

Motion in 1-Dimension



Examples of various types of motion illustrating translational, rotational, and vibrational.

<https://www.youtube.com/watch?v=E-4uyXruTxs>

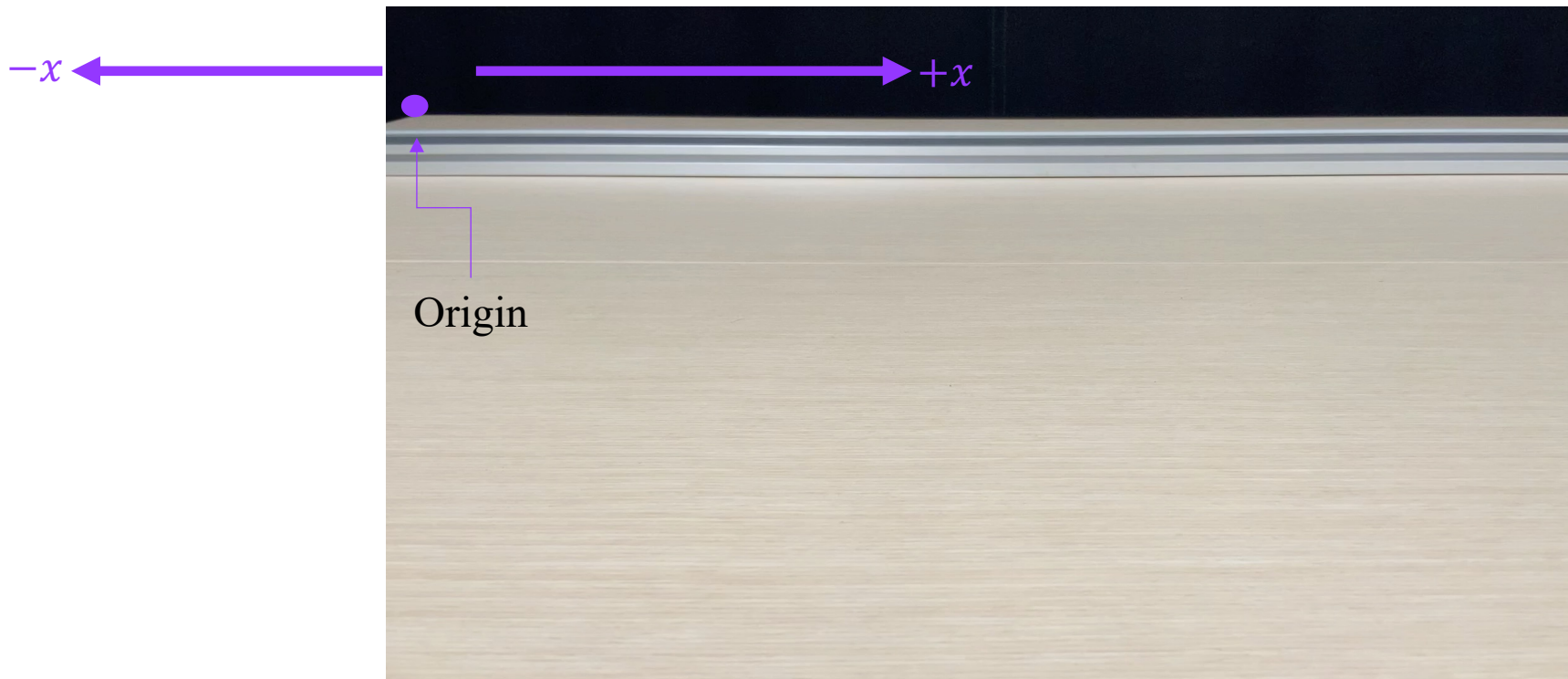
<https://www.youtube.com/watch?v=bUS8aCmytZE>



Motion in 1D

- Position vs. time

Here's a video showing a cart moving in one dimension.



