Lecture 15: Negative Feedback

0. Review
1. Intro
2. Improved Amplifier Properties

- Exam #2 re-do due next Tue (Nov 19)
- HW7 due next Thu (Nov 21) (box outside my office)
- Next Tue: Course Eval!
- Lab 5 report due Nov 26 (Tue) (1 per team)

Textbook Reading:

17-2 VCVS Voltage Gain

17-3 Other VCVS Equations

0. Review

Active Filters ← use op amps without inductors

① Butterworth:

$$Q = \frac{1}{2}\sqrt{\frac{C_2}{C_1}} = \frac{1}{\sqrt{2}}$$

$$f_c = \frac{1}{2\pi R\sqrt{C_1 C_2}}$$

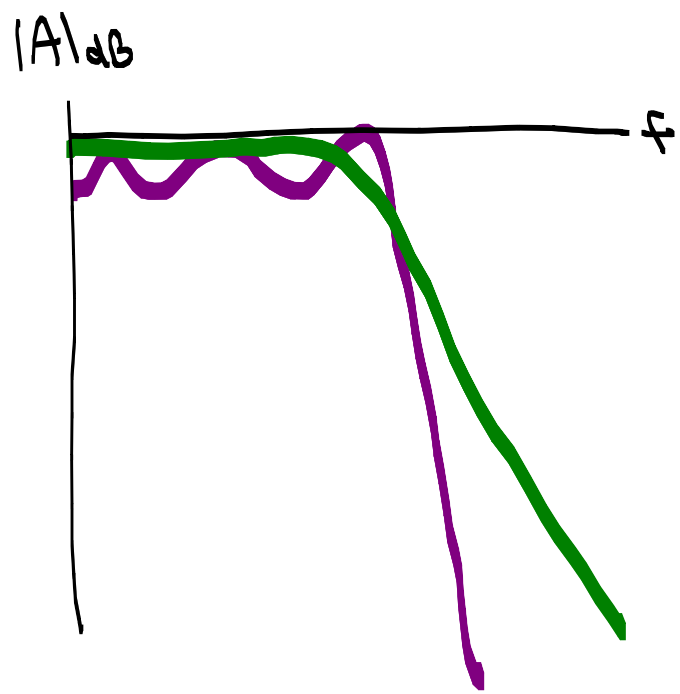
② Chebyshev:

$$Q = \frac{1}{2}\sqrt{\frac{C_2}{C_1}} > \frac{1}{\sqrt{2}}$$

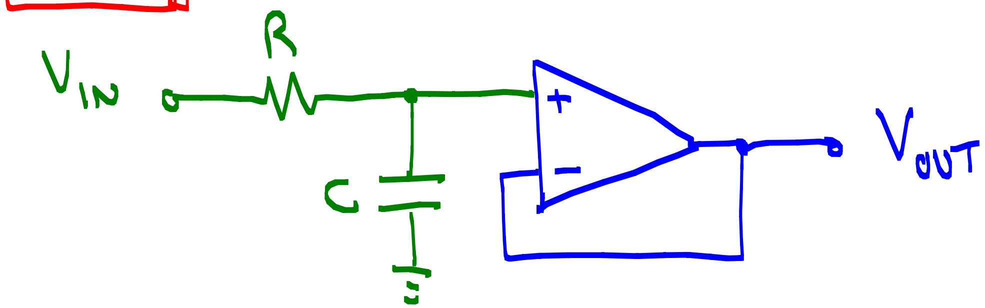
$$f_p = \frac{1}{2\pi R\sqrt{C_1 C_2}}$$

$$f_c = K_c f_p$$

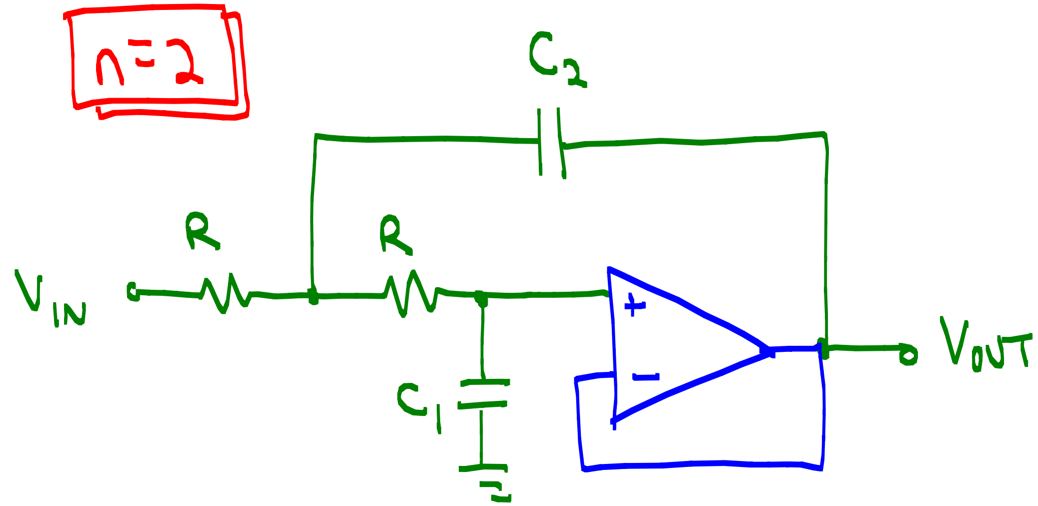
(Low Pass)



