The division of labor and the growth of government

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Abstract

This paper develops a dynamic, general equilibrium model of specialization-driven growth in which private coordination costs are decreasing in public expenditure on physical and institutional infrastructure. The model provides an explicitly economic explanation of the secular rise of government. In addition, endogenous specialization decisions imply the existence of four development stages, characterized by distinct outcomes regarding the division of labor, the role of government and the return to capital. Growth is characterized by capital accumulation, market integration, the division of labor and the growth of government. The effectiveness of government plays a central role in determining whether an economy converges to a high or low level equilibrium. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

While it is well known that among industrialized countries government’s share of output has risen dramatically over the past century, this stylized fact, known as Wagner’s law, has not found its way into the formal literature on economic growth. This omission appears to be due both to the ready availability of arguments that attribute the rise of government to political or socio-economic mechanisms¹ and to the tendency

¹For example, Higgs (1987) explains the rise of the US government in terms of a ratchet effect involving periodic crises and subsequent institutional rigidity; Hughes (1993) argues that rising incomes increase the taste for redistributional policies.
of formal, dynamic analysis to focus on the characteristics of the steady state.\textsuperscript{2} In the steady state, government’s share of output is the ratio of two variables with a common growth rate and is, thus, constant by construction.

While steady state analysis correctly predicts that government’s share of output cannot increase without bound, it does little to explain the US and European experience of a nearly century-long rise of government. This paper suggests an explicitly economic rationale for the secular rise of government by considering the transitional dynamics of a model in which public goods facilitate an ever-finer division of labor.

There are a number of reasons public spending might reduce the private cost of coordinating production and exchange activities among specialized economic agents.

First, the public sector supplies a variety of physical and institutional infrastructure that establish the framework within which economic transactions take place and serve to mitigate private transaction costs. Transportation costs, for example, depend upon the provision of roads, bridges and ports; contract enforcement costs depend on the services of public institutions that define and enforce property rights. Increases in the division of labor, and the resulting increase in the number and complexity of economic interactions, tend to raise the return to public infrastructure and thus to government spending.

Second, increases in the gains to specialization tend to increase the incentive for agents to mitigate transportation and communication costs by reducing interpersonal distances. Urbanization in turn increases the intensity of certain external diseconomies of propinquity, examples being traffic congestion and noise pollution, thereby increasing the scope for positive government intervention.

Third, as argued by North (1990), the division of labor results in the substitution of formal for informal rules governing exchange, increasing the scope and complexity of the legal and justice systems. Models of spontaneous cooperation suggest that the division of labor undermines those group characteristics on which informal coordination depends. Other things equal, an increase in the division of labor raises the number of specialist producers whose efforts must be coordinated. Furthermore, by encouraging the growth of towns and cities the division of labor decreases the density of the social network in which exchange takes place, limiting opportunities for retribution on which informal institutions rely. Finally, specialization increases interpersonal heterogeneity, reducing each agent’s knowledge of other players’ preferences and frustrating attempts to arrive at an appropriate set of “sticks and carrots” for informal enforcement mechanisms.\textsuperscript{3}

The relationship between the division of labor and government spending proposed here is of a more limited nature than Wagner’s (1883) original hypothesis regarding the rise of state activity. In broad accord with the arguments above, Wagner did associate

\begin{itemize}
\item See, for example, Uzawa (1965), Shell (1966), Jones and Manuelli (1990), King and Rebelo (1990), Barro (1990), Rebelo (1991), and Glomm and Ravikumar (1994, 1997). Though not explicitly modeled, the possibility of a positive role for government may be inferred from the work of theorists such as Arrow (1962), Lucas (1988) and Romer (1986, 1990): spillovers to certain private investment activities imply the divergence of private and social returns.
\end{itemize}
the rise of government with industrialization and urbanization, citing their impact on the complexity of economic relations and the demand for law and order. However, he also addressed the increase in redistributive activities and state enterprise. Thus while similar in spirit to certain elements of Wagner’s hypothesis, the argument forwarded here suggests a rationale only for the rise of government expenditures on coordination cost-reducing services and not for the whole of government.

This paper builds on the fundamental insights in Smith ([1776] 1976) and Young (1928). In endogenizing labor specialization by considering the trade-off between the gains to specialization and coordination costs, it follows the lead of Yang and Borland (1991), Tamura (1992, 1996, 2001a, 2001b) and Becker and Murphy (1992). In relation to this literature, the primary theoretical innovation proposed here is the assumption that private coordination costs depend on the provision of public goods.

Section 2 develops and solves the static model. Section 3 constructs a simple dynamic model to address the evolution of the economy and the rise of government. Empirical evidence on Wagner’s law, and its relationship to the model developed here, are examined in Section 4. The final section provides concluding remarks.

