Lecture 14: Filters

0. Review

1. Passive Filters

2. Active Filters

- PreLab 5b (Multisim) due at lab session
- HW 6 due this Fri (Nov 08)
- Exam #2 next Tue (Nov 12)

→ HW4, 5, 6

→ Q24, 5, 6 ← sample on website

→ Sample exam on course website

Textbook Reading:

19-1 Ideal Responses

19-2 Approximate Responses

19-3 Passive Filters

19-4 First order stages

19-5 VCVS unity-gain 2^{nd} order LPF

19-6 Higher order filters

0. Review

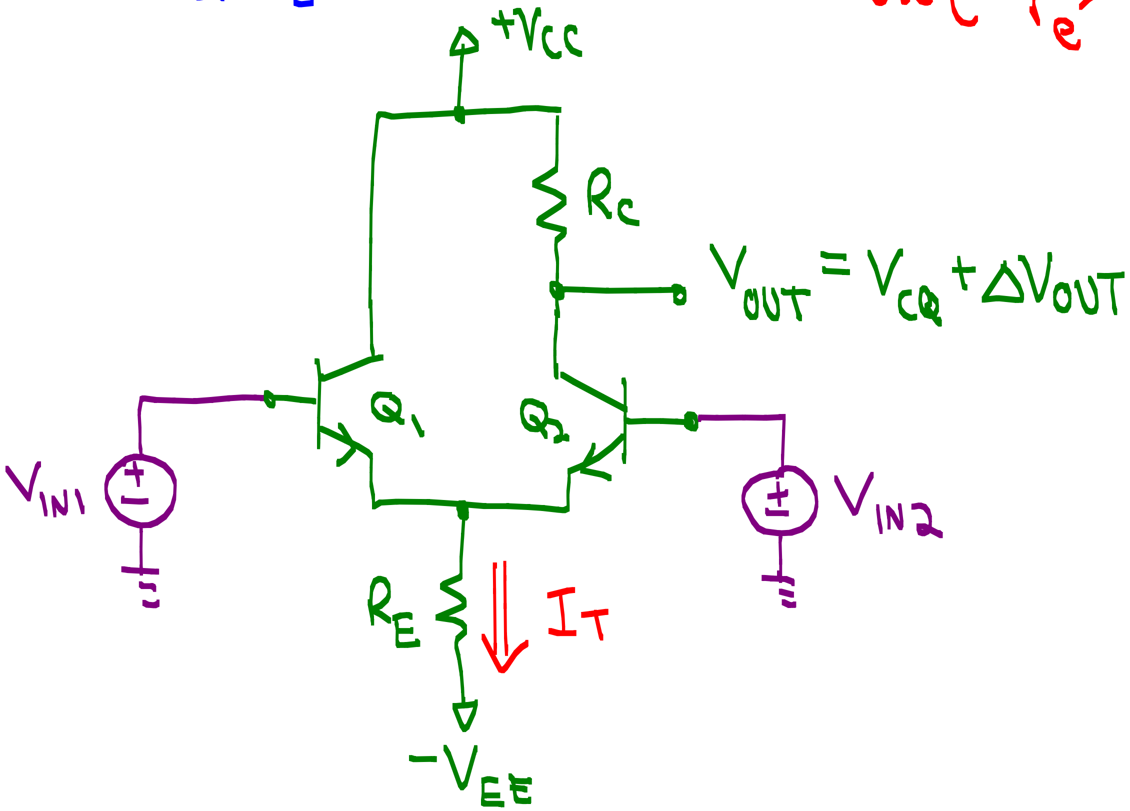
$$V_{OUT} = V_{CQ} + \underbrace{\boxed{A_d}\,\Delta V_{IN} + \boxed{A_{CM}}\,V_{CM}}_{\Delta V_{OUT}}$$

$\Delta V_{IN} = V_{IN1} - V_{IN2}$

$V_{CM} = \frac{1}{2}(V_{IN1} + V_{IN2})$

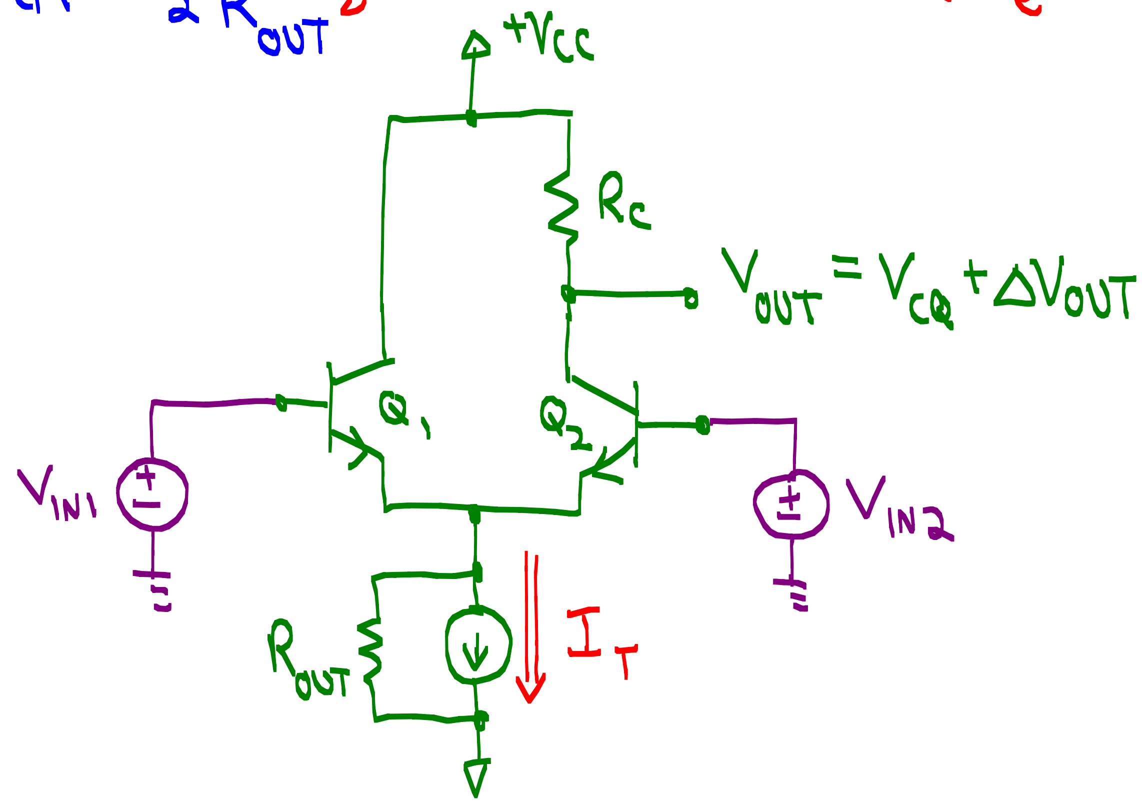
$$A_d = \frac{\alpha R_C}{2r_e'}, \quad A_{CM} = -\frac{\alpha R_C}{2R_E}$$

$$CMRR = 20\log_{10}\left|\frac{A_d}{A_{CM}}\right| = 20\log_{10}\left(\frac{R_C}{r_e'}\right)$$



$$A_d = \frac{\alpha R_C}{2r_e'}, \quad A_{CM} = -\frac{\alpha R_C}{2R_{OUT}}$$

$$CMRR = 20\log_{10}\left|\frac{A_d}{A_{CM}}\right| = 20\log_{10}\left(\frac{R_{OUT}}{r_e'}\right)$$