$\boxed{n=1}$



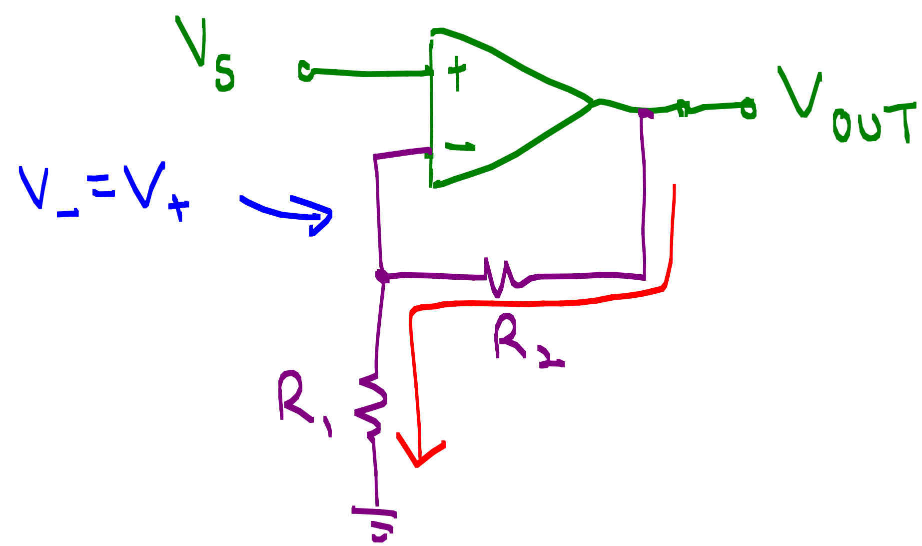
$\boxed{n=2}$



1. Intro

Recall an op amp's <u>Golden Rules</u>:

1. $V_- = V_+$
2. Inputs draw zero current

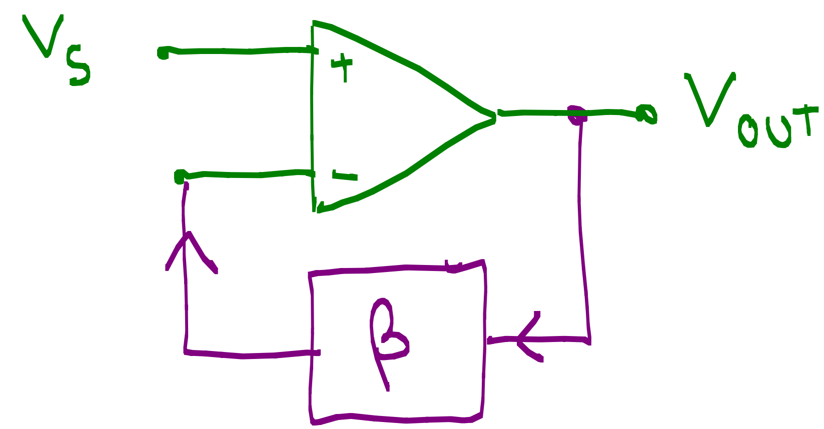


Q: How is Rule #1 ($V_+ = V_-$) possible?

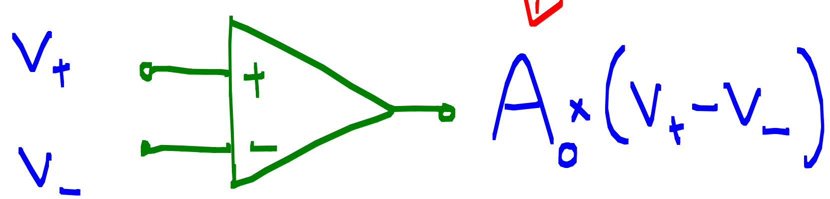
A: Negative feedback

+

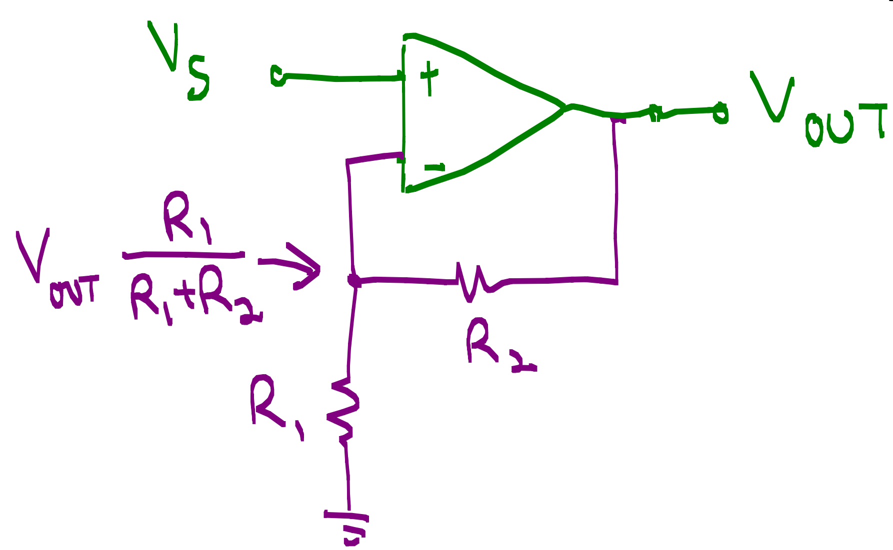
High open-loop gain



$\sim 10^6$!