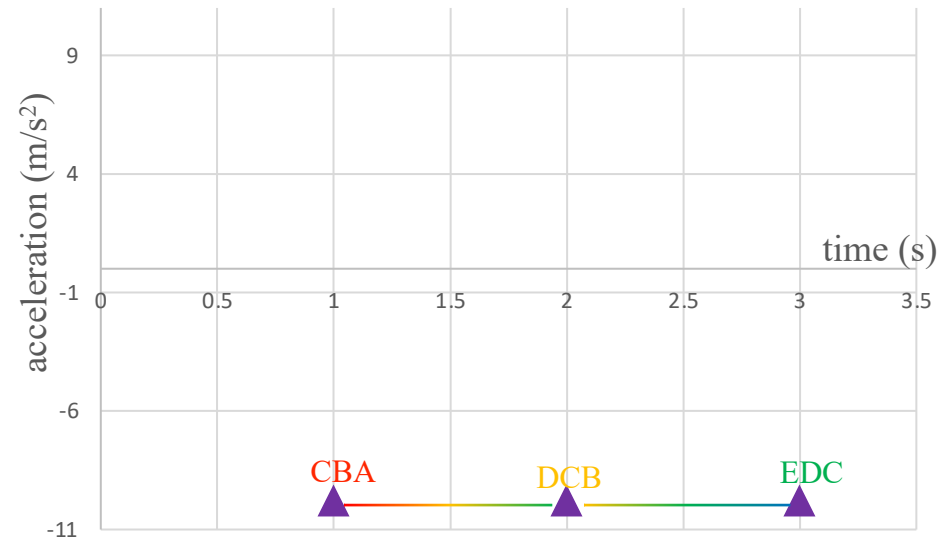
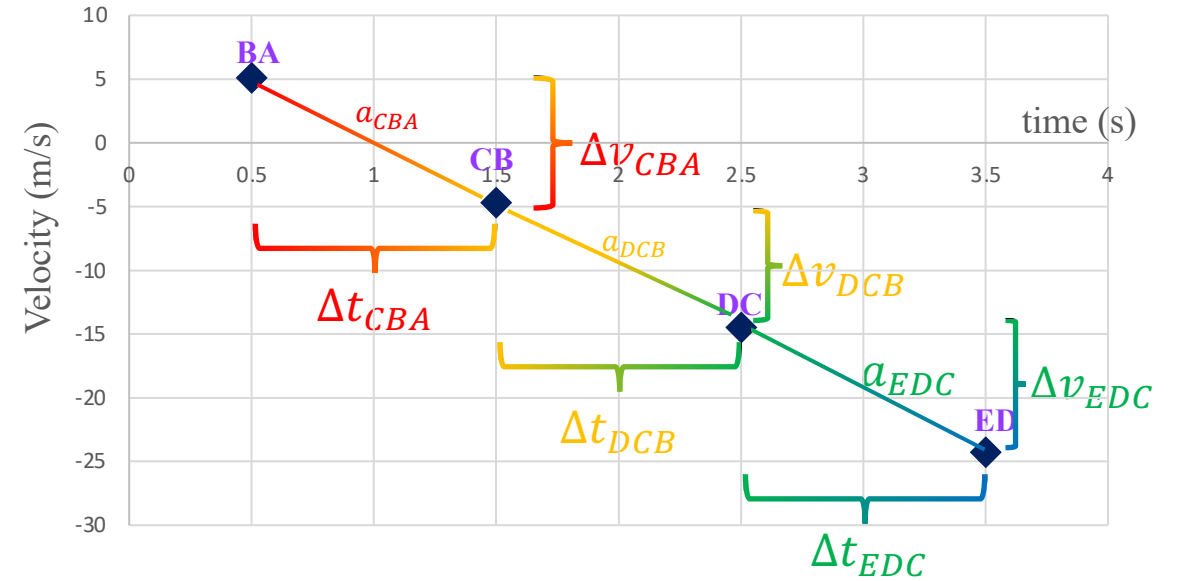
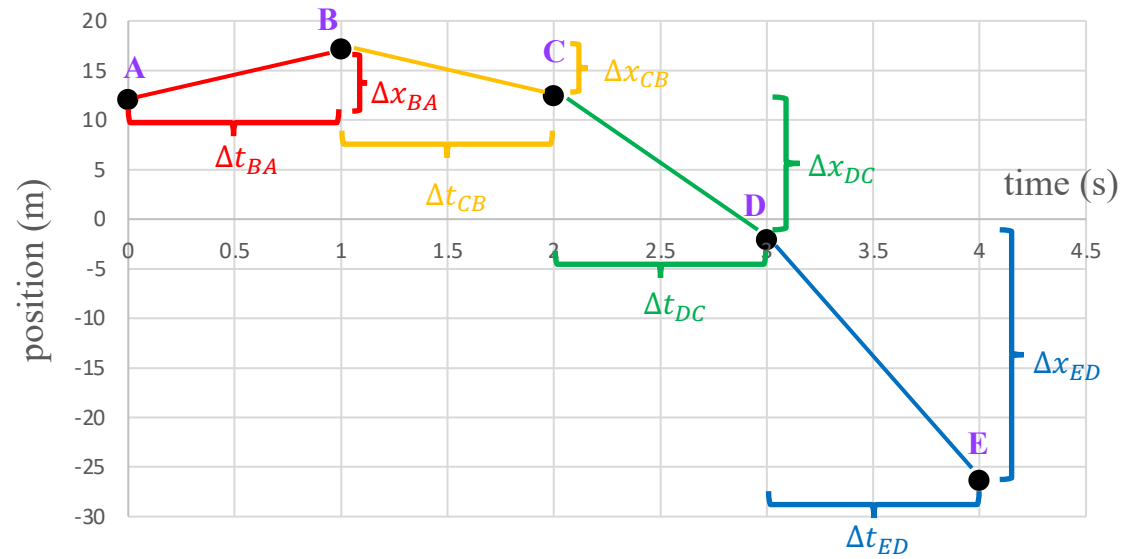
Some “data”

