Lecture 16: Amplifier Stability

0. Review
1. Frequency Response
2. Feedback Stability

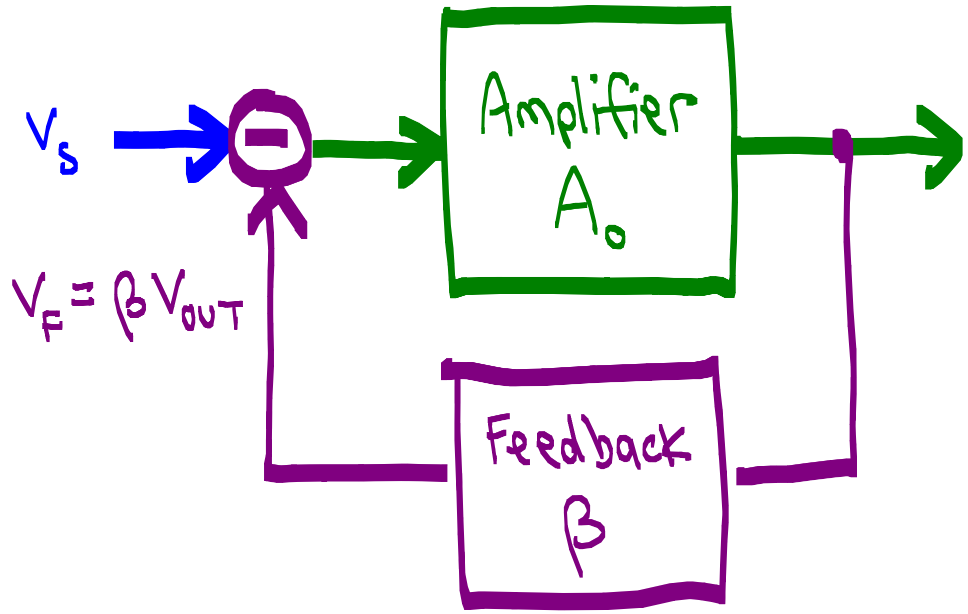
- Exam #2 re-do due today
- HW7 due Thu (Nov 21)
  (box outside my office)
- Lab 5 report due Nov 26 (Tue)
  (1 per team)
- Final Exam Nov 26 (Tue)
  ↳ Location TBD
  (N100 not available)
  ↳ see course website for sample exam, solutions

Textbook reading:

Ch 14.6 Bode plots
14.7 More Bode plots

0. Review

① Negative feedback for voltage amplifier



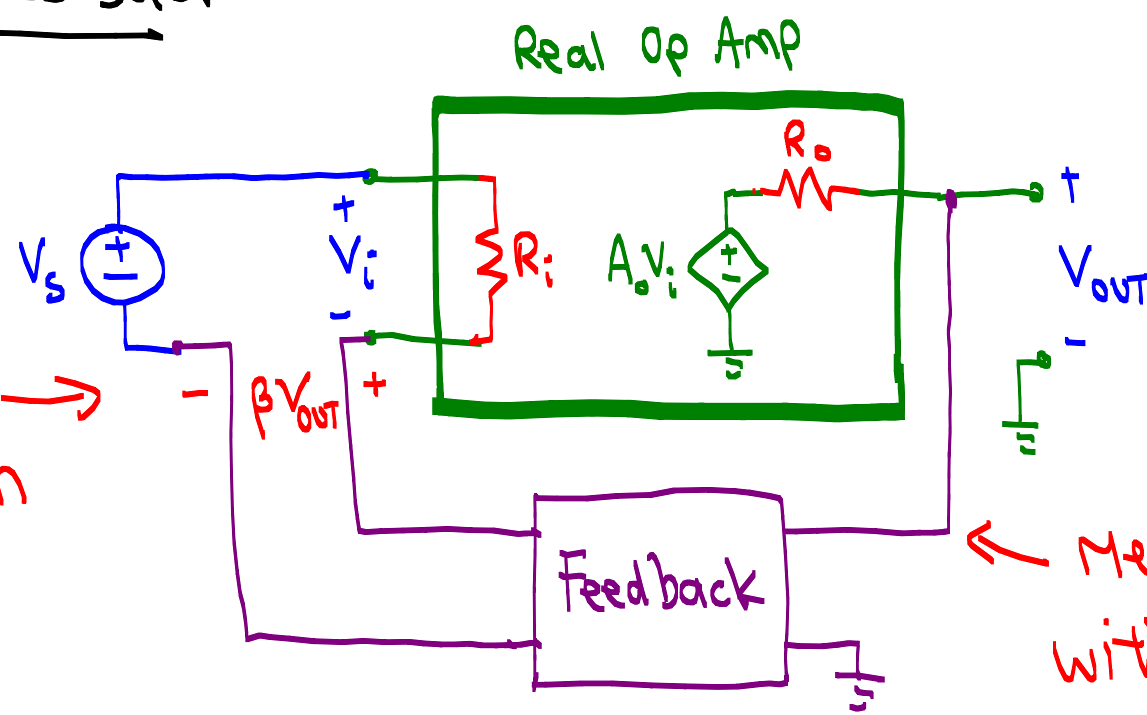
$$V_{OUT} = A_o (V_s - V_F)$$
$$= G\, V_s$$