- Let's analyze the non-inverting amplifier WITHOUT Golden Rule #1!



$$V_{OUT} = A_o(V_+ - V_-) = A_o\left(V_S - V_{OUT}\frac{R_1}{R_1+R_2}\right)$$

$$V_{OUT}\left(1 + A_o\frac{R_1}{R_1+R_2}\right) = A_o V_S$$

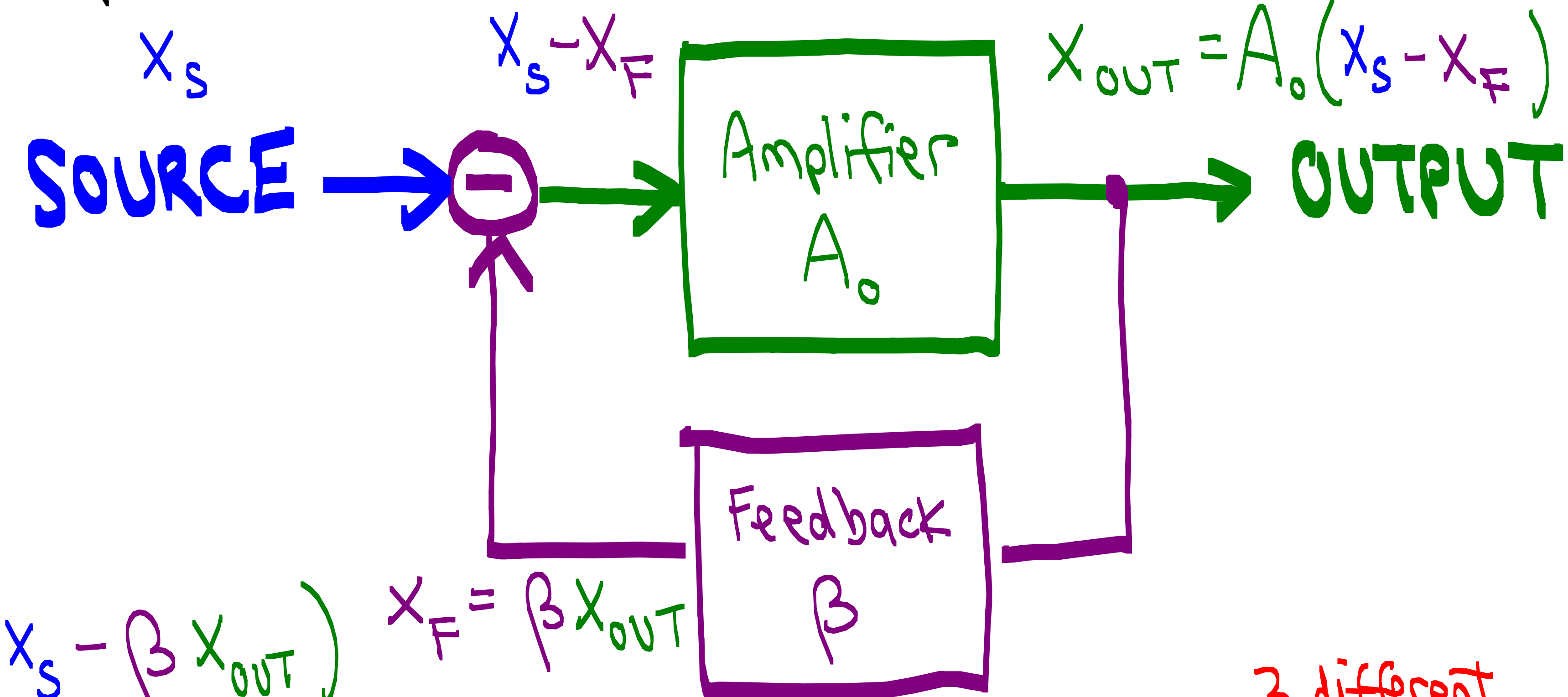
$$\frac{V_{OUT}}{V_S} = \frac{A_o}{1 + A_o\left(\frac{R_1}{R_1+R_2}\right)} \xRightarrow{\text{When } A_o \text{ is huge}} \frac{V_{OUT}}{V_S} \approx \frac{A_o}{A_o\frac{R_1}{R_1+R_2}} = \frac{R_1+R_2}{R_1} = \boxed{1 + \frac{R_2}{R_1}} \checkmark$$

Also, $\boxed{V_-} = V_{OUT}\frac{R_1}{R_1+R_2} = \left[V_S\cancel{\left(1+\frac{R_2}{R_1}\right)}\right]\cdot\cancel{\frac{R_1}{R_1+R_2}} = V_S = \boxed{V_+}$ ← $V_- = V_+$ requires neg feedback and huge A_o

- we can generalize this concept of <u>negative feedback</u>.

① <u>Sample</u> a portion of the output

② <u>Subtract</u> it from the source input



$$X_{OUT} = A_o(X_S - X_F) = A_o(X_S - \beta X_{OUT})$$

$$\rightarrow (1 + A_o\beta) X_{OUT} = A_o X_S$$

$$\Rightarrow \boxed{\frac{X_{OUT}}{X_S} = G = \frac{A_o}{1 + A_o\beta}}$$

When $A_o\beta \gg 1$,

$$G \approx \frac{A_o}{A_o\beta} = \frac{1}{\beta}$$

No longer depends on A_o!

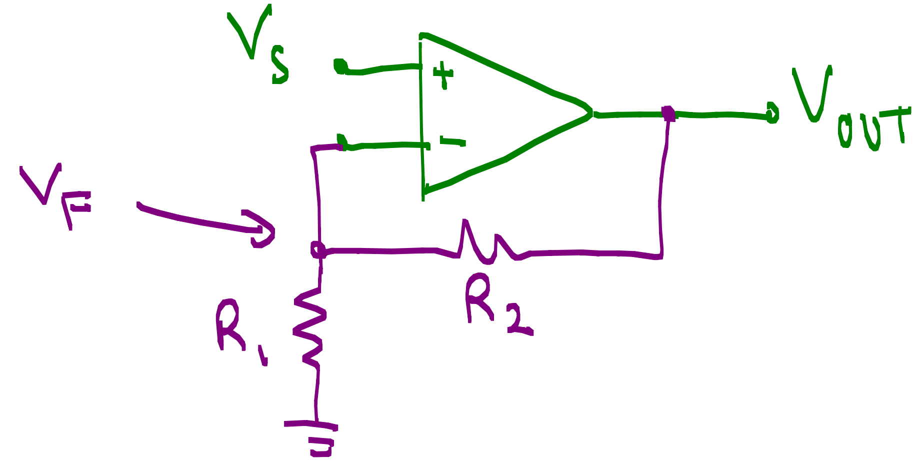
3 different gains:

A_o = OPEN loop gain

G = CLOSED loop gain

$A_o\beta$ = Loop gain

- Example: Non-inverting amplifier



$X_s = V_s$

$X_F = V_F = \boxed{\dfrac{R_1}{R_1 + R_2}} V_{OUT}$ ← β (Feedback factor) $\beta < 1$

$X_{OUT} = V_{OUT}$

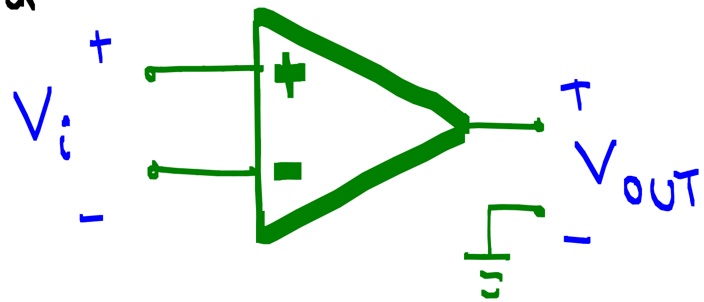
$$\Rightarrow \quad G = \frac{A_o}{1 + A_o\beta} = \frac{A_o}{1 + A_o \dfrac{R_1}{R_1 + R_2}}$$

When $\underbrace{A_o\beta}_{\text{Loop gain}} \gg 1 \Rightarrow G \approx \dfrac{1}{\beta} = \dfrac{R_1 + R_2}{R_1} = 1 + \dfrac{R_2}{R_1}$ ✓

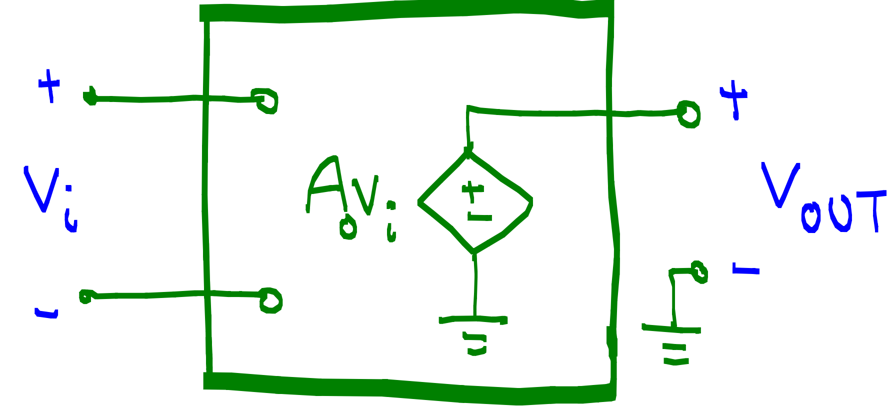
✡ Closed loop gain only depends on feedback network