I. Passive Filters

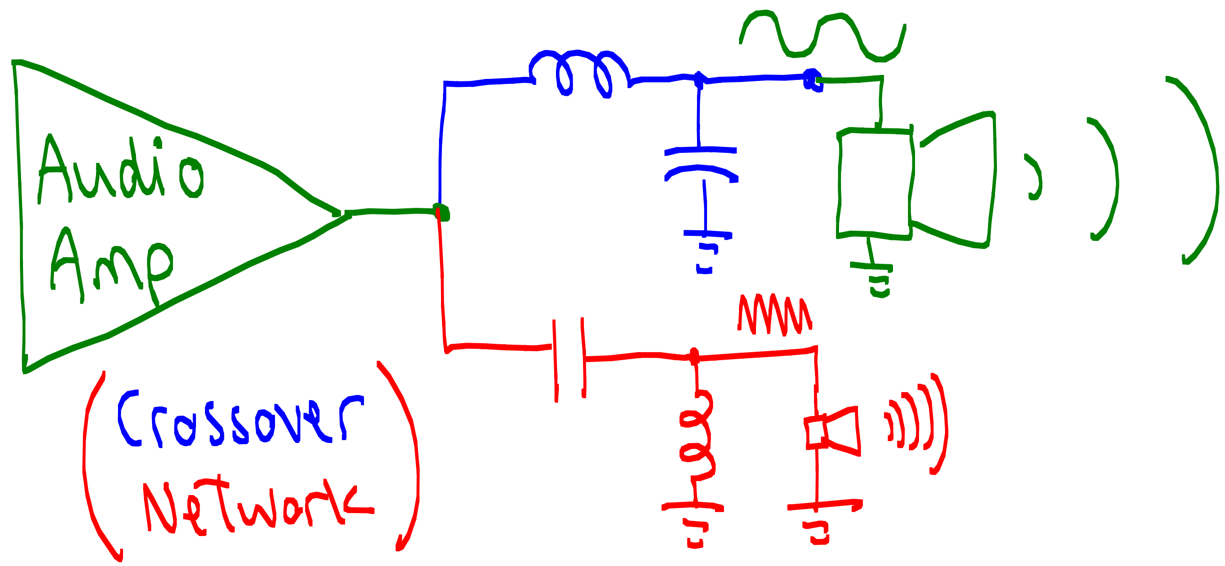
A. Intro

- In many applications, the desired signal competes with <u>noise</u>.

Need a filter to clean up the signal!

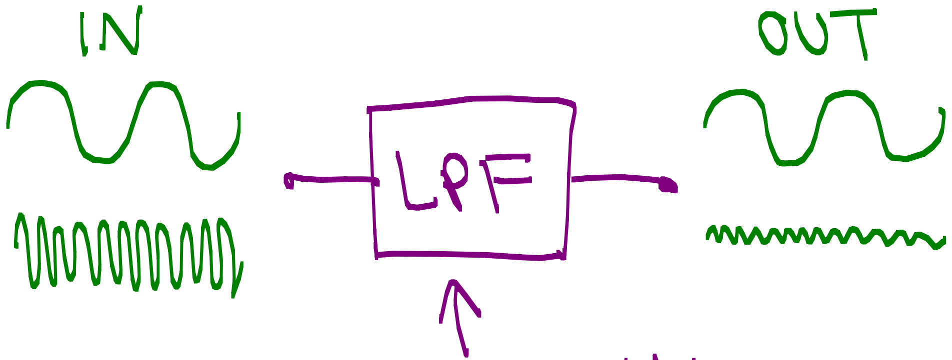
Ex: ECG

- Other applications use filters to separate signals.