2. The static model

2.1. Basic equations

The model proceeds from a “nano-economic” foundation in which production is disaggregated to the level of individual productive tasks. In particular, there is a continuum of productive tasks, which are arranged along the unit interval and indexed by \( a \), and a one-to-one relationship between tasks and intermediate goods: performing a task produces a quantity of the intermediate good of the same index number.

There are \( N \) identical individuals. Each is endowed with \( h \) units of capital and one unit of time, which are allocated uniformly across \( n \) tasks. The time and human capital allocated to producing each intermediate good are given by \( \ell_a = 1/n \) and \( h_a = h/n \). An individual’s output of an intermediate good is given by \( y_a = f(\ell_a, h_a) \), where \( f(\cdot) \) conforms to the Inada conditions, is increasing and concave in both arguments, and is homogeneous of degree \( r \). Labor specialization is defined as the inverse of the number of intermediate goods an individual produces, \( s \equiv 1/n \).

Per capita output, \( y \), is found by integrating \( y_a \) over the set of productive tasks: \( y = \int_0^1 y_a \, da = nf(1/n, h/n) = s^{r-1}f(1,h) \), where the last equality follows from Euler’s Theorem. Gains to specialization result from increasing returns in the production of intermediate goods, \( r > 1 \), which is assumed to hold. Hereafter, we express per capita output as a function of capital per head and labor specialization:

\[
y = y(s, h),
\]

where \( y(0,0) = 0, y_s(s, h) > 0, y_h(s, h) > 0, y_{ss}(s, h) < 0, y_{hh}(s, h) < 0, \) and \( y_{hs}(s, h) > 0 \). 

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4 See Bird (1971) for an interpretation of Wagner’s writing on the growth of government and Peacock and Scott (2000) for an evaluation of the recent empirical work in this area.
The derivatives of Eq. (1) imply that per capita output is increasing and concave in both labor specialization and the capital–labor ratio. The positive cross-partial captures Rosen’s (1983) insight that the accumulation of specialized capital goods increases the gains to labor specialization and plays a critical role in generating increasing returns in the dynamic model: increases in specialization raise the return to capital and capital accumulation increases the gains to specialization.

Intermediate goods are complements in the production of a composite good that may be used either for consumption or investment. One unit of the composite good is produced in Leontief fashion by combining one unit of each of the intermediate goods. A worker may produce the full range of intermediate goods, \( n = 1 \), and combine them to produce the composite good. Alternately, she may specialize in the production of a subset of intermediate goods, in which case she must coordinate her production with other specialists in order to produce the composite good.

The strong complementarity embodied in the final good technology requires that every intermediate good be produced within the economy. This places an upper limit on the level of labor specialization: \( s \leq N \). For example, if there are 10 workers, then \( n \geq \frac{1}{10} \) and \( s \leq 10 \). At the lower end, labor specialization is restricted by the number of intermediate goods. As a worker may produce at most the full range of intermediate goods, we have \( n \leq 1 \) and \( s \geq 1 \).

Coordination costs may arise for a variety of reasons. The emphasis in Smith ([1776] 1976) is on transportation costs, which he uses to explain the relative backwardness of inland Asia and Africa and a low division of labor among Scottish farmers, and on legal barriers to exchange, particularly in reference to China. Becker and Murphy (1992) cite the recent work on principal-agent conflicts and free-rider problems. Arrow (1974) and Coase (1991) argue that information costs associated with market transactions rise with increases in the number of transactors. Coase (1991) and Williamson (1975) call attention to the bureaucratic costs of coordinating production within firms, and Williamson (1979) argues that specialized capital goods give rise to opportunistic behavior and, thereby, increase contracting costs between firms. In each of these cases, coordination costs are increasing in the number of specialist producers involved.\(^5\)

A group of specialists who coordinate their production is a team. The number of team members, \( m \), is associated with Smith’s notion of the extent of the market. Positive coordination costs imply that teams consist of agents who produce non-overlapping sets of intermediate goods. Team membership is given by \( m = 1/n = s \). Integer problems regarding the number of agents in a team are ignored.

In addition, private coordination costs depend on publicly provided physical and institutional infrastructure. Following Barro (1990), it is assumed that the prevalence of congestible public goods (highways, courts, and the like) makes per capita government expenditures the appropriate measure of public services available to a given agent.\(^6\)

\(^5\) This definition of coordination costs includes both market transaction costs and intra-firm management costs. Davis (2000) treats these separately, addressing the trade-off between firms and markets in organizing the division of labor.