2. Improved amplifier properties

- So far, we have assumed an <u>ideal</u> op amp



Ideal op amp



$R_i = \infty$

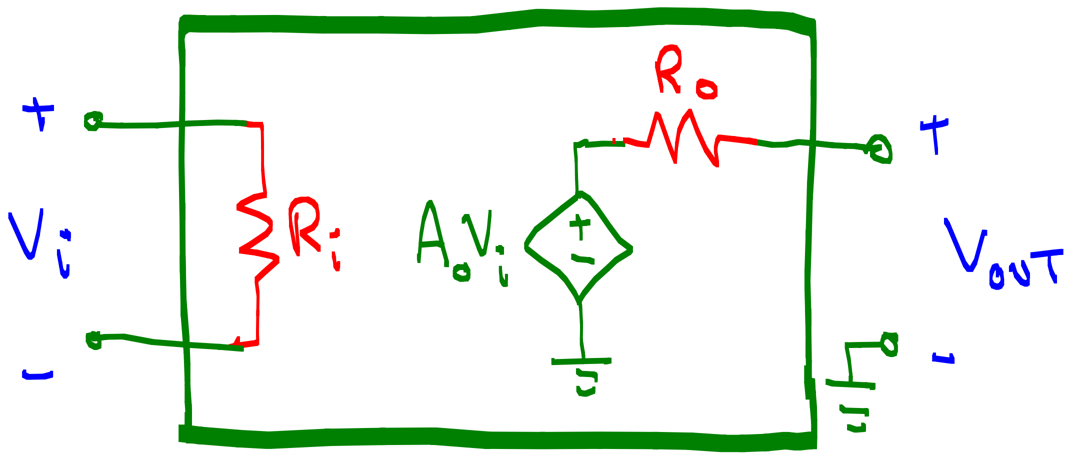
$R_o = 0$

Typically ~ few MΩ

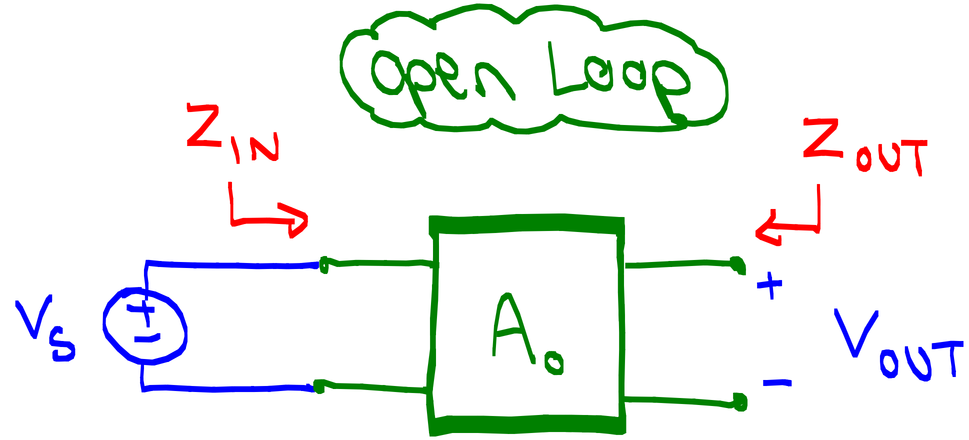
- A <u>real</u> op amp has finite R_i and non-zero R_o

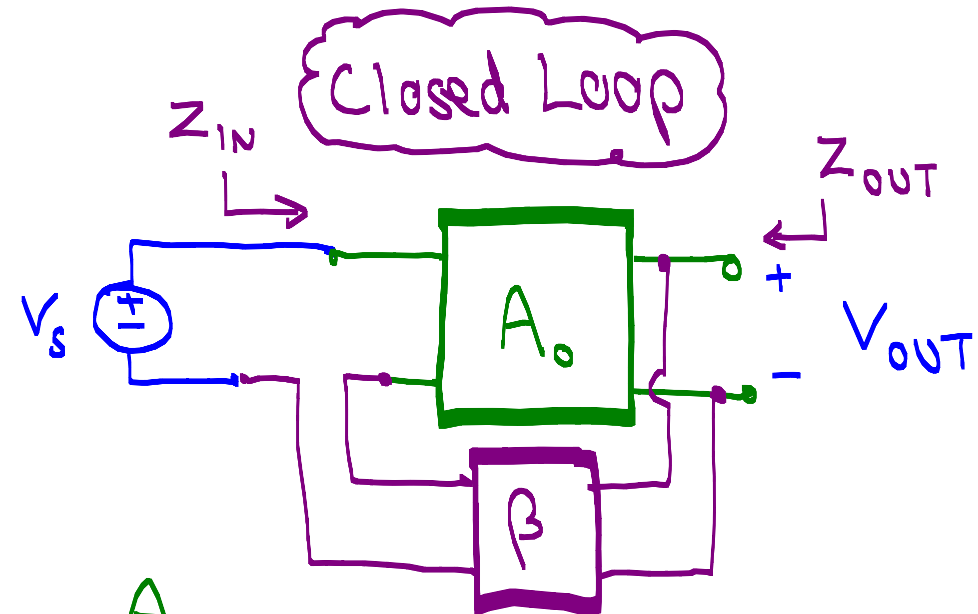
Typically ~100Ω

Real Op Amp



- Negative feedback is a good thing!





	Open Loop	Closed Loop
GAIN:	A_o	$\dfrac{A_o}{1+A_o\beta}$
Z_{IN}:	R_i	$(1+A_o\beta)R_i$
Z_{OUT}:	R_o	$\dfrac{R_o}{1+A_o\beta}$
BANDWIDTH:	Δf_o	$(1+A_o\beta)\Delta f_o$

- Open-Loop amplifier may not have best properties

☆ Negative feedback improves properties by $1+A_o\beta$

- **Input impedance:** $Z_{IN} = \dfrac{V_s}{I_{IN}}$

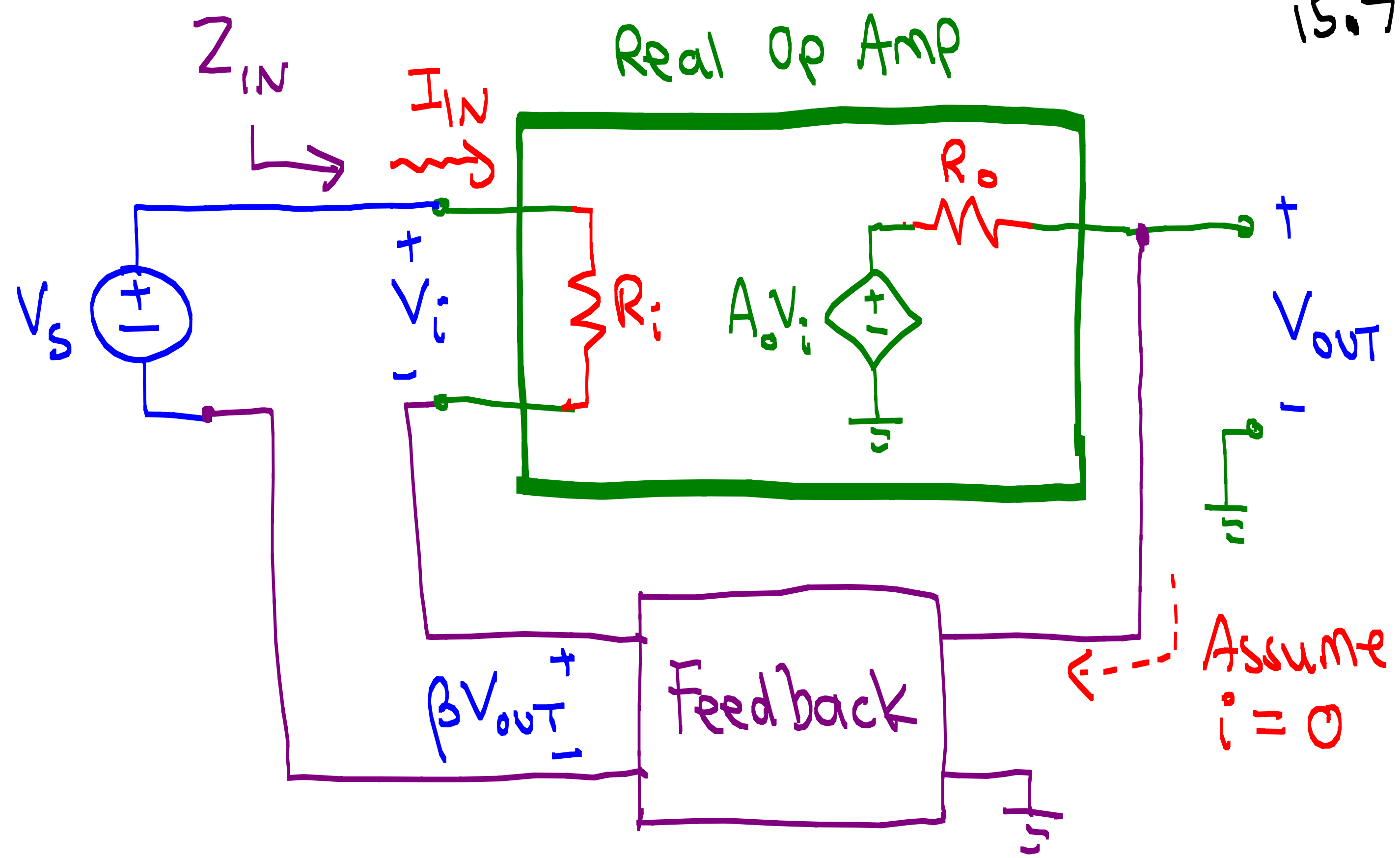
$$I_{IN} = \frac{V_i}{R_i} = \frac{V_s - \beta V_{OUT}}{R_i}$$