☆ $G = \dfrac{A_o}{1 + A_o \beta}$

☆ $Z_{IN} = (1 + A_o \beta) R_i$

☆ $Z_{OUT} = \dfrac{1}{(1 + A_o \beta)} R_o$

② "Series-shunt" Feedback

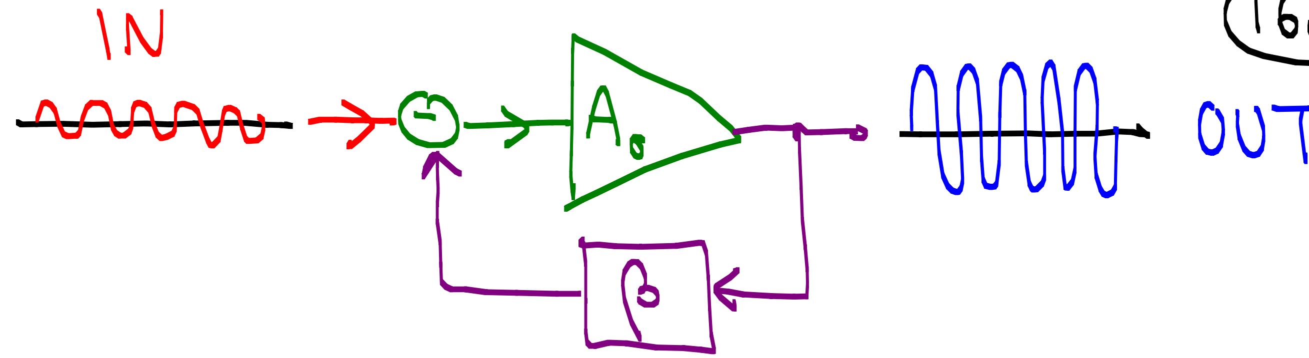


Subtract voltage with series connection →

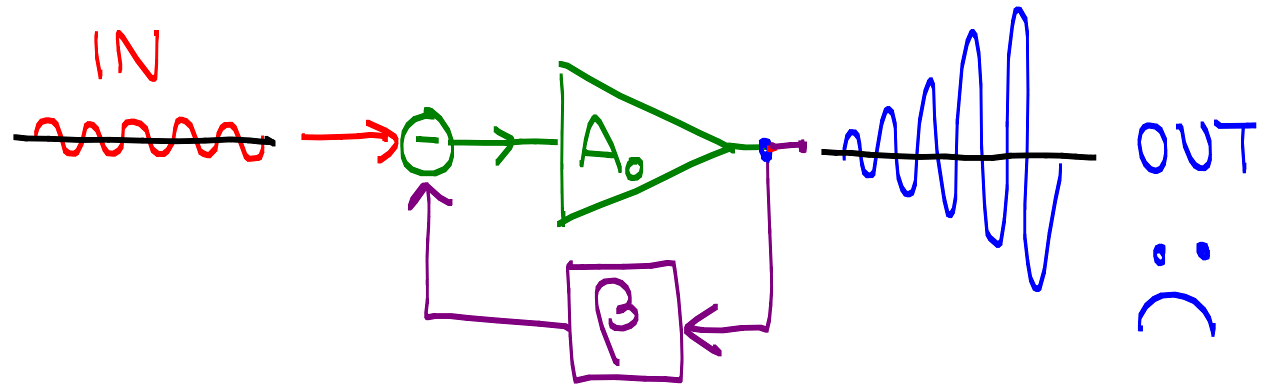
← Measure voltage with shunt connection

1. Frequency Response

- Negative feedback usually improves circuit properties



- HOWEVER, both A and β can depend on freq!



$$G(f) = \frac{A(f)}{1 + \boxed{A(f)\,\beta(f)}}$$

"Loop Gain"

Instability occurs if $A\beta = -1$!

$$\frac{A(f)}{1+(-1)} = \infty!$$

① Magnitude: $|A| = 1/\beta$

② Phase: $\measuredangle(A\beta) = -180°$

BOTH conditions must be met!

☆ We must take a closer look at the <u>frequency response</u> of A!

A. Dominant Pole

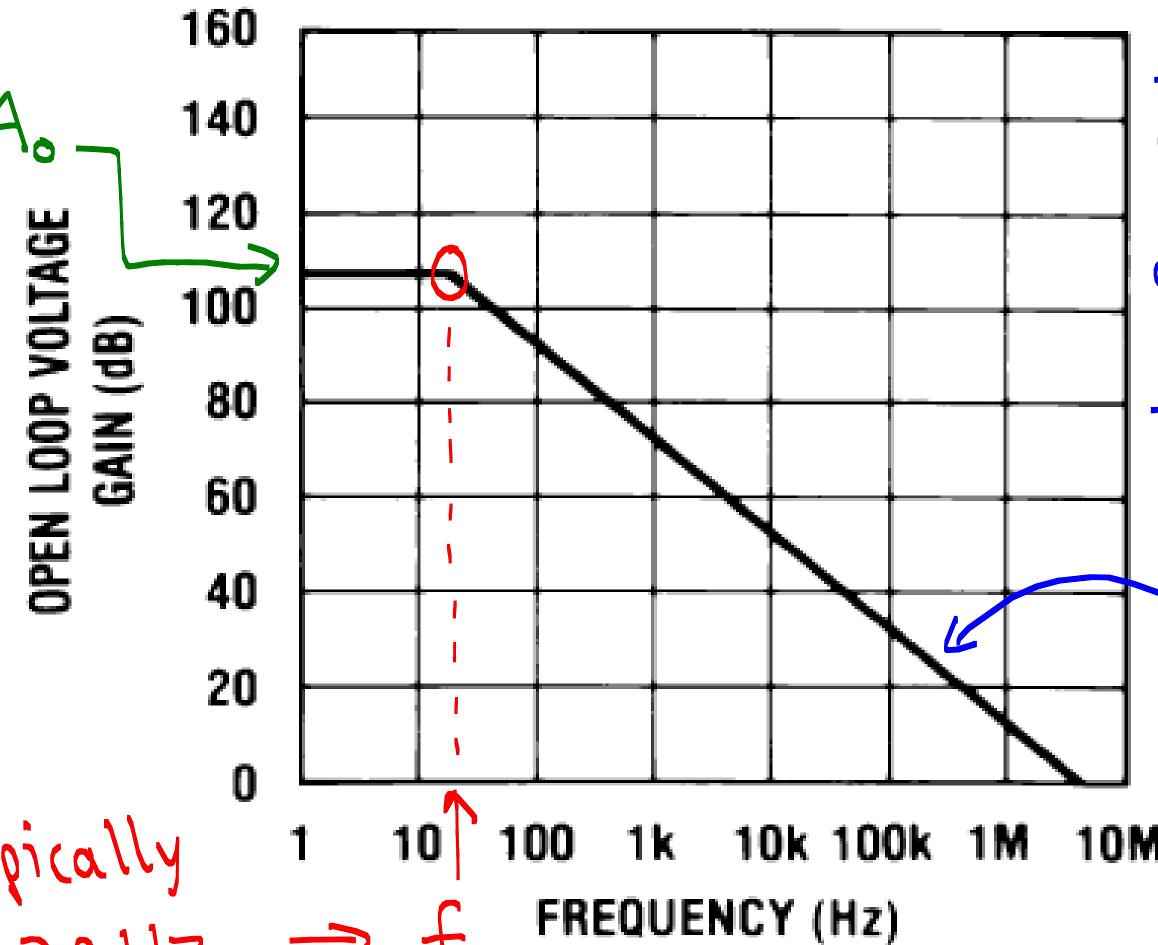
$\boxed{\text{LF411}}$

- For most op amps,
  A is really high (e.g. 10^5)
  only at very low freq!