\(^6\) Note that with a fixed population size, the use of total, rather than per capita, government expenditure would influence the results of the model only through the introduction of a constant.
Defining \( g \) as per capita government spending on coordination cost reducing services, the function governing per capita coordination costs is given by

\[
c = c(s, g),
\]

where \( c_s(s, g) > 0, c_{ss}(s, g) > 0, c_g(s, g) < 0, c_{gg}(s, g) > 0 \) and \( c_{sg}(s, g) < 0 \). The derivatives imply that coordination costs are increasingly increasing in labor specialization, that successive increases in government spending reduce coordination costs but with diminishing effect, and that an increase in labor specialization increases the return to the provision of public goods.

Two additional restrictions are placed on the coordination cost function. First, it is assumed that self-coordination of tasks is costless: \( c(1, g) = 0 \). Second, to capture the notion that the gains to third party intervention are low among small groups, it is assumed that the return to public spending is less than one for economically self-sufficient workers: \(-c_g(1, 0) < 1\).

The public sector maintains a balanced budget at every point in time. The government budget constraint is

\[
g = \tau, \tag{3}
\]

where \( \tau \) is net per capita, lump sum tax revenues.\(^7\)

Workers choose labor specialization and government spending to maximize net per capita income, defined as per capita income less taxes and private coordination costs, taking the capital–labor ratio as given.\(^8\) Employing (3), the representative agent’s optimization problem is given by

\[
\max_{1 \leq s \leq N, g \geq 0} z(s, g, h) = y(s, h) - g - c(s, g). \tag{4}
\]

The Kuhn–Tucker conditions for maximization with respect to \( s \) and \( g \) are, respectively,

\[
s \geq 1, y_s - c_s \leq 0 \quad \text{and} \quad (s - 1)(y_s - c_s) = 0 \tag{5a}
\]

or

\[
s \leq N, y_s - c_s \geq 0 \quad \text{and} \quad (s - N)(y_s - c_s) = 0 \tag{5b}
\]

and

\[
g \geq 0, -1 - c_g \leq 0 \quad \text{and} \quad g(-1 - c_g) = 0. \tag{6}
\]

Let the AA curve, shown in Fig. 1, denote the locus of points \((s, g)\) satisfying (5). For values of \( g \) for which the lower constraint on labor specialization is binding, the AA schedule consists of a vertical line at \( s = 1 \). If the upper constraint on \( s \) is binding, the AA schedule is given by \( s = N \). For intermediate values of \( g \), agents choose \( s \) by

\(^7\) Clearly the use of lump-sum taxes is a departure from reality. They are employed here in a effort to focus more narrowly on the primary subject of the paper, the interaction of labor specialization and government spending. For investigations of the relationship between tax structure and growth, see Barro (1990), Lucas (1990) and Stokey and Rebelo (1995).

\(^8\) The first-order conditions are identical if government spending is chosen by a social planner to maximize net per capita income subject to worker specialization decisions.
equating the marginal gains to specialization and the marginal cost of coordinating the division of labor: \( y_s = c_s \).

Differentiating \( \delta z/\delta s = 0 \) with respect to \( s \) and \( g \) indicates that for \( 1 < s < N \) the slope of the AA schedule is given by

\[
\frac{dg}{ds}|_A = \frac{y_{ss} - c_{ss}}{c_{sg}} > 0,
\]

(7)

where the sign follows from (1) and (2). Intuitively, an increase in government spending decreases the marginal costs of specialization, inducing workers to become more specialized.
The BB curve is the locus of points \((s, g)\) satisfying (6). If unconstrained, government spending is optimal provided \(-c_g(s, g) = 1\). That is, the reduction in coordination costs due to an additional dollar of government spending equals the marginal cost of taxation. At low levels of the division of labor, the return to government spending is less than the cost of taxation, and optimal government spending is zero. In particular, optimal government spending is zero for \(s \leq s_0\), which is defined implicitly by \(-c_g(s_0, 0) = 1\).

The slope of the BB schedule is found by differentiating \(\delta z/\delta g = 0\) with respect to \(g\) and \(s\), implying

\[
\frac{dg}{ds}|_B = \begin{cases} 
0, & \text{for } s < s_0, \\
-c_{sg}/c_{gg} > 0, & \text{for } s > s_0,
\end{cases}
\]

where the sign follows from (2). On the upward sloping portion of BB, an increase in specialization raises the impact of marginal public spending on coordination costs to greater than one. An increase in government spending is required to re-equate the marginal gains to government spending and the marginal cost of taxation.