using $V_{OUT} = \dfrac{A_o}{1+A_o\beta} V_s$

$$V_s - \beta V_{OUT} = V_s - \frac{A_o\beta}{1+A_o\beta} V_s$$

$$= \left(1 - \frac{A_o\beta}{1+A_o\beta}\right) V_s$$

$$= \frac{1}{1+A_o\beta} V_s$$



$$\Rightarrow I_{IN} = \frac{V_s}{(1+A_o\beta) R_i}$$

R_i is magnified by $1+A_o\beta$!

$$\Rightarrow \boxed{Z_{IN} = \frac{V_s}{I_{IN}} = \underbrace{(1+A_o\beta)}_{\text{Typically} > 1000} R_i}$$

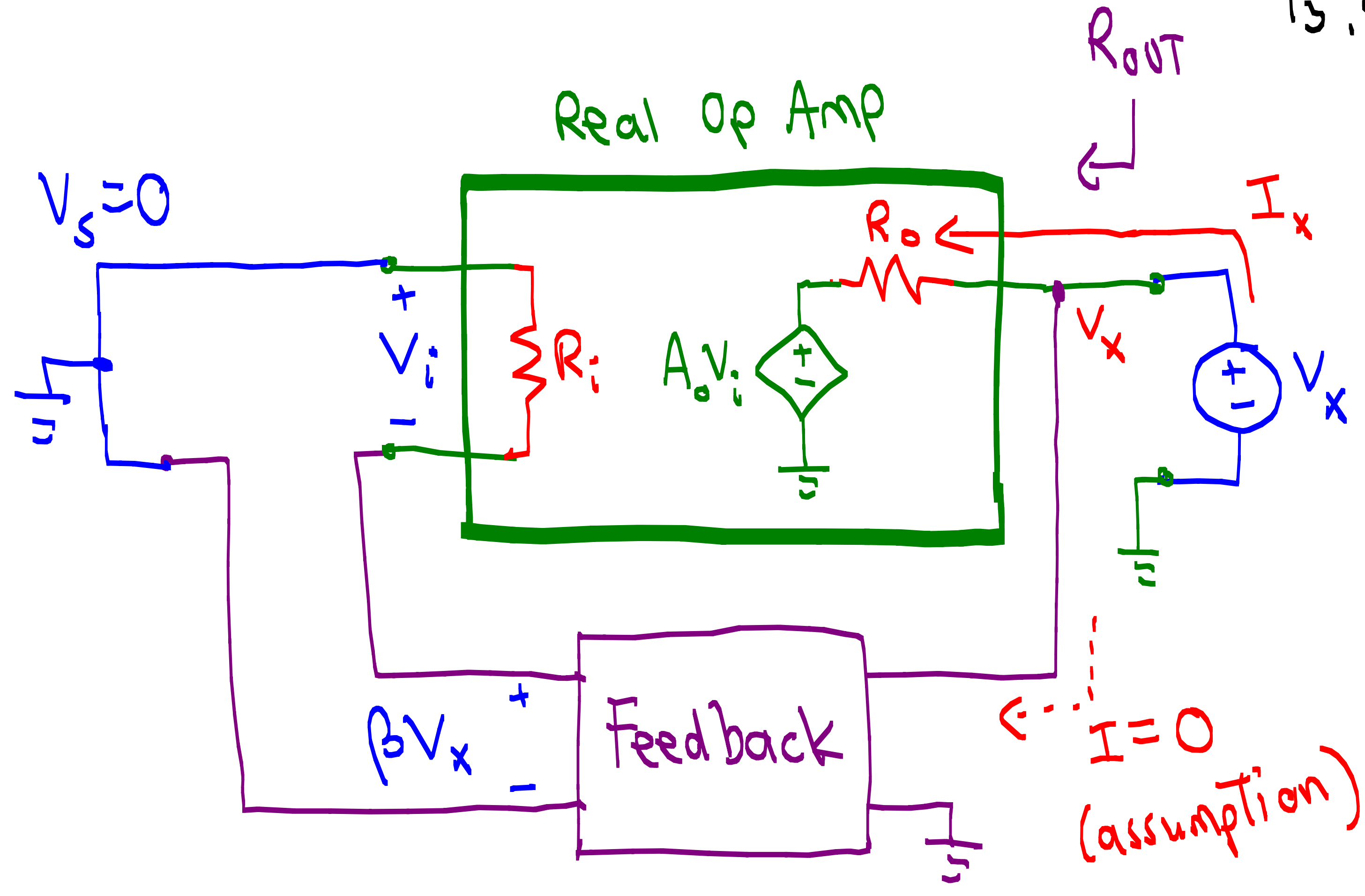
:)