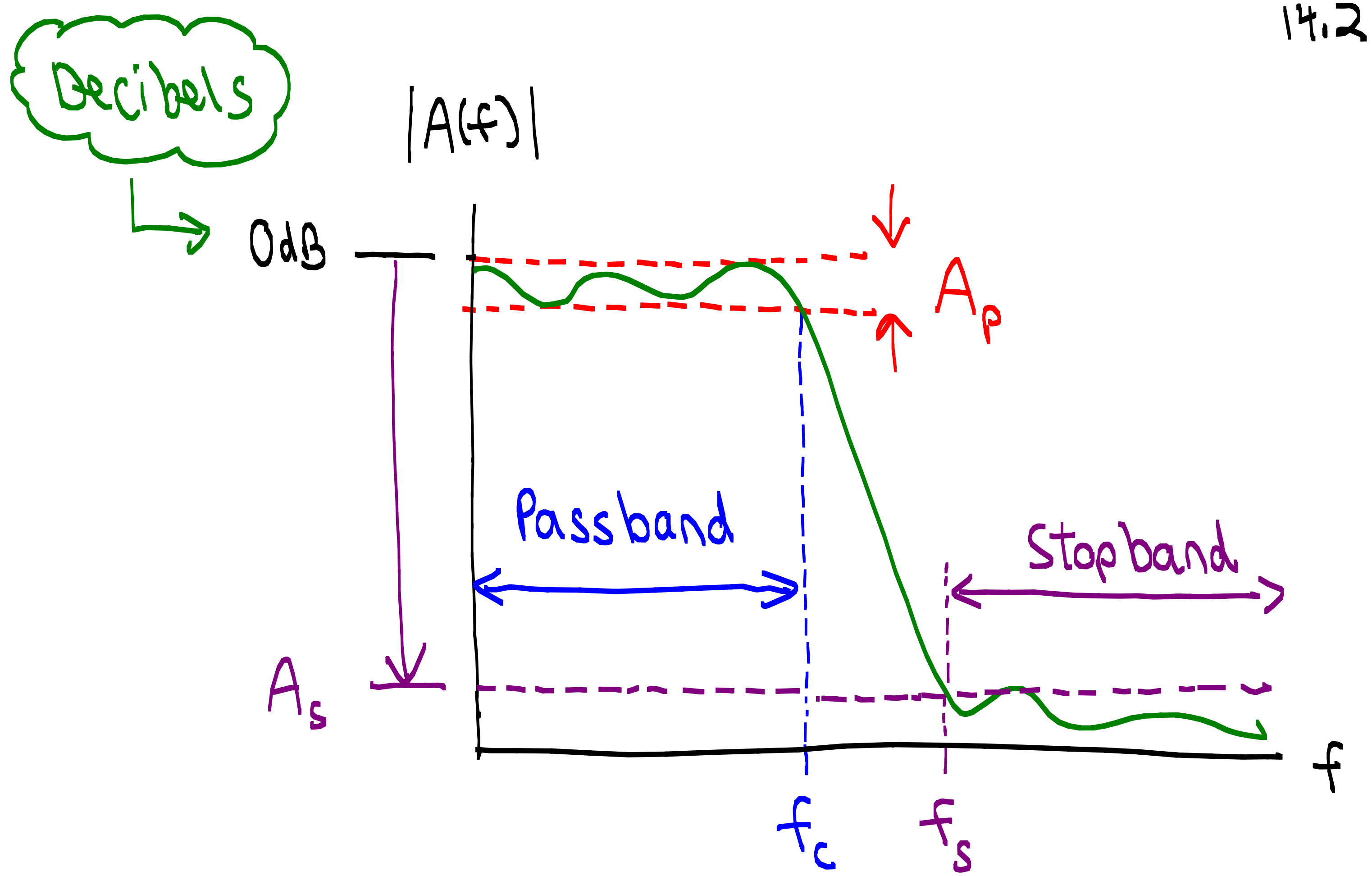


- We will only discuss low-pass filters, but most concepts apply to high-pass and band-pass filters.

B. Filter Parameters



Filter gain $= \left| \frac{V_{OUT}}{V_{IN}} \right|$



- The gain of a low-pass filter has three "frequency zones":

Pass band

$f_c \equiv$ cut-off frequency (or passband edge)

$A_p \equiv$ passband ripple

Transition

Steepness (roll off) depends on filter order

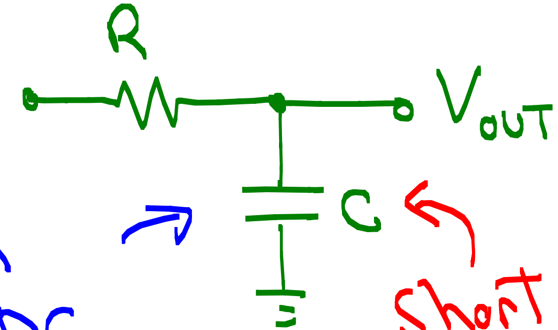
Stopband

$f_s \equiv$ Stopband edge

$A_s \equiv$ Stopband gain

C. Passive RC Filter

(1st order filter)



open at DC

Short at high freq

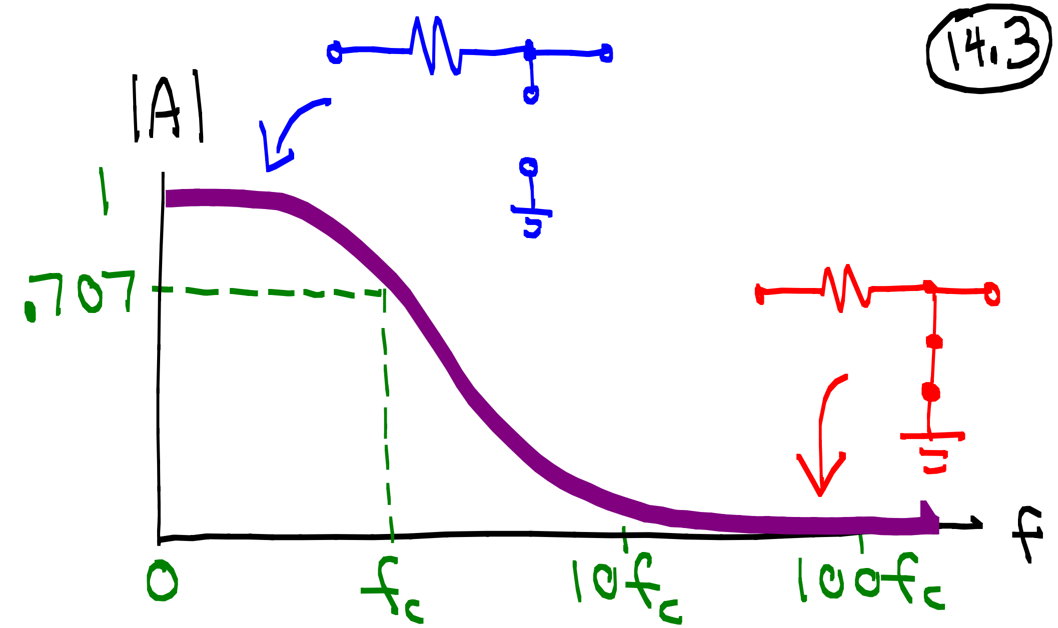
$$V_{OUT} = V_{IN}\,\frac{Z_c}{Z_c + R}$$