t (s)	x (m)
0	12
1	17.1
2	12.4
3	-2.1
4	-26.4

The cart is at specific locations at every instant of time and what we'd like is to be able to describe the position of the cart as a function of time and from this describe the complete motion of the cart.

Motion in 1D

- Motion Graphs



Motion in 1D with Constant Acceleration

- Uniform Motion (Constant Speed)

Example 1 (KJF 2.11):

I'm leaving my house in Albany at 8:00am to drive to Boston 170 miles away and I'm going to obey the speed limit of $65 \frac{mi}{hr}$. My brother is leaving my house an hour later but is not going to obey the speed limit and drives at $75 \frac{mi}{hr}$. Assume that there is no traffic so that we both can travel unabated from Albany to Boston.

- Who gets to Boston first?
- How long does the first person to arrive have to wait for the second person to arrive?

Motion in 1D with Constant Acceleration

- Uniform Motion (Constant Speed)

Example 1 (KJF 2.11) Solution:

- Who gets to Boston first?

$$v_{me} = \frac{\Delta x}{\Delta t_{me}} \rightarrow \Delta t_{me} = \frac{\Delta x}{v_{me}} = \frac{170mi}{65\frac{mi}{hr}} = 2.6hrs$$

→ $t_{me} = 2.6hrs$

$$v_{bro} = \frac{\Delta x}{\Delta t_{bro}} \rightarrow \Delta t_{bro} = \frac{\Delta x}{v_{bro}} = \frac{170mi}{75\frac{mi}{hr}} = 2.3hrs$$

→ $t_{bro} = 2.3hrs + 1hr = 3.3hrs$

- How long does the first person to arrive have to wait for the second person to arrive?

Since my brother waited an extra hour to leave, I get to Boston ahead of him by

$$3.3hrs - 2.6hrs = 0.7hrs \times \frac{60min}{1hr} = 42min$$

Motion in 1D with Constant Acceleration

- Uniform Motion (Constant Acceleration)

Example 2 (KJF 2.31):

In car crashes, large accelerations of the head can lead to severe injuries or even death. A driver can probably survive an acceleration of $50g$ that lasts for less than $30ms$, but in a crash with an acceleration of $50g$ lasting longer than $30ms$, a driver is unlikely to survive.

- What is the highest speed that the car could have such that the driver survived?
- What is the shortest survivable distance over which the driver's head could have come to rest?

Motion in 1D with Constant Acceleration

- Uniform Motion (Constant Speed)

Example 2 (KJF 2.31) Solution:

- What is the highest speed that the car could have such that the driver survived?

$$v_{fx} = v_{ix} - a_x t \rightarrow v_{ix} = a_x t = 50g \cdot \frac{9.8 \frac{m}{s^2}}{g} \cdot 30 \times 10^{-3} s = 14.7 \frac{m}{s} \times \frac{1mi}{1600m} \times \frac{3600s}{1hr} = 33 \frac{mi}{hr}$$

- What is the shortest survivable distance over which the driver's head could have come to rest?

$$v_{fx}^2 = v_{ix}^2 - 2a_x \Delta x \rightarrow \Delta x = -\frac{v_{fx}^2 - v_{ix}^2}{2a_x} = \frac{v_{ix}^2}{2a_x} = \frac{\left(14.7 \frac{m}{s}\right)^2}{2 \cdot 50g \cdot \frac{9.8 \frac{m}{s^2}}{g}} = 0.22m \times \frac{1in}{0.0254m} = 8.7inches$$

Motion in 1D with Constant Acceleration

- Problems

Example 3:

Suppose you are stopped at a 4-way intersection.