### 2.2. Existence and stability of equilibria

If the AA and BB curves do not intersect for \(s \in (1, N)\), the model produces a stable, boundary equilibrium at either \((1, 0)\) or \((N, g(N))\). An intersection of the two loci, such as indicated in Fig. 1 at \((s^*, g^*)\), is a local maximum provided the second order conditions of the representative agent’s optimization problem are met. These are:

\[
\begin{align*}
\frac{\delta^2 z}{\delta s^2} &= y_{ss} - c_{ss} < 0, \\
\frac{\delta^2 z}{\delta g^2} &= -c_{gg} < 0, \\
(\frac{\delta^2 z}{\delta s^2})(\frac{\delta^2 z}{\delta g^2}) - (\frac{\delta^2 z}{\delta s \delta g})^2 &= -c_{gg}c_{sg}[dg/ds|_A - dg/ds|_B] > 0.
\end{align*}
\]

(9a) and (9b) follow from Eqs. (1) and (2). (9c) is satisfied provided AA is steeper than BB at the point of intersection.

There is nothing in the analysis that precludes the existence of multiple equilibria, which will occur if the AA and BB schedules intersect more than once. Statements about the curvature of AA and BB, however, depend upon the magnitude of third order derivatives of the objective function, about which it is difficult to make reliable assumptions. To simplify the discussion of the model, it is assumed hereafter that AA is steeper than BB for all \(s \in (1, N)\), implying the existence of a unique stable equilibrium (interior or otherwise).

### 2.3. Endogenous organizational stages

The location of an equilibrium on the BB curve determines the economy’s stage of development. Successive stages are associated with higher capital–labor ratios and are characterized by greater organizational complexity. As defined below, the stages are interpersonal autarky, the traditional economy, the industrializing economy and the mature economy.
As illustrated in Fig. 2, increases in the capital–labor ratio raise the gains to specialization and shift the AA curve to the right, so that the equilibrium moves from left to right along the BB curve. Since the BB curve is weakly monotonic, we may define equilibrium government spending as a function of specialization: \( g^* = g^*(s) \), such that \([s, g^*(s)] \in BB\). Using this function, we define \( h_0, h_1 \) and \( h_2 \) as follows

\[
\begin{align*}
    h_0 &: y_s(1, h_0) = c_s(1, 0), \\
    h_1 &: y_s(s_0, h_1) = c_s(s_0, 0), \\
    h_2 &: y_s(N, h_2) = c_s(N, g^*(N)).
\end{align*}
\]

These values of \( h \) mark the transitions between development stages. They are, respectively, the highest value of \( h \) for which labor is unspecialized, the highest value of \( h \) for which government spending is zero, and the lowest value of \( h \) for which the economy consist of a single market.

For \( h < h_0 \) the gains to specialization are less than marginal coordination costs, implying an equilibrium at \((s^*, g^*) = (1, 0)\). This equilibrium corresponds to interpersonal autarchy: each agent produces the full range of intermediate goods, consumes her own output and is, thus, economically independent. In particular, this stage is marked by the absence of government and interpersonal trade.

\( h_0 \) indicates the threshold at which labor specialization, the interpersonal coordination of production and market formation begin. For \( h \in [h_0, h_1] \), the equilibrium lies on the flat portion of BB. These equilibria are characterized by the existence of many small teams, low levels of the division of labor and the absence of government, teams being sufficiently small for coordination to rely on informal enforcement mechanisms. The
economic fragmentation and absence of government that characterize these equilibria suggests that they correspond to a *traditional economy*.

For \( h \in (h_1, h_2) \), the equilibrium lies on the upward sloping portion of the BB curve. These equilibria correspond to higher levels of labor specialization, larger markets and positive government spending, with \( h_1 \) marking the threshold for government formation. Along this portion of the BB curve, capital accumulation increases labor specialization, market size and per capita government spending. These characteristics and the possibility of increasing marginal returns to capital (see next section) suggest that these equilibria correspond to an *industrializing economy*.

Finally, for \( h > h_2 \), the AA curve lies below the BB curve for \( s < N \). Equilibrium occurs at \((s^*, g^*) = (N, g^*(N))\). The economy consists of a single team incorporating the entire population of \( N \) individuals, each of whom performs \( 1/N \)th of the productive tasks. This equilibrium is interpreted as corresponding to an organizationally *mature economy*. A mature economy differs from an industrializing economy in that the gains available from the expansion and integration of domestic markets have been exhausted. In a mature economy the division of labor is constrained by population size, and government spending is constant at a level consistent with facilitating an \( N \)-person division of labor.

As noted above, comparative statics of capital accumulation differ across development stages. In particular, we have

\[
\begin{align*}
\frac{ds^*}{dh} &= \begin{cases} 
0, & \text{for } h \in [0, h_0) \\
\frac{-y_{sh} - c_{ss}}{y_{ss} - c_{ss}} > 0, & \text{for } h \in [h_0, h_1) \\
\frac{-c_{ag} y_{sh}}{A} > 0, & \text{for } h \in [h_1, h_2) \\
0, & \text{for } h \geq h_2
\end{cases}
\end{align*}
\]

and

\[
\begin{align*}
\frac{dg^*}{dh} &= \begin{cases} 
\frac{c_{ag} y_{sh}}{A} > 0, & \text{for } h \in [h_1, h_2) \\
0, & \text{otherwise}
\end{cases}
\end{align*}
\]

where \( A = (y_{ss} - c_{ss})c_{ag} + c_{sg}^2 < 0 \).