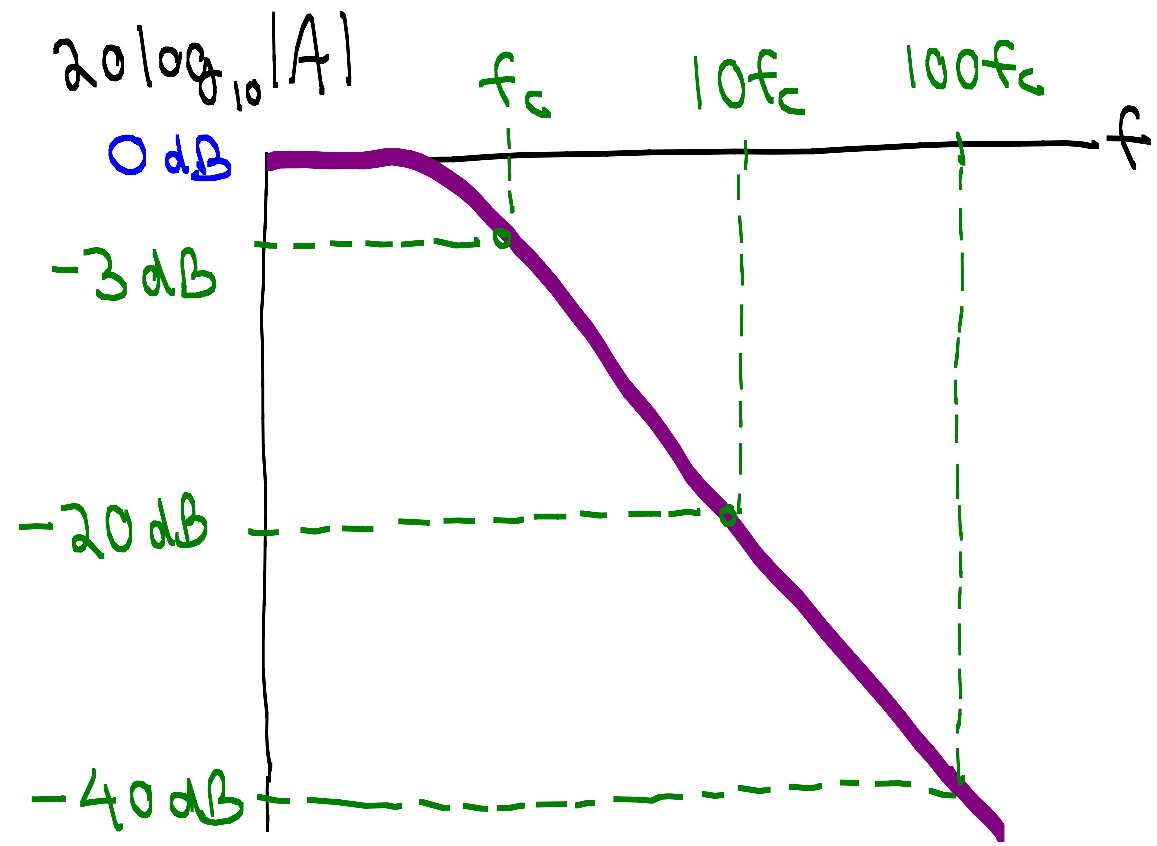
$$= V_{IN}\,\frac{1/j\omega C}{1/j\omega C + R} = V_{IN}\,\frac{1}{1 + j\omega RC}$$

$$\text{Filter Gain} \equiv A(f) = \frac{V_{OUT}(f)}{V_{IN}(f)} = \frac{1}{1 + j\, f/f_c}$$

"Cut-off" frequency $\boxed{f_c = \frac{1}{2\pi RC}}$

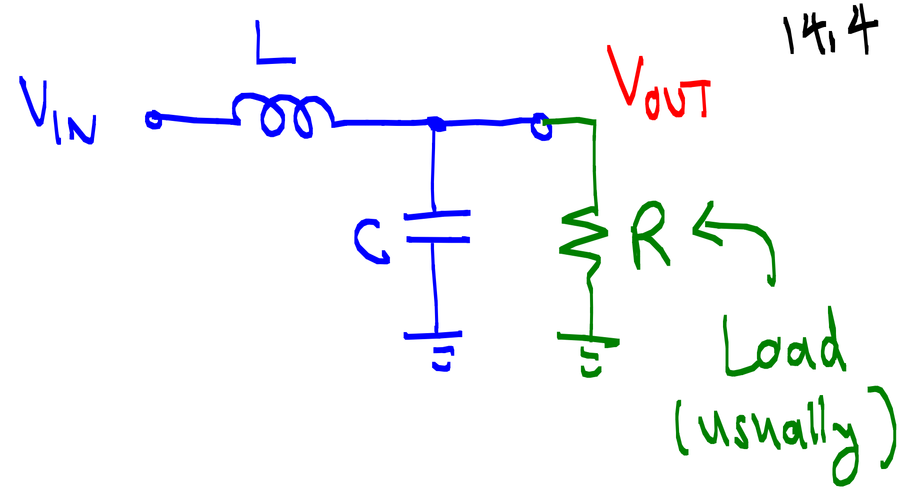
$$|A(f)| = \frac{1}{\sqrt{1 + (f/f_c)^2}} \approx \begin{cases} 1, & f \ll f_c \\ \frac{1}{(f/f_c)}, & f \gg f_c \end{cases}$$



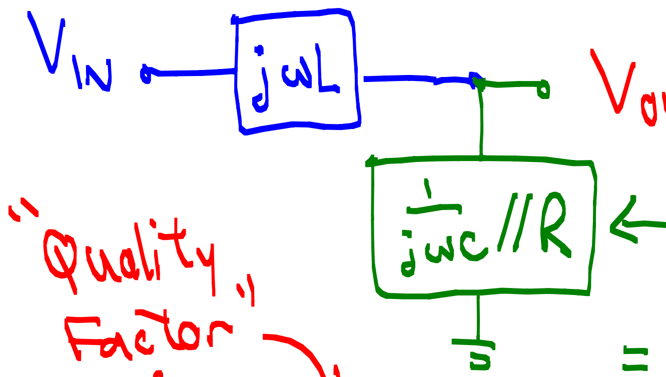


D. Passive LC filter

An LC circuit can produce faster freq roll off.



$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{\frac{1}{j\omega C} /\!/ R}{\frac{1}{j\omega C} /\!/ R + j\omega L} = \frac{\frac{R}{1+j\omega RC}}{\frac{R}{1+j\omega RC} + j\omega L}$$



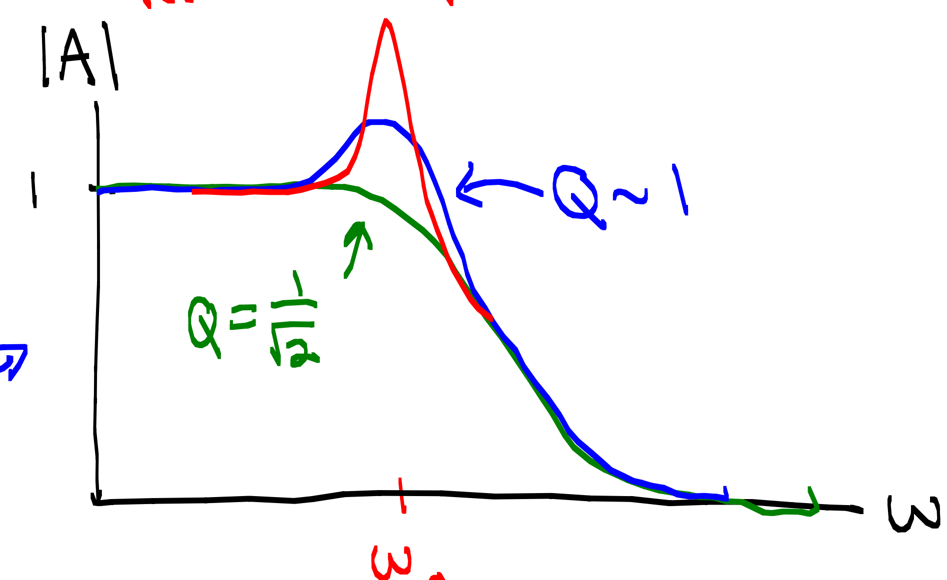
$$= \frac{R}{1+j\omega RC}$$

$$= \frac{1}{1 + j\omega \frac{L}{R}(1+j\omega RC)} = \frac{1}{1 + j\omega\frac{L}{R} - \omega^2 LC}$$

$$= \boxed{\frac{1}{1 + j\frac{\omega}{\omega_0 Q} - \omega^2/\omega_0^2}} \leftarrow \begin{cases} \omega_0 = \frac{1}{\sqrt{LC}} \\ Q = \frac{1}{\omega_0} R/L \end{cases}$$