- A(f) typically resembles
  1st order low-pass filter

Open Loop Frequency Response ← $|A(f)|$

A_o



$\lvert A\rvert$	f
110 dB	10 Hz
90 dB	10^2 Hz
70 dB	10^3 Hz

-20 dB per decade

Typically ~20 Hz → f_p

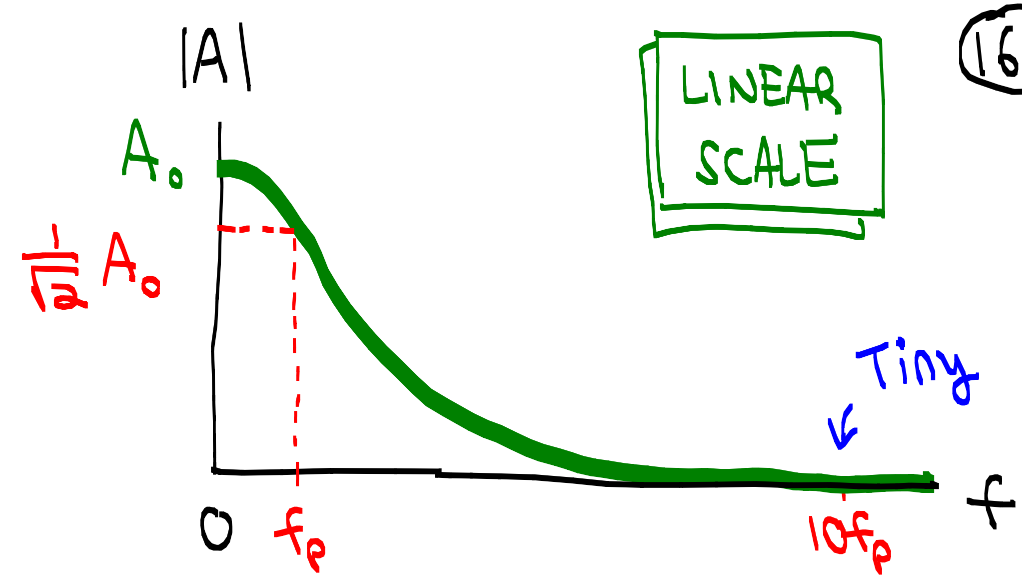
$$A(f) = \frac{A_o}{\boxed{1 + j\,f/f_p}}$$ ← DC gain

$$|A(f)| = \frac{A_o}{\sqrt{1 + (f/f_p)^2}}$$

"Dominant" Pole ← When $|A(f)|$ is 3 dB lower than A_o

B. Bode Plots ← Textbook Ch 14.6 is useful

- Linear scale is NOT very useful
  - → Cannot show large freq range
  - → cannot show both tiny and large $|A|$



- Log-Log scale is most common.

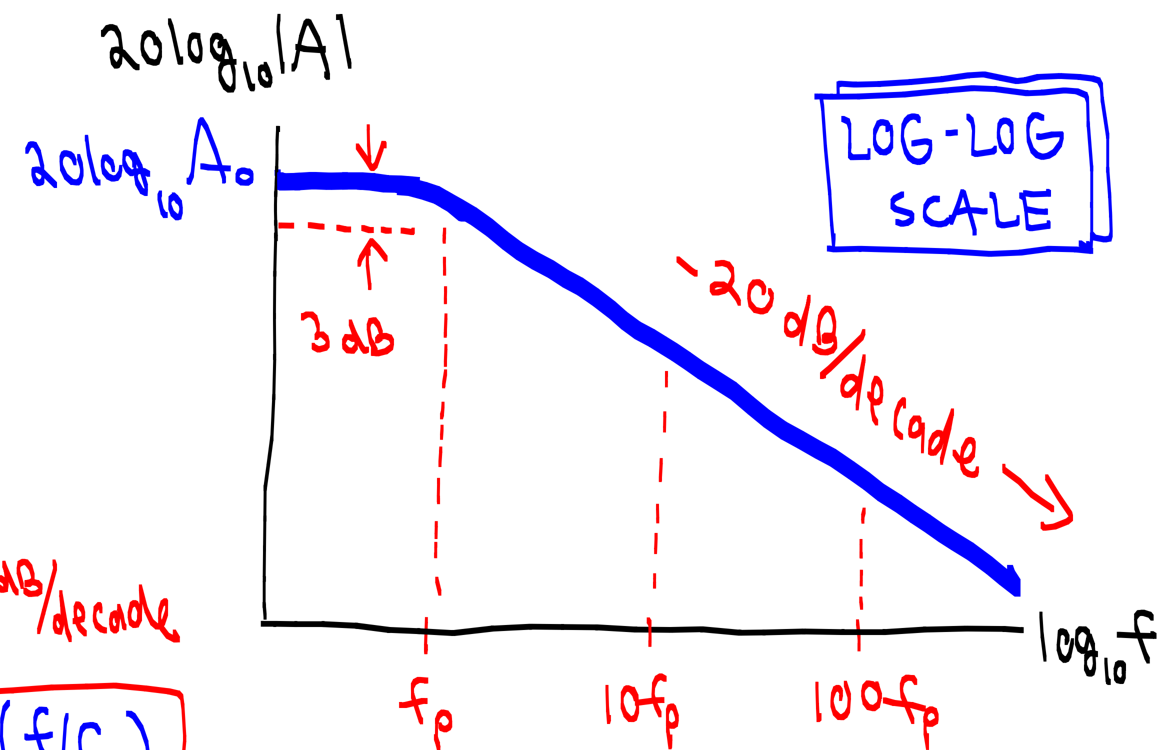
Ex: $|A(f)| = \dfrac{A_0}{\sqrt{1+(f/f_p)^2}}$ $\underbrace{\begin{matrix}\sim 1 & (f \ll f_p) \\ \sim f/f_p & (f \gg f_p)\end{matrix}}$

$$20\log_{10}|A| = 20\log_{10}A_0 - 20\log_{10}\sqrt{1+(f/f_p)^2}$$

$f \ll f_p$: $20\log_{10}|A| \approx 20\log_{10}A_0$

$f \gg f_p$: $20\log_{10}|A| \approx 20\log_{10}A_0 - \boxed{20\log_{10}(f/f_p)}$

-20 dB/decade



- What about phase?

$$A = \frac{A_o}{1+jf/f_p} = \frac{A_o}{\sqrt{1+(f/f_p)^2}\, e^{j\tan^{-1}(f/f_p)}}$$

$$= \underbrace{\frac{A_o}{\sqrt{1+(f/f_p)^2}}}_{\text{Magnitude}}\; e^{j\overbrace{\left[-\tan^{-1}(f/f_p)\right]}^{\text{Phase}}}$$