- Starting from rest, how long does it take you to cross the intersection if your car accelerates at $3\frac{m}{s^2}$ and the intersection is about $10m$ ($\sim 30ft$) wide?
- After the car crosses the intersection what is its speed? Is this reasonable?

Motion in 1D with Constant Acceleration

- Problems

Example 3 Solution:

- Starting from rest, how long does it take you to cross the intersection if your car accelerates at $3\frac{m}{s^2}$ and the intersection is about $10m$ ($\sim 30ft$) wide?

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_xt^2 \rightarrow t = \sqrt{\frac{2x_f}{a_x}} = \sqrt{\frac{2 \cdot 10m}{3\frac{m}{s^2}}} = 2.6s$$

- After the car crosses the intersection what is its speed? Is this reasonable?

$$v_{fx} = v_{ix} + a_xt = 3\frac{m}{s^2} \cdot 2.6s = 7.8\frac{m}{s} \times \frac{1mi}{1600m} \times \frac{3600s}{1hr} = 17.4\frac{mi}{hr}$$

The car's traveling about $17\frac{mi}{hr}$.
That's probably reasonable for crossing an intersection.

Motion in 1D with Constant Acceleration

- Problems

Example 4 (KJF 2.64):

Suppose instead of a 4-way stop, you are driving down a road at a constant velocity of $20\frac{m}{s}$ when you notice that the traffic light ahead is turning red. At the instant the light turns red, you take your foot off the gas and apply the brakes. We'll assume that your reaction time is negligible, and you know (I don't know how) that your car is $70m$ ($\sim 210ft$) from the intersection and light.

- What is $20\frac{m}{s}$ in miles per hour?
- What acceleration would be needed to bring your car to rest at the intersection and how long would it take to come to rest from $20\frac{m}{s}$?

Motion in 1D with Constant Acceleration

- Problems

Example 4 (KJF 2.64). Solution:

- What is $20\frac{m}{s}$ in miles per hour?

$$20\frac{m}{s} \times \frac{1mi}{1600m} \times \frac{3600s}{1hr} = 45\frac{mi}{hr}$$

- What acceleration would be needed to bring your car to rest at the intersection and how long would it take to come to rest from $20\frac{m}{s}$?

$$v_{fx}^2 = v_{ix}^2 - 2a_x\Delta x \rightarrow a_x = -\frac{v_{fx}^2 - v_{ix}^2}{2\Delta x} = \frac{v_{ix}^2}{2\Delta x} = \frac{\left(20\frac{m}{s}\right)^2}{2 \cdot 70m} = 2.9\frac{m}{s^2}$$

Motion in 1D with Constant Acceleration

- Problems

Example 5:

Suppose instead that you are driving down the highway at a constant velocity of $45 \frac{m}{s} \sim \left(97 \frac{mi}{hr}\right)$ when you pass a law-enforcement officer (LEO) at rest on the side of the highway. One second after you pass, the LEO starts out from rest and accelerates at $3 \frac{m}{s^2}$.

- How long does it take the LEO to catch you?
- How far down the road does the LEO catch up with you?
- What will be the LEO's speed when they catch you?

Motion in 1D with Constant Acceleration

- Problems

Example 5 Solution:

- How long does it take the LEO to catch you?

$$x_{f,LEO} = x_{i,LEO} + v_{i,LEO}t + \frac{1}{2}a_{LEO}t^2 = \frac{1}{2}a_{LEO}t^2 = \frac{1}{2}\left(3\frac{m}{s^2}\right)t^2$$

$$x_{f,you} = x_{i,you} + v_{i,you}t + \frac{1}{2}a_{you}t^2 = x_{i,you} + v_{i,you}t = 45m + 45\frac{m}{s} \cdot t$$

We need to pick an origin for our coordinate system and what the initial time is. I'm going to put it at the LEO's position. Thus, $x_{i,LEO} = 0m$. For the initial time, I'm going to use when the LEO starts to move. So, I need to know where I am when the Leo starts moving. This will be $x_{i,you}$ above. Since I'm traveling at a constant speed, when I pass the LEO $x_{i,you} = 0m$ and $x_{f,you} = x_{i,you} + v_{i,you}t + \frac{1}{2}a_{you}t^2 = v_{i,you}t = 45\frac{m}{s} \cdot 1s = 45m$

When the LEO catches me, we have to be at the same final position at the same time.