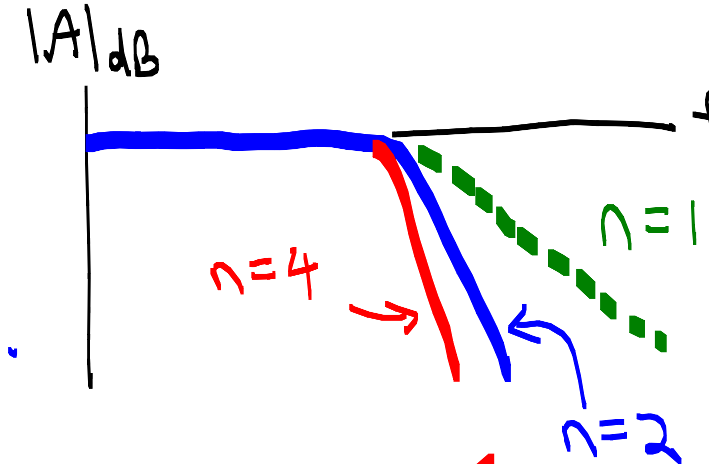
"Quality Factor" of resonance $Q \sim 3$

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1 - \left(2 - \frac{1}{Q^2}\right)\frac{\omega^2}{\omega_0^2} + \frac{\omega^4}{\omega_0^4}}} \leftarrow \text{2}^{\text{nd}} \text{ order response}$$



- Simple RC filter (1st order filter) is OK if a gradual rolloff is acceptable.

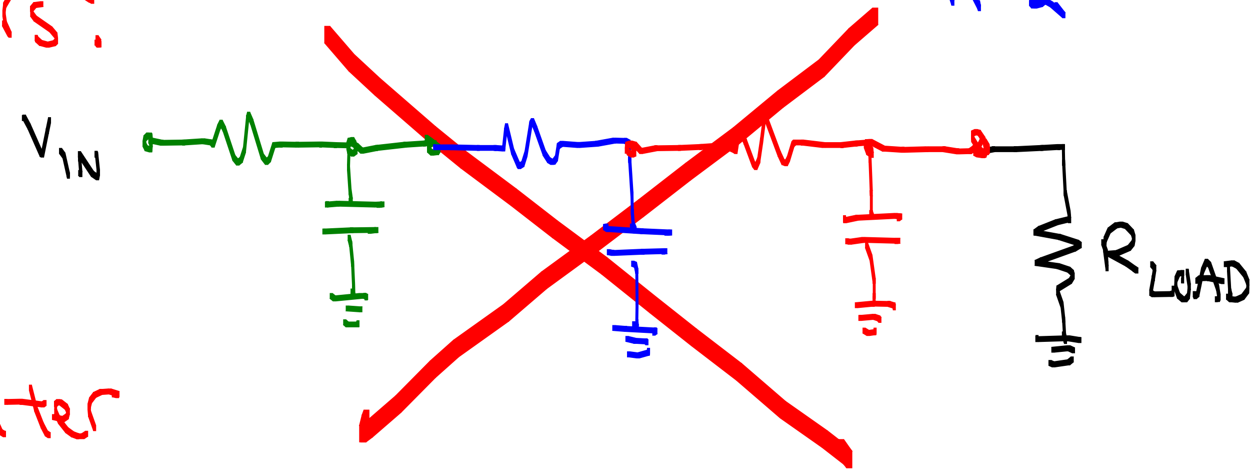
→ Higher order filters have sharper cut-off.



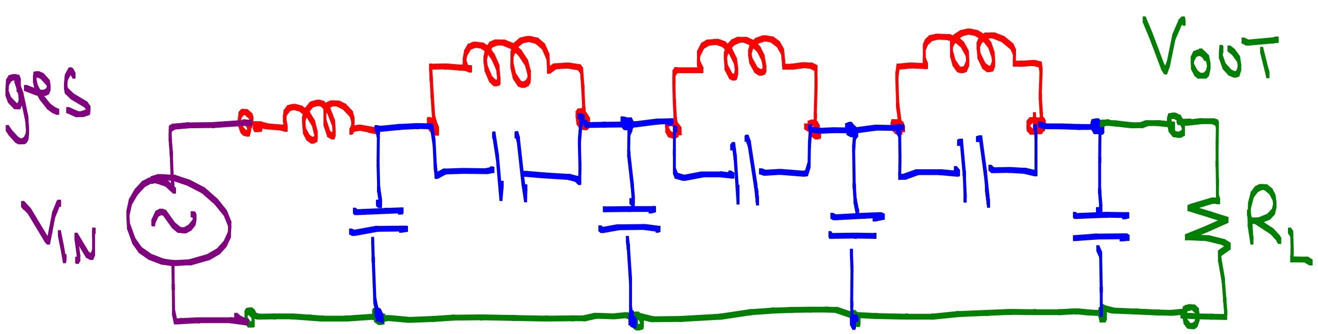
Q: Higher order = Multiple passive filters?

A: NO! Each section's input impedance loads the previous section :(

→ Difficult to design a high order filter



- Passive RLC filters with multiple stages can achieve very sharp cut-off.