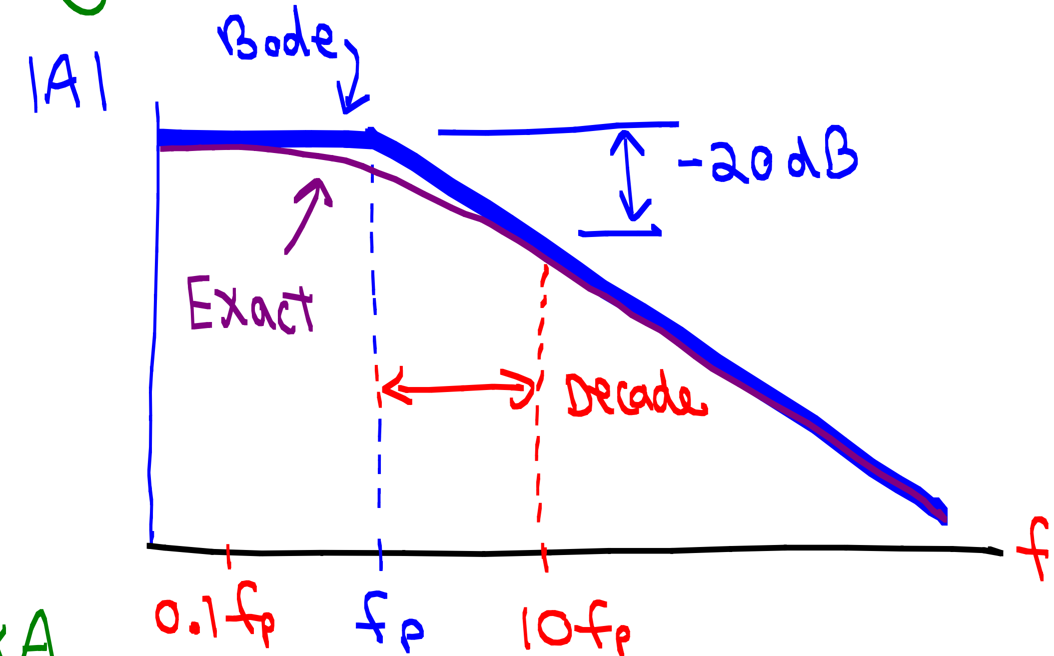
$$\underbrace{-\tan^{-1}(f/f_p)}_{\phi < 0}$$

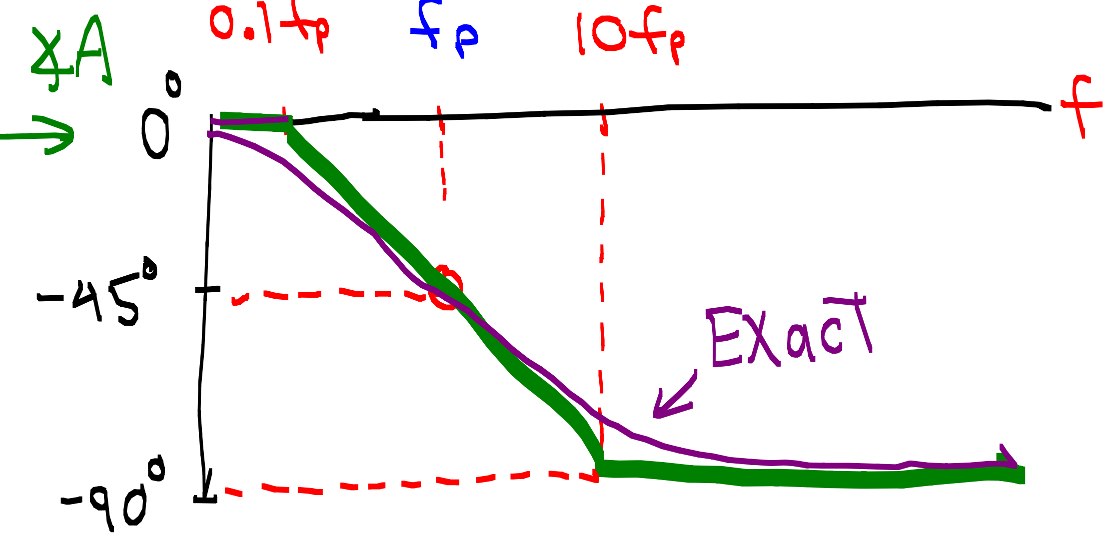
$\phi \approx 0°$, $f \le \frac{1}{10} f_p$