- Output Impedance $R_{out} = \frac{V_x}{I_x}$

Theoretical method:

① Ground the input

② Attach test source to output



Assuming feedback network draws negligible current

$$\Rightarrow I_x = \frac{V_x - A_oV_i}{R_o}$$

using $V_i + \beta V_x = 0 \Rightarrow V_i = -\beta V_x$

$$\Rightarrow I_x = \frac{V_x + A_o\beta V_x}{R_o} = V_x \frac{(1+A_o\beta)}{R_o}$$

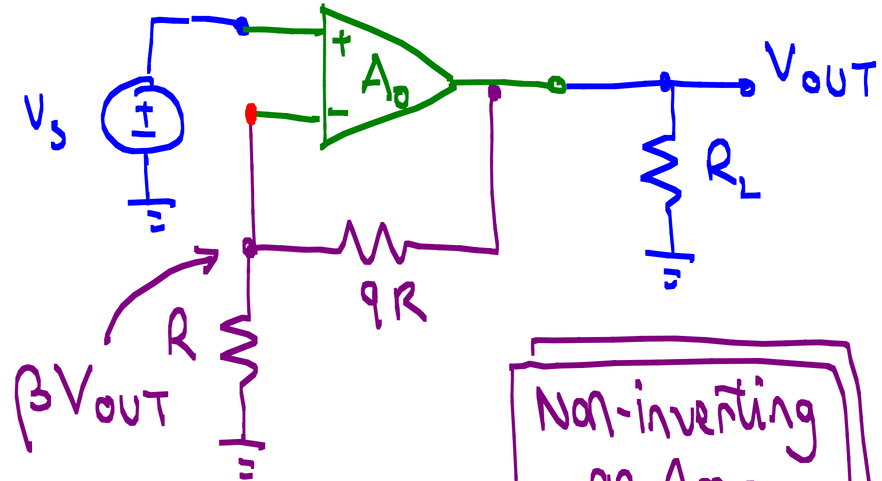
$$\Rightarrow \boxed{R_{OUT} = \frac{V_x}{I_x} = \frac{R_o}{1+A_o\beta}}$$

R_o is reduced by $1+A_o\beta$!

Example

Suppose $A_o = 10^4$ varies by +/- 50%.
What is the variation in G?

Textbook uses A_f



Non-inverting op Amp

STEP 1: Find β

$$\beta V_{OUT} = \underbrace{\frac{R}{R+9R}}_{\beta} V_{OUT} \rightarrow \beta = \frac{R}{10R} = \underline{\underline{0.1}}$$

STEP 2: Find G

$$G_o = \frac{A_o}{1+A_o\beta} = \frac{10^4}{1+10^4 \times 0.1}$$

$$= \underline{\underline{9.990}}$$

"nominal" gain

STEP 3: Let A_o vary

$$A_{max} = 1.5\times10^4 \rightarrow G_{max} = \frac{1.5\times10^4}{1+1.5\times10^3} = \underline{\underline{9.993}}$$

$$A_{min} = 0.5\times10^4 \rightarrow G_{min} = \frac{0.5\times10^4}{1+0.5\times10^3} = \underline{\underline{9.980}}$$

$\boxed{\text{STEP 4: Compute } \Delta G/G}$

$$\frac{\Delta G}{G} = \frac{G_{MAX} - G_{MIN}}{G_o} = \frac{9.993 - 9.980}{9.990} = .0013$$

$\Rightarrow \boxed{0.13\%}$

very stable! :)

$\underbrace{\hspace{4cm}}$

Tiny change in closed-loop gain

VS.

$$\frac{\Delta A_o}{A_o} = \frac{1.5A_o - .5A_o}{A_o} = 1$$

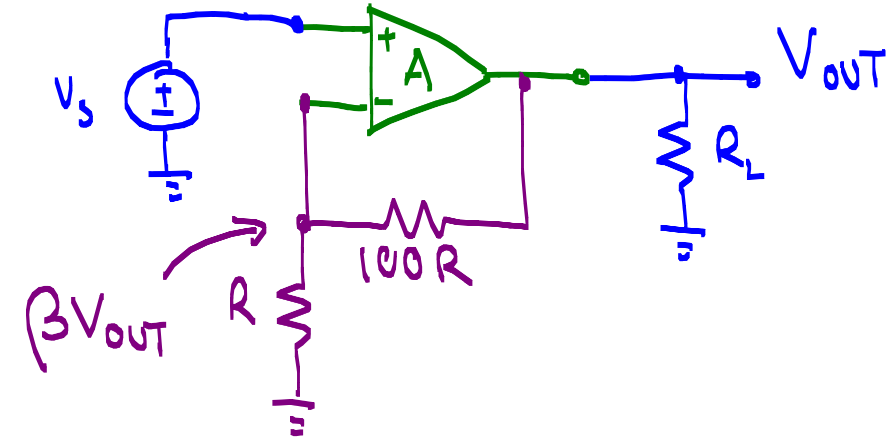
$$= 100\% !$$

$\underbrace{\hspace{6cm}}$

Huge change in open-loop gain

☆ (−) feedback improves gain stability!