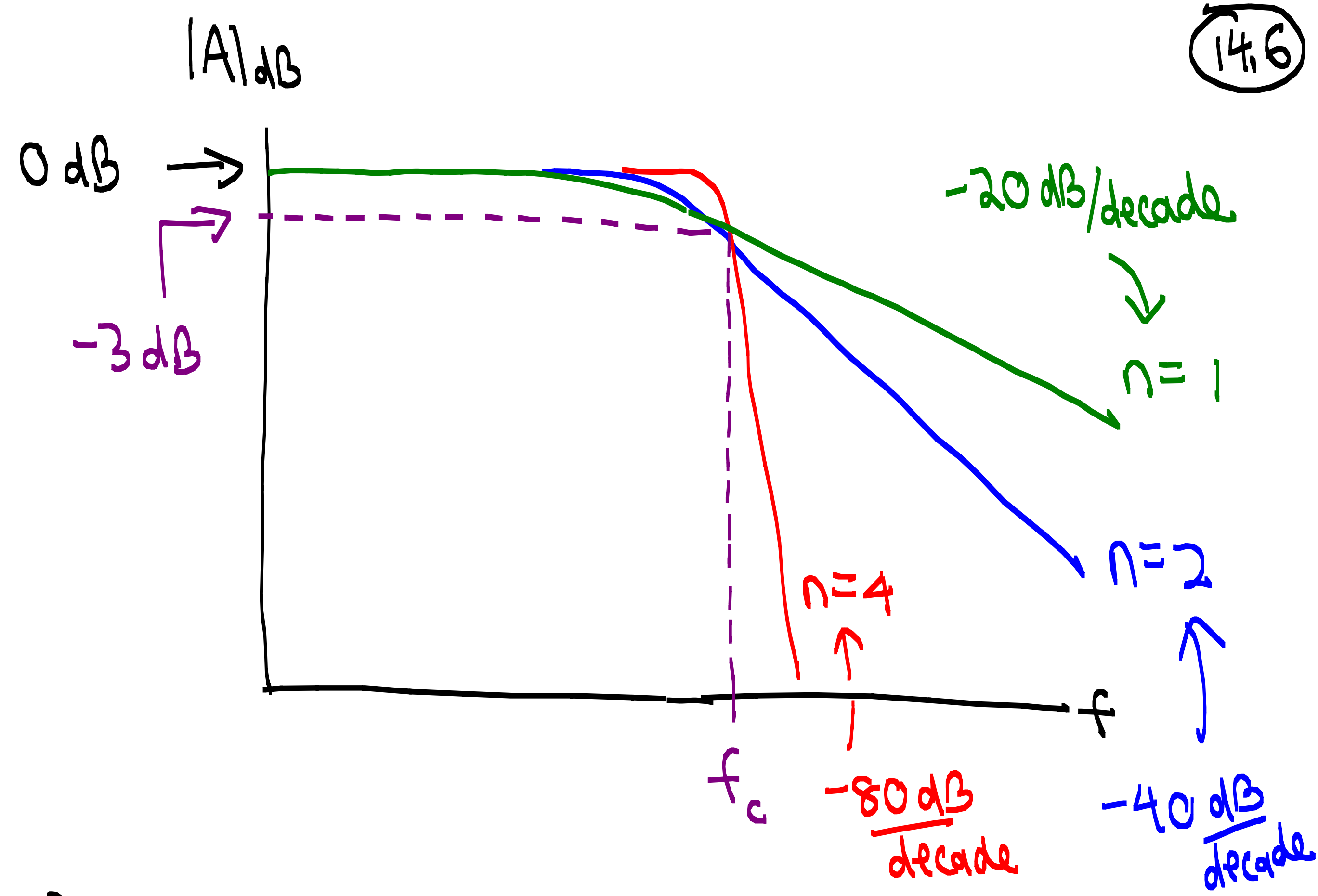


<u>Disadvantage</u>: Inductors are not easy to use.

Q: Can we make <u>inductorless</u> filters with same properties as RLC circuits?

2. Active Filters

→ Active filters can produce very steep roll-off without inductors!

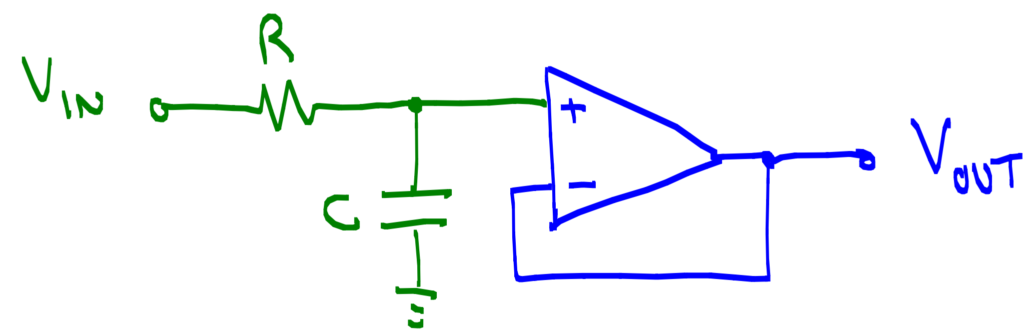


- **Butterworth Filter**

$$\boxed{|A(f)| = \frac{1}{\sqrt{1 + (f/f_c)^{2n}}}}$$

→ Produces the flattest passband ($A_p = 0$ dB)

→ BUT, transition steepness is less than some other filter types.



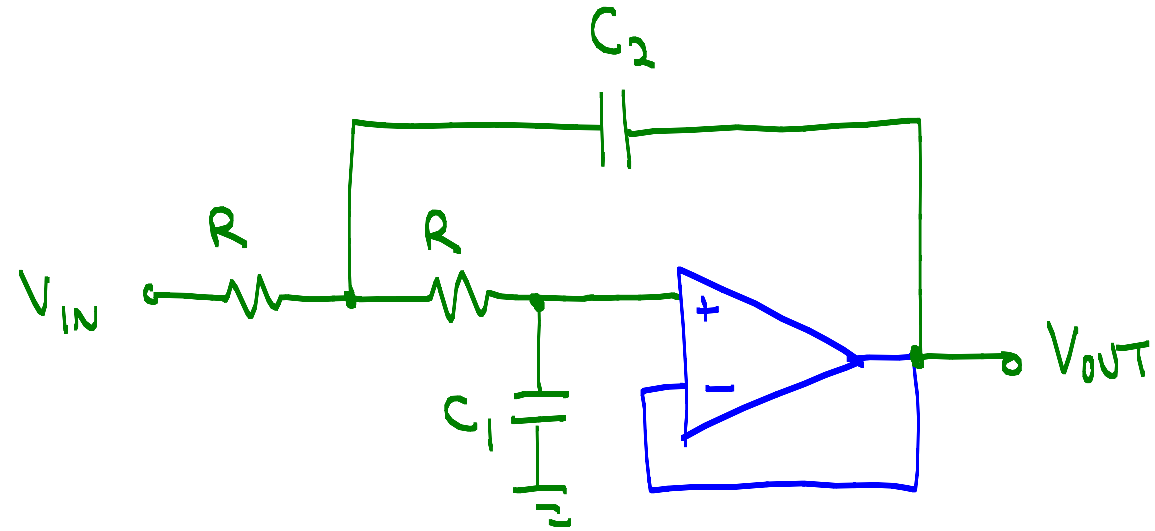
$\boxed{1^{st} \text{ order Butterworth}}$ ($n = 1$)

Just add a buffer!

$$\Rightarrow |A| = \frac{1}{\sqrt{1 + (f/f_c)^2}}, \quad f_c = \frac{1}{2\pi RC}$$

2nd order Butterworth

$$|A| = \frac{1}{\sqrt{1 + (f/f_c)^4}}$$



- A simple implementation is to use a "Sallen-and-Key" design.