$= -45°$, $f = f_p$

$\approx -90°$, $f \ge 10 f_p$

★ Bode plot is an approximation using straight line segments





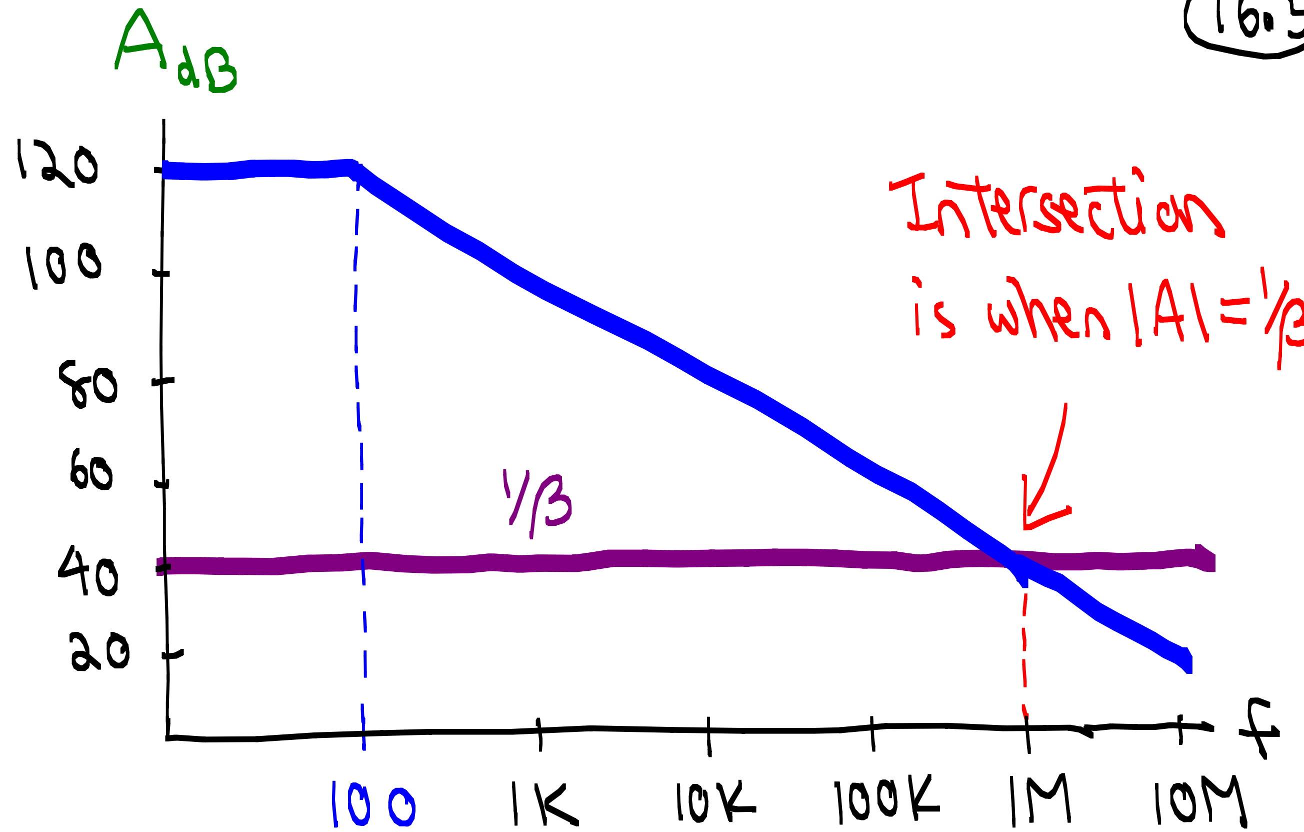
2. Amplifier Stability

Example 1-pole amplifier

Consider an amplifier with $G = 100$

and $A(f) = 10^6 \dfrac{1}{1 + j(f/100)}$.

Is the amplifier stable?



STEP 1 Draw Bode plot for $|A(f)|$

- $20\log_{10} A_o = 120\text{ dB}$ ($f = 0$)
- One pole freq $\rightarrow$ $f_{p1} = 100\text{Hz}$
- -20 dB/decade after 1st pole

STEP 2 Find Ω $\leftarrow$ when $|A| = 1/\beta$

$$G = 100 = \frac{10^6}{1 + 10^6\beta} \Rightarrow \frac{1}{\beta} = 100.01$$

$$20\log_{10}(1/\beta) = \underline{40\text{ dB}} \Rightarrow \boxed{\Omega = 1\text{MHz}}$$

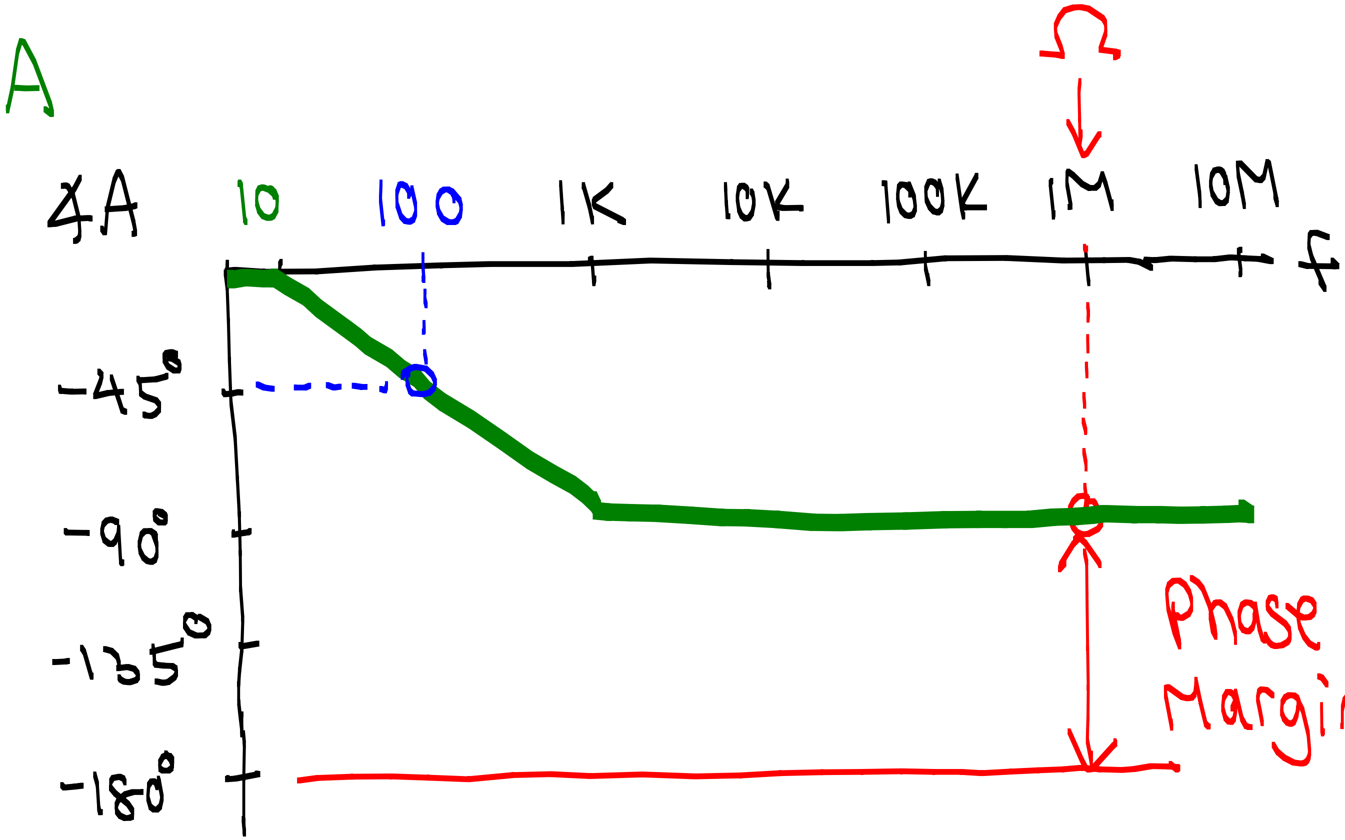
STEP 3 Draw Bode plot for $\measuredangle A$

$$\measuredangle A = -\tan^{-1}(f/100)$$

$\rightarrow$ $\measuredangle A = 0°$ @ $f < \frac{1}{10} f_{p1}$

$\measuredangle A = -45°$ @ f_{p1}

$\measuredangle A = -90°$ @ $f \gtrsim 10 f_{p1}$



STEP 4 Phase Margin

- Bode plot shows quick estimate
- Compute exact phase margin: like "head room"

At $f = \Omega$: $\measuredangle A = -\tan^{-1}\left(\frac{10^6}{10^2}\right) = -89.99°$

Phase margin $= \measuredangle A - (-180°) = \underline{\underline{90.01°}}$

☆ Phase Margin $\geq 45°$ is stable

$\rightarrow$ This amplifier is stable

☆ Any single-pole amplifier is stable!