In traditional and industrializing economies, labor specialization is unconstrained. An increase in the capital–labor ratio increases the gains to specialization and, thus, equilibrium labor specialization. As indicated by the first and fourth lines of (11), in autarkic and mature economies labor specialization is unresponsive to increases in the capital–labor ratio. Either the gains to specialization are too low for an incremental increase in the capital–labor ratio to induce specialization or labor specialization is constrained by population size.

Comparing the second and third lines of (11), note that the effect of capital accumulation on labor specialization is larger in industrializing than traditional economies. This reflects the ability of industrializing economies to offset rising coordination costs through an increase government spending. Note also that in industrializing economies

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9 See Appendix A for derivation.
this effect will be larger the more sensitive coordination costs are to government spending; that is, the larger is \(|c_{sg}|\).

Rearranging terms in (12), we see that for an industrializing economy the increase in government spending is given by \(dg^*/dh = (dg/ds)_{B}(ds^*/dh)\). Both terms are increasing \(|c_{sg}|\). Thus, the greater the effectiveness of government spending in reducing marginal coordination costs, the more an increase in the capital–labor ratio impacts the level of government spending.

3. The dynamic model

It is the possibility of increasing marginal returns to capital that makes the static model developed above interesting from a growth theoretic perspective. As is well known from the Solow growth model, given diminishing returns to reproducible factors accumulation driven growth is self-limiting. To generate sustained growth, theorists have argued that the properties of knowledge or human capital make the assumption of diminishing returns inappropriate (for example, Romer, 1986, 1990; Lucas, 1988). In contrast, here increasing returns result from the endogenous evolution of economic organization, despite diminishing returns to capital in production. In particular, growth proceeds by the mutually reinforcing processes of accumulation and specialization.

The potential for a virtuous cycle of accumulation and specialization varies across development stages. With specialization constant in autarkic and mature economies, these stages exhibit diminishing marginal returns and support stationary equilibria. In the intermediate stages, however, specialization increases with accumulation, and these stages may exhibit increasing marginal returns, with its attendant implications for threshold effects and multiple equilibria. We begin by considering the interaction of labor specialization, public spending and the return to capital, followed by analysis of a simple dynamic model and its implications for the growth of government.

3.1. Specialization, government spending and increasing returns

Defining equilibrium labor specialization and government spending as functions of the capital–labor ratio, \(s^*(h)\) and \(g^*(h)\), net per capita income can be represented in reduced form:

\[
z(h) = z[s^*(h), g^*(h), h].
\]

Differentiating (13) twice with respect to \(h\), it follows that \(z(h)\) exhibits increasing marginal returns to capital provided

\[
\left(\frac{ds^*}{dh}\right) y_{sh}(s^*, h) > -y_{hh}(s^* h).
\]

The right hand side of (14) measures the strength of the direct effect of accumulation, diminishing returns to capital in production. The left hand side measures the strength of an indirect effect due to the impact of capital accumulation on economic organization.

Here, the first term second shows the effect of capital accumulation on equilibrium specialization derived in Eq. (11). The second term shows the effect of specialization
on the return to capital: an increase in labor specialization implies greater utilization of task-specific human capital and, thus, an increase in the return to capital. Thus, the indirect effect captures the increase in the return to capital due to the induced increase in labor specialization.

An economy exhibits increasing marginal returns to capital provided the interaction between capital accumulation and labor specialization, is strong relative to the direct effect. In autarchic and mature economies, equilibrium specialization is constant, implying that the left hand side of (14) equals zero and that these stages of development exhibit diminishing marginal returns to capital.

In traditional and industrializing economies, however, equilibrium specialization is increasing in the capital–labor ratio, implying the possibility of increasing returns. From (11), the induced increase in labor specialization is given by

\[
d s^*/d h = \left[ \frac{y_{sh}}{\epsilon_{ss} - y_{ss}} \right] \left[ 1 - \frac{d g/d s|_B}{d g/d s|_A} \right]^{-1}.
\]

Here, the first term gives the horizontal shift in the AA curve. The second term captures a multiplier effect reflecting the additional increase in specialization fostered by the rise in government spending.

Taken together, Eqs. (14) and (15) reveal a role for effective government in generating increasing returns. In a traditional economy, government spending is constrained at zero, so \(d g/d s|_B = 0\) and the change in equilibrium specialization is limited to the first term of (15). In an industrializing economy, induced labor specialization and thus the indirect effect of accumulation are larger the greater the effectiveness of government spending in reducing the marginal cost of specialization: \(d g/d s|_B = -\epsilon_{sg}/\epsilon_{gg}\).