① $Q = \frac{1}{2}\sqrt{\frac{C_2}{C_1}} = 0.707 \Rightarrow \boxed{C_2 = 2C_1}$

"Quality Factor"

② $\boxed{f_c = \frac{1}{2\pi R\sqrt{C_1 C_2}}}$

Example $R = 30\,K$, $C_1 = 820\,pF$

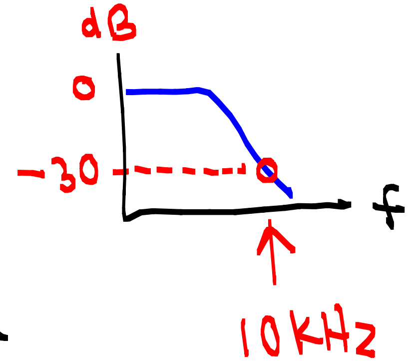
$C_2 = 1.64\,nF$

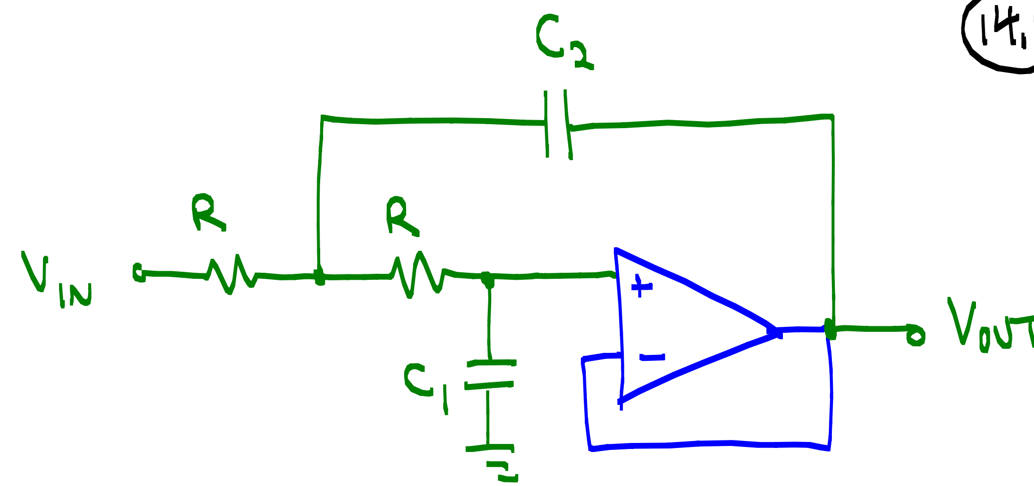
- Is Q correct? $\frac{C_2}{C_1} = 2 \Rightarrow Q = \frac{\sqrt{2}}{2} = \underline{\underline{.707}}$ ✓

- f_c? $f_c = \underline{\underline{4.58\,KHz}}$

Design Example

2nd order Butterworth LPF with $-30\,\text{dB}$ gain at $10\,\text{KHz}$





① $A_{dB} = -30\,\text{dB} = 20\log_{10}|A|$

$\rightarrow |A| = 10^{-30/20} = .0316$ at $f = 10\,\text{KHz}$

② $$|A| = \frac{1}{\sqrt{1+\left(\frac{10^4\,\text{Hz}}{f_c}\right)^4}} = .0316$$

$$\rightarrow 1000 = 1 + \left(\frac{10^4\,\text{Hz}}{f_c}\right)^4$$

$$f_c = \frac{10^4}{999^{1/4}} = \underline{\underline{1778.7\,\text{Hz}}}$$

$C_2 = 2C_1$

③ $$f_c = \frac{1}{2\pi R\sqrt{C_1 C_2}} = \frac{1}{2\sqrt{2}\,\pi R C_1}$$

Let $C_1 = 1\,\text{nF} \rightarrow R = \dfrac{1}{2\sqrt{2}\,\pi(10^{-9})(1778.7)}$

$C_2 = 2\,\text{nF}$

$= 63270.7\,\Omega$

5% value $\rightarrow$ Choose $\underline{\underline{R = 62K}}$

- **Higher Order Butterworth**

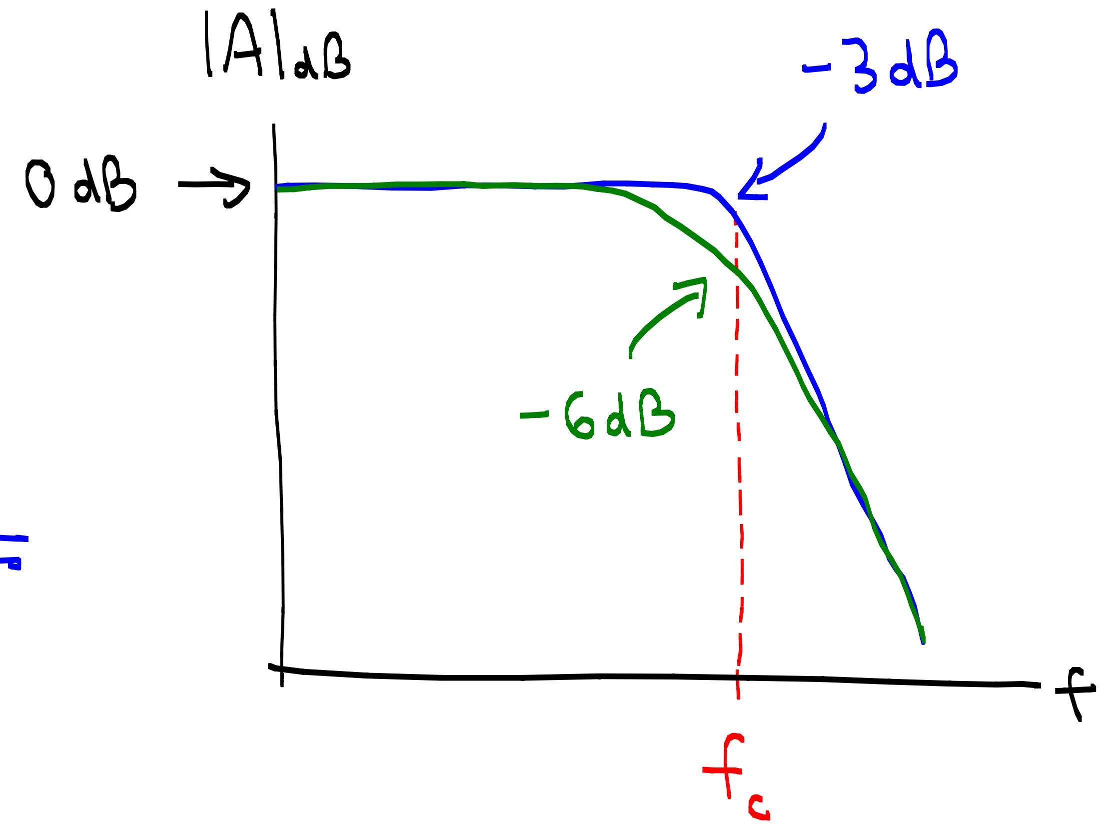
How to make a 4^{th} order Butterworth?