Example 2-pole amplifier with $G = 100$

$$A = 10^6 \frac{1}{1 + j(f/100)} \cdot \frac{1}{1 + j(f/10K)}$$

STEP 1 Draw Bode plot for $|A|$

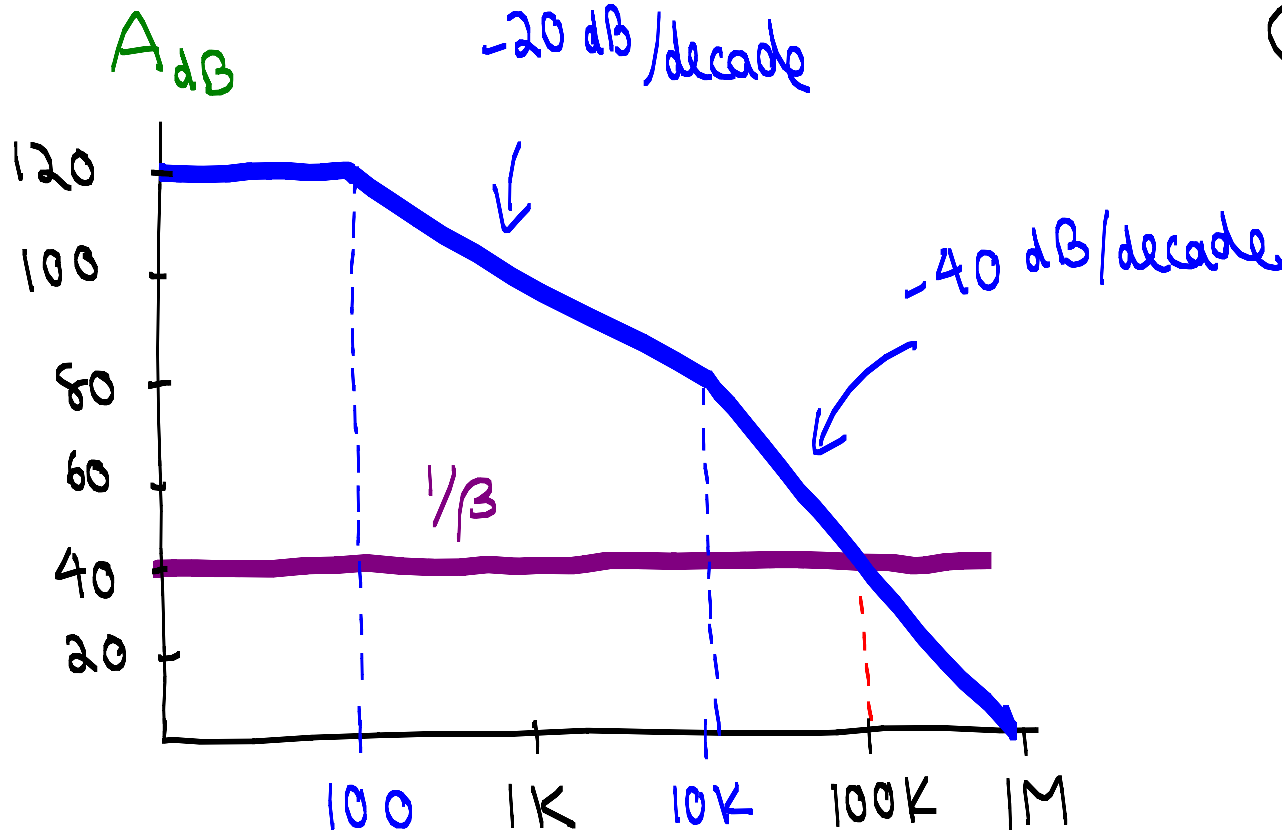
- $20\log_{10}|A_o| = 120\text{ dB}$

  ↑ $f = 0$

- Two pole freq: $f_{p1} = 100\text{ Hz}$

  $f_{p2} = 10\text{ KHz}$

✱ Extra −20 dB/decade after each pole!