By extension, the realization of increasing marginal returns may be undermined by inefficiencies in the provision of public goods. This might be the case, for example, if the public sector is corrupt or lacking in skilled officials or, as Olson (1982) suggested, if the allocation of public spending is distorted by the influence of narrow private interests.

While an economy may experience increasing return in either of the intermediate stages of development, below attention is restricted to the case in which they occur only during industrialization. It follows, as illustrated in Fig. 3 that the marginal product of
capital, \( r(h) = \frac{dz(h)}{dh} \), is decreasing in \( h \) during the autarchic and traditional stages of development, increasing in \( h \) during industrialization, and decreasing again for mature economies. \( r(h) \) has a local minimum at \( r_1 = r(h_1) \), marking the transition between traditional and industrializing stages, and a local maximum at \( r_2 = r(h_2) \), the transition from an industrializing to a mature economy.

### 3.2. Dynamic optimization

Because we are interested in the roles of accumulation and endogenous organizational change in the generation of economic growth, the model abstracts from other dynamic factors, including population growth and technical progress.\(^{10}\) The dynamic model uses a discrete-time framework in which the variables of the preceding section are interpreted as functions of time. In addition, we assume that specialization is separable over time, so that we may use the reduced-form equation for per capita net output, (13), generated above. Agents maximize an infinite horizon utility function with logarithmic instantaneous utility

\[
U = \sum_{t=0}^{\infty} B^{-t} \ln (c(t)),
\]  

(16)

where \( B > 1 \), subject to the initial level of the capital–labor ratio, \( h(0) \), and an accumulation equation with full-depreciation of capital\(^{11}\)

\[
h(t + 1) = z(h(t)) - c(t),
\]  

(17)

resulting in the first-order condition

\[
1 + g(t) = c(t + 1)/c(t) = \frac{r(h(t + 1))}{B}.
\]  

(18)

### 3.3. Steady state equilibria

The model produces stationary equilibria for values of \( h \) for which \( r(h) = B \). As the return to capital function is piecewise defined, there are up to three stationary equilibria, one during the first two stages of development and one in each of the last two. The realization a particular equilibrium depends on the initial level of the capital–labor ratio and the position of the return to capital function relative to the discount rate, \( B \), as summarized in following proposition:

**Proposition 1.**

A. If \( r(h_2) < B \), the economy converges to a low level equilibrium at \( h_L < h_1 \) in the autarchic or traditional stage of development.

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\(^{10}\) See Davis (2001) for a division of labor growth model with population growth.

\(^{11}\) With multiple dynamic equilibria, the use of a discrete time framework with full depreciation of capital each period sharpens the analysis by allowing an analytic solution. These assumptions may be justified intuitively if each period is interpreted as a separate generation and one ignores intergenerational human capital spillovers.
B. If \( r(h_1) > B \), the economy converges to a high-level equilibrium at \( h_H > h_2 \), in the mature stage of development.

C. If \( r(h_1) < B < r(h_2) \), the model supports stable equilibria at \( h_L < h_1 \) and \( h_H > h_2 \) and an unstable equilibrium at \( h_M \in [h_1, h_2] \). \( h_M \) also serves as a threshold level of the capital–labor ratio, marking the boundary between the basins of attraction for the stable equilibria.

These equilibria are illustrated in Fig. 4, with arrows indicating the direction of motion of the capital–labor ratio.

Proposition 1 may be interpreted as indicating a central role for history and preferences, notably the discount rate, in determining which equilibrium obtains. However, attention may be refocused on effectiveness government by considering its role in determining the shape of \( r(h) \). In an industrializing economy an increase in government efficiency, \( |c_{sg}| \), increases the slope of \( r(h) \) for \( h \in (h_1, h_2) \). This decreases \( h_M \) and thus increases the range of values of \( h(0) \) for which an economy converges to the high level equilibrium. By corollary, a change in government structure or policy that reduces its effectiveness in offsetting private coordination costs may cause a growing economy to reverse its course, leading to market disintegration, de-specialization and falling levels of capital and output per worker as it converges to the low level equilibrium.