Example Suppose $A_o = 10^4$ varies by +/- 50%.
We want an amplifier with gain ~100.
Single or double stage more stable?



① Single Stage:

$$\beta V_{OUT} = \frac{R}{R + 100R} V_{OUT} \rightarrow \beta = \frac{R}{101R} = .0099$$

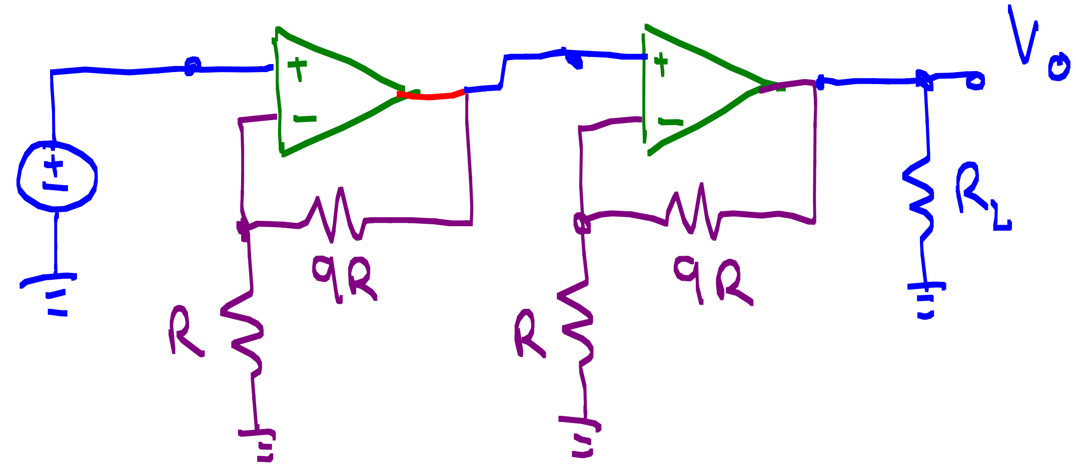
$$G = \frac{A_o}{1 + A_o\beta} : \qquad A_o = 10^4 \longrightarrow G_o = \frac{10^4}{1 + 99} = 100$$

$$A_{max} = 1.5 \times 10^4 \longrightarrow G_{MAX} = 100.33$$

$$A_{min} = .5 \times 10^4 \longrightarrow G_{MIN} = 99.01$$

$$\frac{\Delta G}{G} = 1.3\ \%$$

② Two stage: $\beta = \frac{R}{R+9R} = 0.1$



$$G_{Total} = G_1 \times G_2$$
$$= \left(\frac{A}{1+A\beta}\right)^2$$

$$A_o = 10^4 \rightarrow G_{Total} = (9.99)^2 = 99.8$$

Worst-case variation:

$$A_{max} = 1.5\times10^4 \rightarrow G_{Total} = \left[\frac{1.5\times10^4}{1+.15\times10^4}\right]^2 = [9.993]^2 = 99.87$$

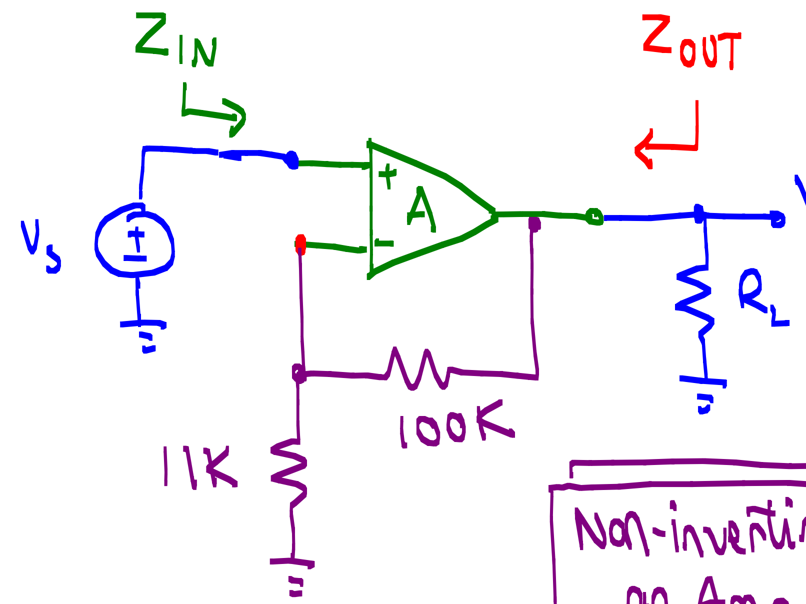
$$A_{min} = .5\times10^4 \rightarrow G_{Total} = \left[\frac{5000}{1+500}\right]^2 = [9.98]^2 = 99.60$$

$$\frac{\Delta G}{G} = 0.27\%$$

WINNER!

Example Suppose $A = 10^5$, $R_{id} = 100K$, $R_o = 100\,\Omega$

Compute Z_{IN} and Z_{OUT} of <u>closed</u>-loop circuit.



Non-inverting op Amp

$$\beta = \frac{11K}{11K + 100K} = .099 \quad , \quad A_o\beta = 9910$$

- $Z_{IN} = R_{id}(1 + A\beta)$

$$= (100K)(1 + 9910)$$

$$= \boxed{991\ M\Omega} \text{ HUGE!}$$

- $Z_{OUT} = \dfrac{R_o}{1 + A_o\beta}$

$$= \frac{100\,\Omega}{1 + 9910} = \boxed{0.01\,\Omega} \text{ tiny!}$$