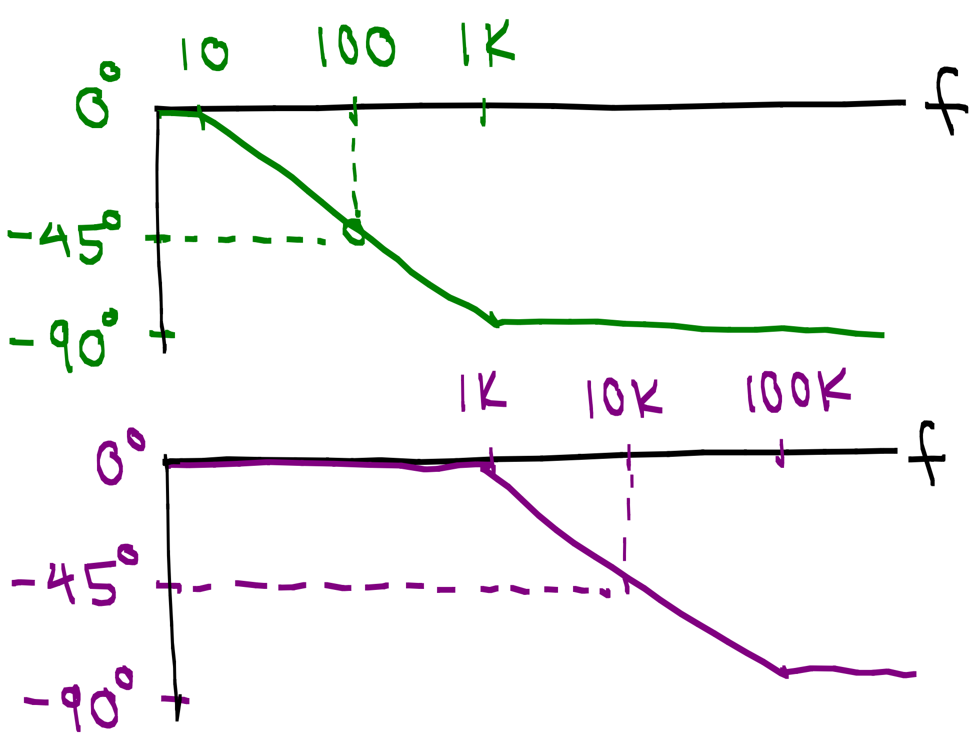
$\underline{\underline{\Omega = 100K}}$

STEP 2 Find Ω

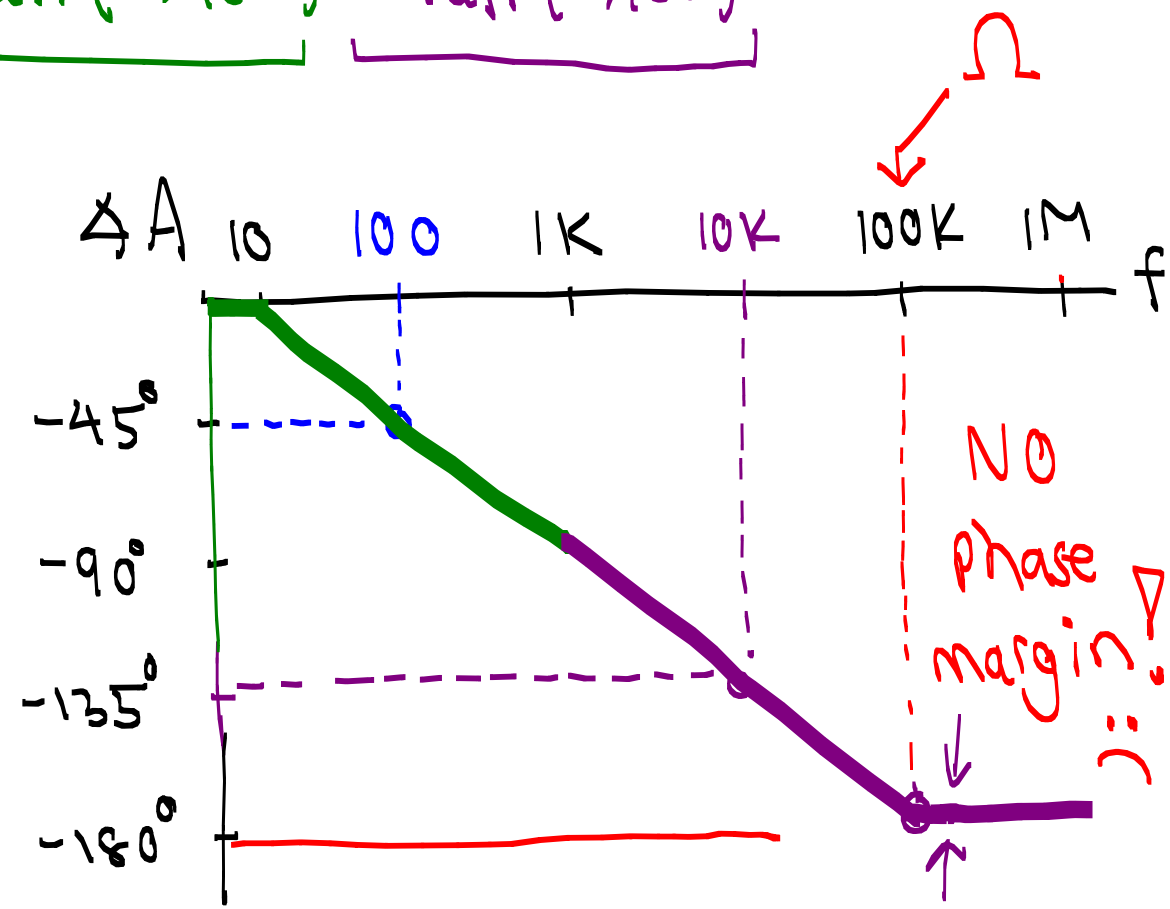
$$G = 100 = \frac{A_o}{1 + A_o\beta} \rightarrow \frac{1}{\beta} = 100.01$$

$$= 40\text{ dB}$$

STEP 3 Draw Bode plot for $\measuredangle A = \underbrace{-\tan^{-1}(f/100)} \underbrace{-\tan^{-1}(f/10K)}$