3.4. On the growth of government

As noted in the introduction, government’s share of output will be constant in the steady state. During industrialization, however, an economy converging to the high level equilibrium at \( h_H \) will devote ever greater resources to public goods in order to facilitate the coordination of the division of labor. For government’s share of output to rise during this stage of development, the elasticity of government spending with respect to \( h \) must be greater than that of output: \( \eta(g, h) > \eta(y, h) \), where \( \eta(f, x) \) is the elasticity of \( f \) with respect to \( x \). Assuming a Cobb–Douglas production function, this condition reduces to

\[
\eta[g^*(h), h] > 1, \tag{19}
\]

implying that government’s share of spending rises provided an increase in the capital–labor ratio induces a more-than-proportionate increase in government spending. This condition will tend to be satisfied if the impact of government spending on marginal coordination costs, \( |c_{sg}| \), is large.\(^{12}\)

The rise of government, however, is necessarily a temporary phenomenon. Once the division of labor has expanded to the level of the population, additional accumulation increases income but not government spending and government’s share of output will fall as the economy approaches the high level steady state. The empirical relevance of this outcome is discussed below.

The association between effective governance and secular increases in government’s share of output should not be taken to imply that government size is necessarily an

\(^{12}\) See Appendix B.
Fig. 4. (a) Stationary state for $r(h_2) < B$. (b) Stationary state for $r(h_1) > B$. (c) Stationary states for $r(h_1) < B < r(h_2)$. 
indication of government efficiency. In restricting attention to the socially optimal level of government spending, the possibility of large, inefficient government is precluded from the analysis.

4. Empirical evidence on the rise of government

Loosely interpreted, Wagner’s law is supported by the evidence. The rise of government in industrialized countries, as evidenced by increases in its share of national product, is strikingly apparent in the data. In the US, government’s share of output rose from 7.3% in 1870 to 32% in 1996. Averaging across 14 OECD countries government spending rose from 10.8% to 45% over the same time period (Tanzi and Schuknecht, 2000).

In spite of the strength of this trend, econometric tests of Wagner’s law deliver decidedly mixed results. While older studies, relying on OLS methods, tended to find in its favor, more recent work has argued that because the data under investigation are non-stationary, OLS may lead to spurious correlations. Employing time series methods, these studies tend to reject Wagner’s law. Others, for example Ram (1992) and Peacock and Scott (2000), reject the short run elasticities estimated in time series studies, holding that Wagner’s law refers to a long run relationship. The evidence is further muddied by debate over which of several competing empirical models is appropriate (Mann, 1980).

As noted in the introduction, the argument forwarded here is of a more limited nature than Wagner’s original hypothesis, relating as it does to coordination cost-reducing services and not to the whole of government spending. There is some indirect evidence in support for our more limited hypothesis. First, a number of studies have found that urbanization is significantly related to the growth of per capita government spending in national or US state economies. There is also support for the model’s prediction of distinct stages in the responsiveness of government spending to rising per capita incomes. For example, Abizadeh and Gray (1985) find that in the post-war years Wagner’s law holds for developing but not underdeveloped or developed countries, and Mann (1980) finds that government spending rose more quickly in Mexico following its “take-off”.

The model’s prediction that government’s share of spending falls in organizationally mature economies is less appealing. Instead, the evidence suggests that it levels off. For example, since 1980 government’s share of output has been around 32% for the US and 42% for the UK (Tanzi and Schuknecht, 2000). It may well be that the stylized nature of the model over simplifies the distinction between industrializing and mature economies and that a model in which the scope for additional specialization fell gradually, rather than suddenly being exhausted, would fare better in this regard. Alternately, at some level of development factors not addressed by the model, for

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13 See, for example, Bairam (1992), Bohl (1996) and Henreckson (1993).
example changes in ideology, technology or institutions, may matter more than the
division of labor in determining government spending.

Wallis and North (1986) provide evidence more directly relevant to the relationship
between government spending and the division of labor, by estimating the share of
US government spending that directly or indirectly reduces private coordination costs.
Spending on domestic property rights enforcement, which is taken as directly related
to the division of labor, rose from 1.54% to 2.30% of GNP from 1902 to 1970.
Indirect expenditures include transportation and urban services, which rose from 1.36%
to 3.92% over the same period. While not conclusive, these trends are consistent
with a role for government in facilitating an expanding division of labor.

5. Conclusion

The principle aim of this paper has been to develop a formal model of economic
growth capable of explaining the secular rise of government. In doing so, it departs
from the familiar assumptions that markets function costlessly and that the institutional
context within which market transactions occur may be taken as given and relegated
to the analytical background.

Instead, the model assigns a central place to market and other coordination costs
and to the role that government plays as the provider of the physical and institutional
infrastructure necessary to coordinate economic activity involving an ever-finer division
of labor. Thus, the economic role of government is placed in the foreground, with
optimal public spending at each point in time determined endogenously, along with
labor specialization, by the structure of the production and coordination cost functions
and the capital–labor ratio.

In the dynamic model, gains to specialization give rise to increasing returns, implying
the potential for multiple equilibria. More effective provision of public goods is
associated with convergence to the high rather than low level equilibrium, providing
an argument for the role of effective governance in generating economic growth.

