source,

R_{OUT} is typically several MΩ.





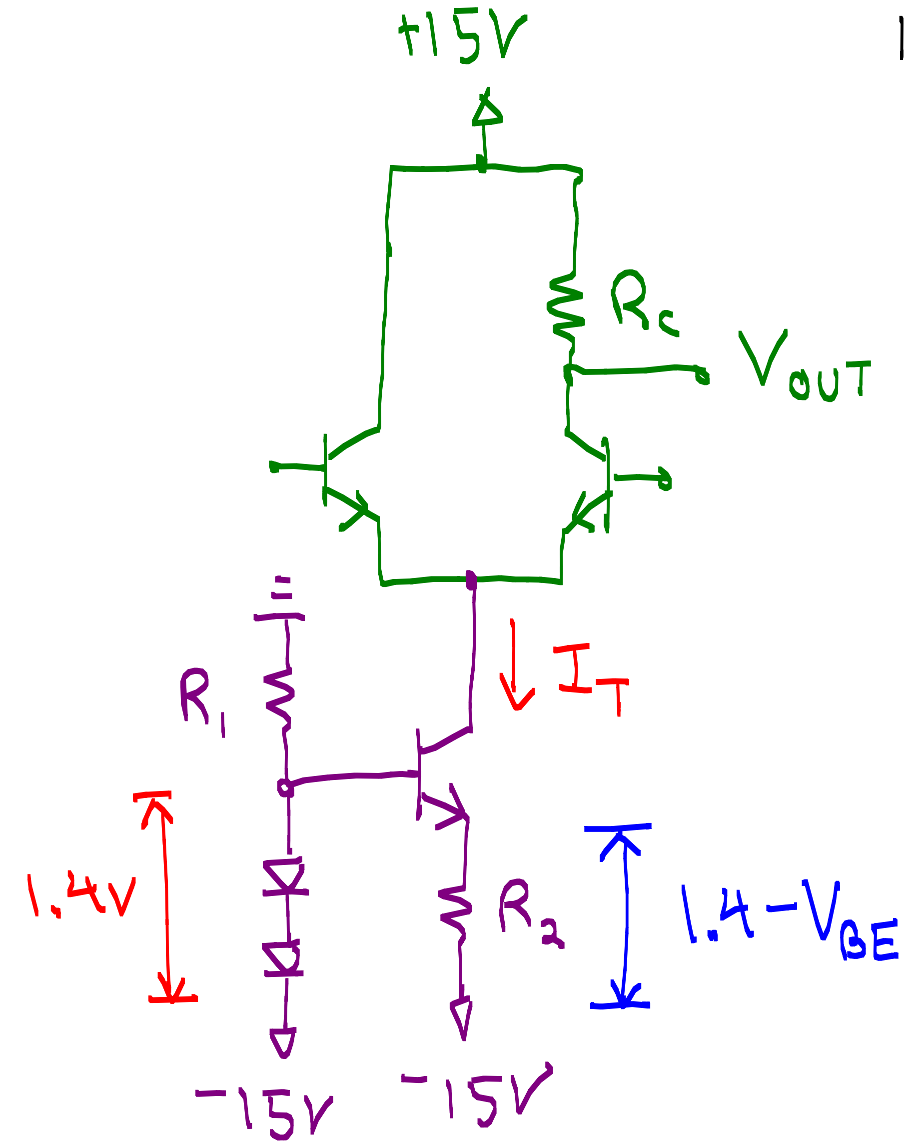
- Design Example ← from Lecture 12

Gain ≥ 50, $R_{IN} \geq 10\,k\Omega$, $V_{CC} = V_{EE} = 15V$

$$R_{IN} = 2(\beta+1) r_e' \geq 10K$$

$$\frac{.026V}{\frac{1}{2} I_T} \Rightarrow r_e' \geq 49.5\Omega$$

★ $I_T \leq 1.06\,mA$



Current Source:

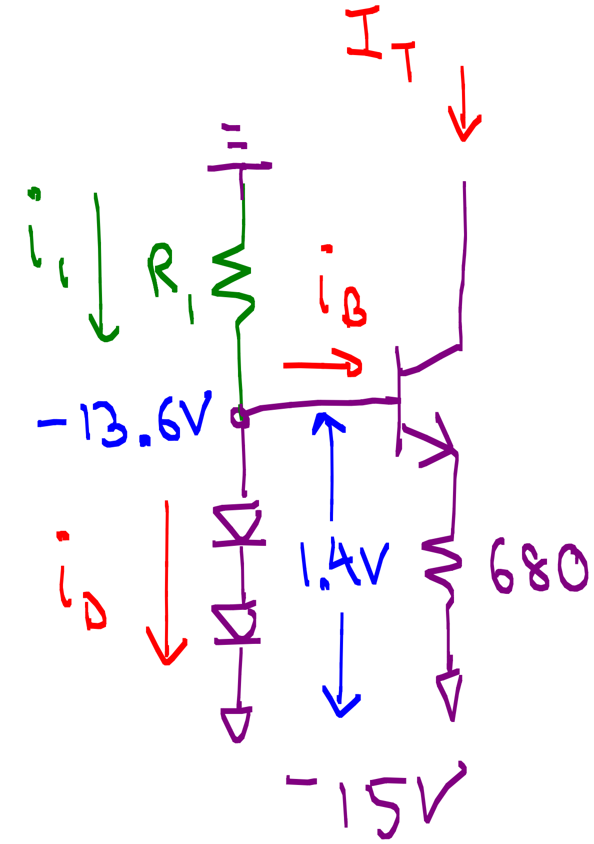
$$I_T = \alpha \frac{1.4 - V_{BE}}{R_2} \leq 1.06\,mA$$

$R_2 \geq 0.65K \Rightarrow$ Choose $\boxed{R_2 = 680\Omega}$

$$\Rightarrow I_T = .99 \frac{0.7V}{680\Omega} = 1.02\,mA \checkmark, \quad r_e' = 51.0\Omega \checkmark$$

• R_1? Want $i_D \approx 1mA$ (not critical)

$$i_D = i_1 - i_B = \frac{0-(-13.6V)}{R_1} - \frac{1}{101}\frac{0.7V}{680\Omega} \approx 1mA$$



$\Rightarrow R_1 = 13.5K$

$\Rightarrow$ Choose $\boxed{R_1 = 13K}$ (15K would also be fine)

• CMRR? Assuming $R_{OUT} \sim 2M\Omega$

$$CMRR = 20\log_{10}\left(\frac{3000K}{.051K}\right) = 91.9\ dB$$

] MUCH higher than using tail resistor (~41 dB)

• R_C? $A_d = \frac{\alpha R_C}{2r_e'} \geq 50 \rightarrow R_C \geq 5.15K$

Choose $\boxed{R_C = 5.6K}$