Add plots together ⇒



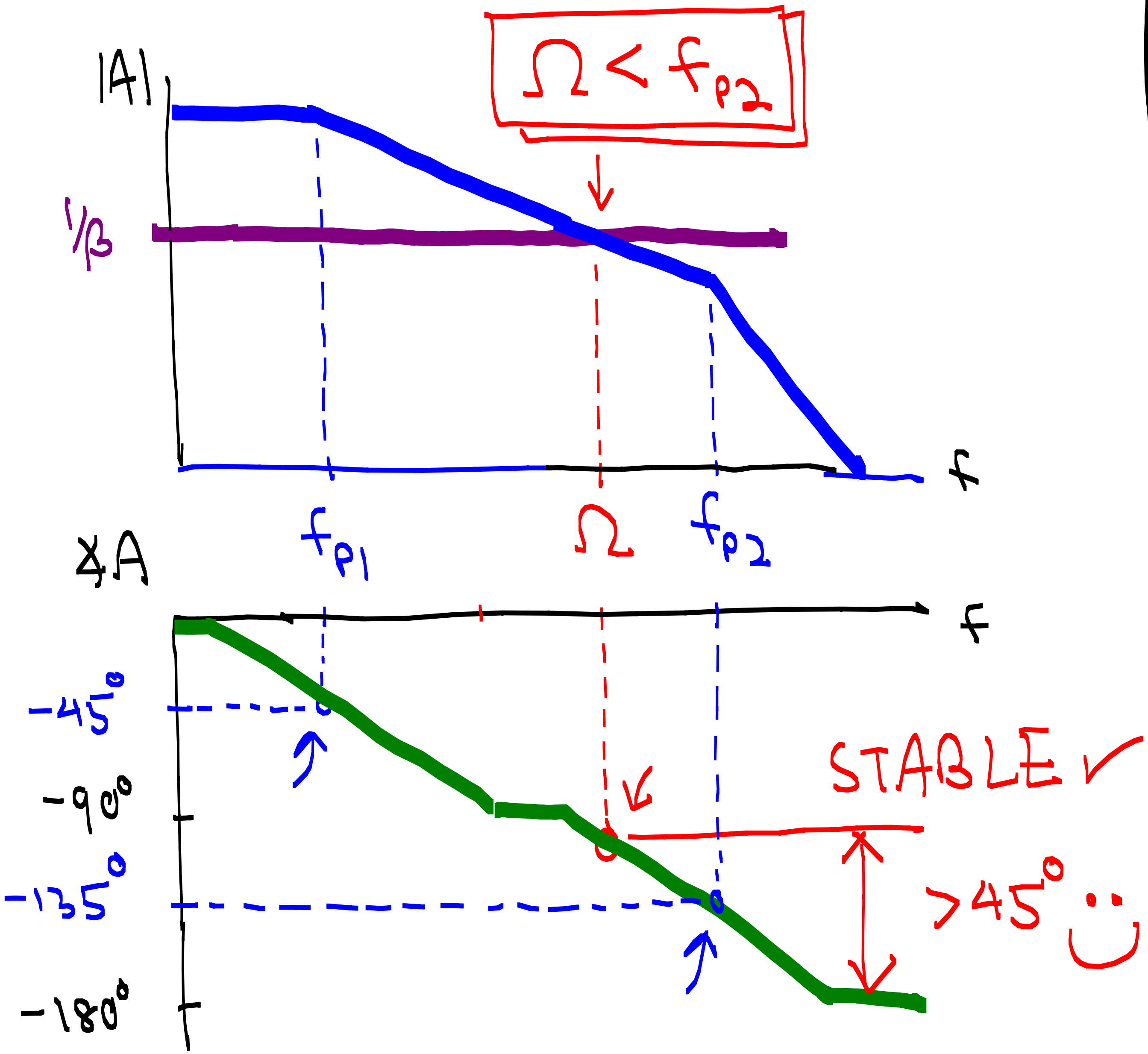
STEP 4 Exact phase margin

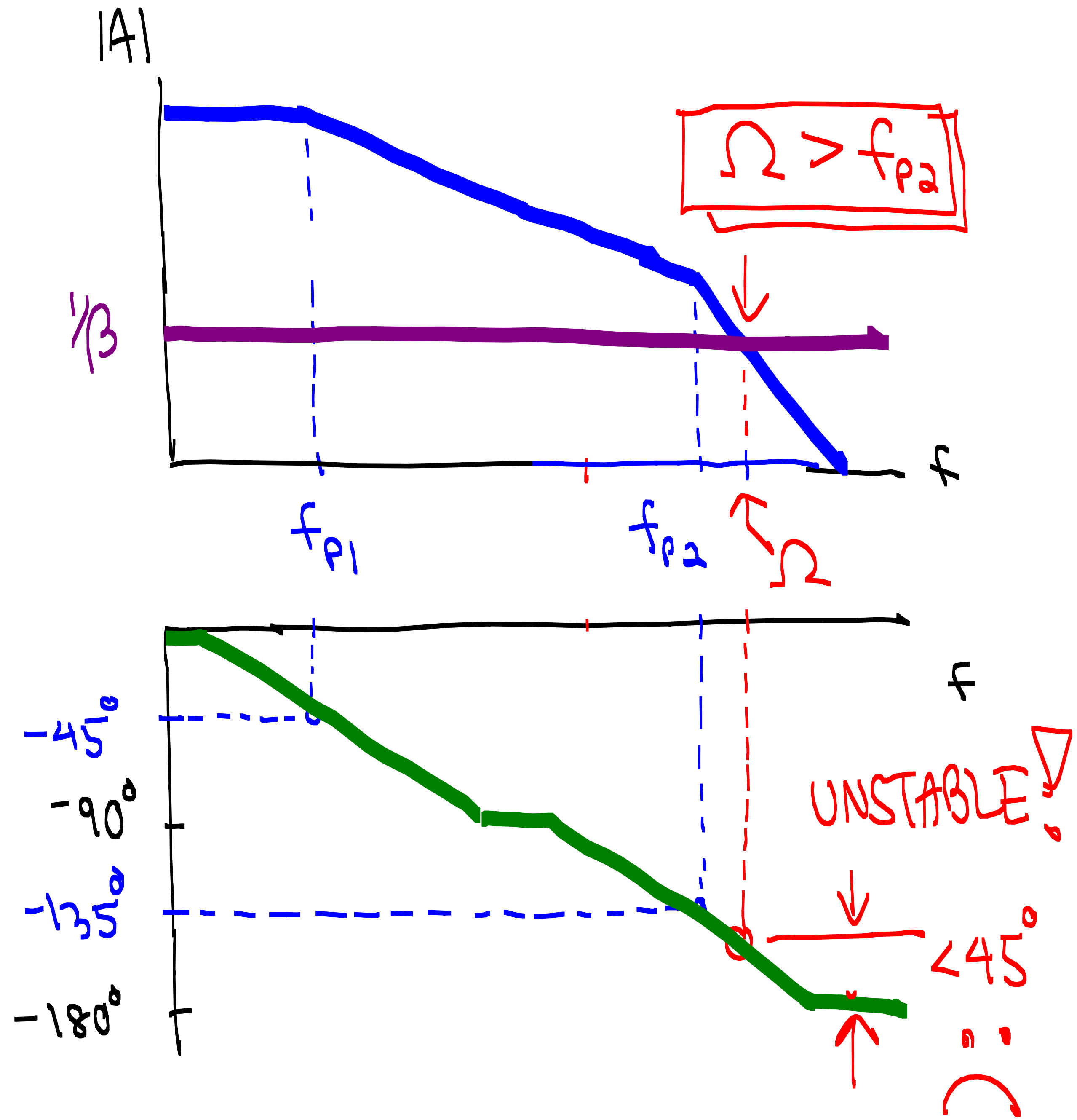
At $f = \Omega$: $\measuredangle A = -\tan^{-1}\left(\frac{10^4}{10^2}\right) - \tan^{-1}\left(\frac{10^4}{10^3}\right) = -89.4^\circ - 84.3^\circ = -173.7^\circ$

Phase margin $= \measuredangle A - (-180^\circ) = \boxed{6.3^\circ} < 45^\circ$ {NOT STABLE! :(

- 2-pole amplifier may or may not be stable!

$$A(f) = A_0 \frac{1}{1+j\,f/f_{p1}} \frac{1}{1+j\,f/f_{p2}}$$ } Depends on Ω vs. f_{p2}





• Same with 3-pole amplifier

$$A_o(f) = A_o \frac{1}{1+j\,f/f_{p1}} \frac{1}{1+j\,f/f_{p2}} \frac{1}{1+j\,f/f_{p3}}$$

$\measuredangle A$	f
-45°	f_{p1}
$\boxed{-135^\circ}$	f_{p2}
-225°	f_{p3}

45° phase margin

★ Amplifier is unstable if $\Omega > f_{p2}$!