The model also suggests an association between effective governance and the secular
rise of government’s share of output. In terms of the formal characteristics of the
model, this owes to the assumption of social optimality in public spending decisions.
By implication, if the rising share of government spending in mature economies is
related to their declining rate of growth, the explanation should be sought in the theory
of non-optimal government behavior.

The approach used here employs much of the conceptual framework of neoinsti-
tutionalist economics. Both are primarily concerned with issues of complexity and
coordination in economic life, employ a “nano-economic” analytical framework which
takes the individual economic transaction as the fundamental unit of analysis, and posit

15 See Wallis and North (1986, Tables 3.8–3.10, pp. 116–117). Unlike North and Wallis, I exclude from
property rights enforcement military spending, which arguably depends on variables unrelated to the domestic
division of labor. Similarly, I do not include education in social overhead capital, though Wallis and North
argue that it plays a critical role in socialization regarding the legitimization of contracts.
a central role for transaction or coordination costs in understanding the organization of economic activity.

An important difference is that the primary focus of neoinstitutionalist attention has been structure of individual transactions, in particular whether a given transaction is best organized through the market or within a firm. The analysis here takes as its subject the organization of economy as a whole and, more importantly, the evolution of organization over time. In addition, while the neoinstitutionalists may be interpreted as saying that “transaction costs matter”, the model developed here suggests why they are important. Transaction costs matter because coordination matters, and coordination matters because of gains to the division of labor.

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Appendix A

This appendix derives Eqs. (11) and (12). To begin with, note that for \( h < h_0 \) and \( h > h_2 \) the lower and upper constraints on \( s \) are binding, implying \( \frac{ds^*}{dh} = \frac{dg^*}{dh} = 0 \). The third line of (11) and first line of (12) are found by solving the system of equations generated by total differentiation of \( \frac{dz}{ds} = 0 \) and \( \frac{dz}{dg} = 0 \):

\[
\begin{bmatrix}
y_{ss} - c_{ss} & -c_{sg} \\
c_{sg} & c_{gg}
\end{bmatrix}
\begin{bmatrix}
ds \\
dg
\end{bmatrix}
= 
\begin{bmatrix}
-y_{sh} \\
0
\end{bmatrix},
\]

which yields

\[
\begin{bmatrix}
ds \\
dg
\end{bmatrix}
= \frac{1}{A}
\begin{bmatrix}
-c_{gg} & y_{sh} \\
c_{sg} & y_{sh}
\end{bmatrix}
\begin{bmatrix}
dh
\end{bmatrix},
\]

where

\[
A = \left| \begin{array}{cc} c_{gg} & c_{sg} \\ -c_{sg} & y_{ss} - c_{ss} \end{array} \right| = (y_{ss} - c_{ss})c_{gg} + c_{sg}^2 < 0,
\]

and the sign of \( A \) follows from (9c). For \( h \in [h_0, h_1) \) government spending is constrained to zero and thus unresponsive to changes in specialization. Therefore, the second line of (11) is equal to the third with \( c_{sg}(s, g) = 0 \):

\[
\frac{ds^*}{dh} = \frac{-y_{sh}}{y_{ss} - c_{ss}}.
\]

Appendix B

This appendix derives the inequalities in Eq. (19). Assume that per capita output is a Cobb–Douglas function of \( h \) and \( s \): \( y = As^x h^\beta \), where \( \alpha, \beta \in (0, 1) \). Since (14) implies
\[ \eta(y, h) > 1, \eta(g, h) > 1 \] is a necessary condition for government’s share to increase. To show this is also a sufficient condition, note that \[ \eta(y, h) = \beta + x\eta(s^*(h), h) \] and, from (11), \[ \eta(g(h), h) = \eta(g(s)|_B, s)\eta(s, h) \], where \( \eta(g(s)|_B, s) \) is the elasticity of \( g \) with respect to \( s \) on the BB curve. It follows that \( \eta(g, h) > \eta(y, h) \) if and only if
\[
\eta(g(s)|_B, s)\eta(s, h) > \beta + x\eta(s, h)
\]
or
\[
\{\eta(g(s)|_B, s) - x\}\eta(s, h) > \beta.
\]
Since (14) implies \( \eta(s, h) > (1 - \beta)/x \), a sufficient condition for this to hold is
\[
\{\eta(g(s)|_B, s) - x\}(1 - \beta)/x > \beta
\]
\[
\eta(g(s)|_B, s) > x/(1 - \beta) > \eta(s, h)^{-1}
\]
where the second inequality again employs (14), or \( \eta(g(s)|_B, s)\eta(s, h) = \eta(g, h) > 1 \). Thus, \( \eta(g, h) > 1 \) is both necessary and sufficient for the growth of government’s share.

References