$$|A| = \frac{1}{\sqrt{1+(f/f_c)^8}}$$

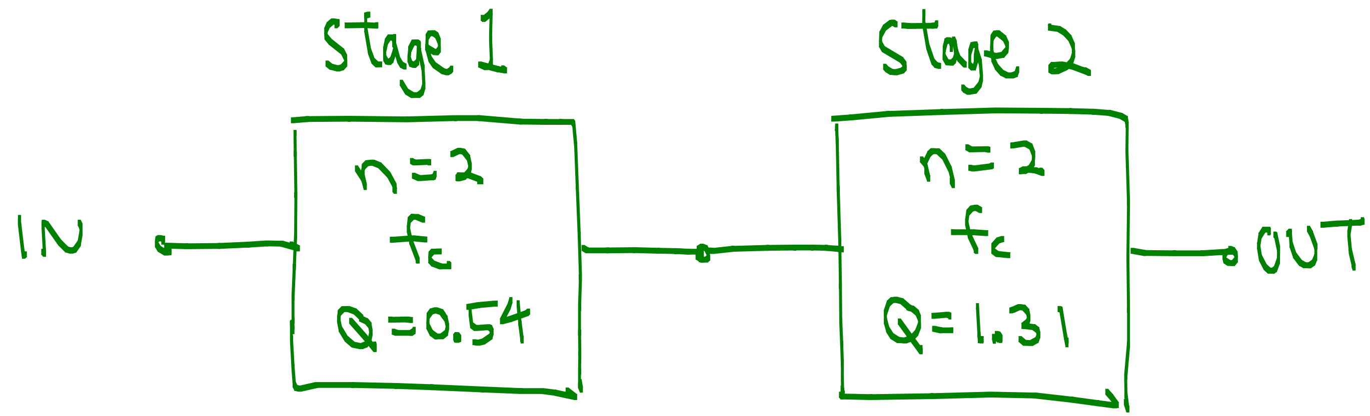
we cannot simply cascade two identical 2^{nd} order Butterworth filters. :(

$$|A| = \frac{1}{\sqrt{1+(f/f_c)^4}} \times \frac{1}{\sqrt{1+(f/f_c)^4}}$$

$$= \frac{1}{1+(f/f_c)^4} \neq \frac{1}{\sqrt{1+(f/f_c)^8}}$$



- Fortunately, we can cascade two $n=2$ Sallen-Key filters with the same f_c but different Q!



- This "staggered Q" technique can be used for higher orders:

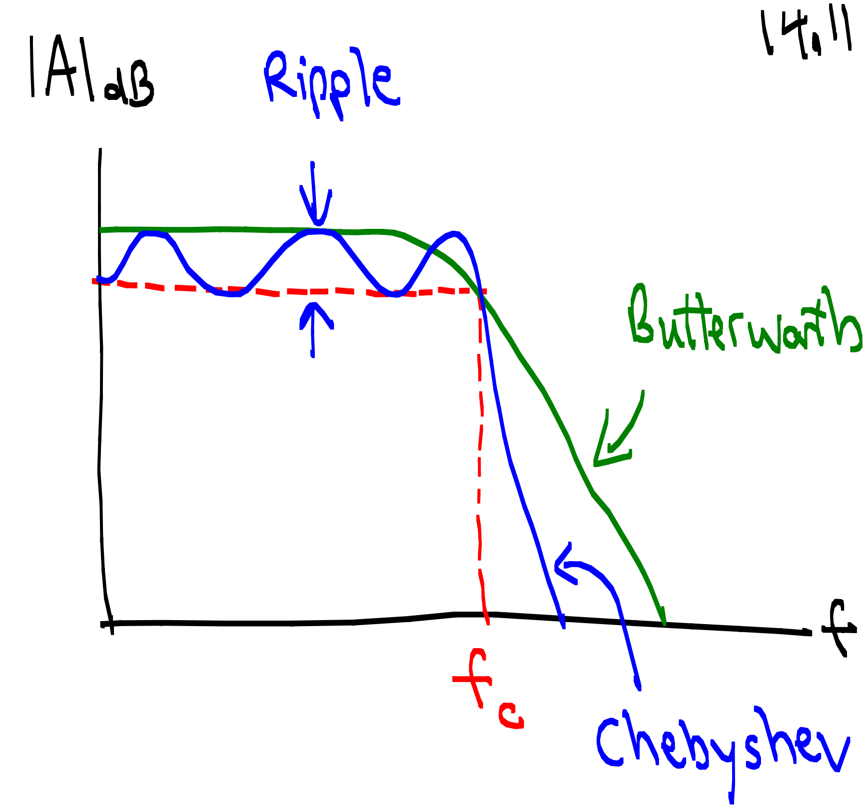
(Table 19-3 in textbook)

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
$n=2$	.707				
$n=4$	.54	1.31			
$n=6$	.52	1.93	.707		
$n=8$	.51	2.56	0.6	0.9	
$n=10$	.51	3.2	.56	1.1	.707

- Chebyshev Filter

→ Produce a steeper transition than Butterworth

BUT, passband has slight ripple!



→ Same op amp circuit, but the filter parameters are slightly more complicated...

$$Q = 0.5\sqrt{\frac{C_2}{C_1}} > 0.707$$

"Pole Freq" → $f_p = \dfrac{1}{2\pi R\sqrt{C_1 C_2}}$

"Cut-off freq" → $f_c = K_c f_p$

A_p : Passband ripple ← Should be < 1 dB

Ex: n=2 Chebyshev

→ $A_p = 0.69\,dB$

$Q = 0.9$

$f_c = 0.874 f_p$