Therefore, $x_{f,LEO} = x_{f,you}$.

$$x_{f,LEO} = x_{f,you} \rightarrow \frac{1}{2}a_{LEO}t^2 = x_{i,you} + v_{i,you}t \rightarrow \frac{1}{2}\left(3\frac{m}{s^2}\right)t^2 = 45m + 45\frac{m}{s} \cdot t$$

$$\longrightarrow t^2 - 30t - 30 = 0 \rightarrow t = \frac{-(-30) \pm \sqrt{(-30)^2 - 4 \cdot 1 \cdot (-30)}}{2 \cdot 1} \rightarrow t = \begin{cases} -1s \\ 31s \end{cases}$$

The time it takes the LEO to catch me is $t = 31s$. We note that the other time, $t = -1s$ corresponds to when I passed the LEO one second before the LEO started moving.

Motion in 1D with Constant Acceleration

- Problems

Example 5 Solution:

- How far down the road does the LEO catch up with you?

$$x_{f,LEO} = x_{i,LEO} + v_{i,LEO}t + \frac{1}{2}a_{LEO}t^2 = \frac{1}{2}a_{LEO}t^2 = \frac{1}{2}\left(3\frac{m}{s^2}\right)t^2 = \frac{1}{2}\left(3\frac{m}{s^2}\right)(31s)^2 = 1441.5m$$

$$x_{f,you} = x_{i,you} + v_{i,you}t + \frac{1}{2}a_{you}t^2 = x_{i,you} + v_{i,you}t = 45m + 45\frac{m}{s} \cdot t = 45m + 45\frac{m}{s} \cdot 31s = 1440m$$

- What will be the LEO's speed when they catch you?

$$v_{f,LEO} = v_{i,LEO} + a_{LEO}t = a_{LEO}t = 3\frac{m}{s^2} \cdot 31s = 93\frac{m}{s} \times \frac{1mi}{1600m} \times \frac{3600s}{1hr} \sim 210\frac{mi}{hr}$$

The difference is due to rounding of the time from the quadratic equation.

This seems a bit excessive for the LEO to try to catch me, but that's ok.

Motion in 1D with Constant Acceleration

- Problems

Example 6 (KJF 2.39 & KJF 2.64):

You're driving down the road late one night at $20\frac{m}{s}$ ($\sim 45\frac{mi}{hr}$) when a deer steps out onto the road $35m$ ($\sim 105feet$) in front of you. Your reaction time, the time it takes you to get your foot off the accelerator and onto the brake, is $0.25s$. You step as hard as you can on the brakes and your car decelerates at a maximum rate of $10\frac{m}{s^2}$.

- How much distance is between you and the deer when you come to a stop (assuming the deer doesn't move)?
- What is the maximum speed you could have and still not hit the deer?

Motion in 1D with Constant Acceleration

- Problems

Example 6 (KJF 2.39 & KJF 2.64) Solution:

- How much distance is between you and the deer when you come to a stop (assuming the deer doesn't move)?

$$x_{total} = x_{react} + x_{stop} = v_{ix}t_{react} + v_{ix}t_{stop} + \frac{1}{2}a_x t_{stop}^2$$

$$x_{total} = \left(20\frac{m}{s} \cdot 0.25s\right) + 20\frac{m}{s} \cdot 2s - \frac{1}{2} \cdot 10\frac{m}{s^2}(2s)^2 = 25m$$

$$\longrightarrow \Delta x = x_{deer} - x_{you} = 35m - 25m = 10m$$

$$v_{fx} = v_{ix} + a_x t_{stop} \rightarrow t_{stop} = \frac{v_{fx} - v_{ix}}{a_x} = \frac{0\frac{m}{s} - 20\frac{m}{s}}{-10\frac{m}{s^2}} = 2s$$

The total distance traveled by you is the distance traveled in the time it takes you to react (assuming your car travels at a constant speed during this reaction time) plus the distance it takes you to come to rest after you put on the brakes.

To find the time it takes you to stop your car after the brakes are applied, we know the initial and final velocities and the acceleration needed, so we can use our time dependent velocity equation to determine the stopping time.

Motion in 1D with Constant Acceleration

- Problems

Example 6 (KJF 2.39 & KJF 2.64) Solution:

- What is the maximum speed you could have and still not hit the deer?

$$v_{fx} = v_{ix} + a_x t_{stop} \rightarrow t_{stop} = \frac{v_{fx} - v_{ix}}{a_x} = \frac{v_{ix}}{a_x}$$

Assuming the reaction time remains constant; I need to know how long it takes me to stop from some initial velocity v_{ix} . In this, I assumed the acceleration is negative (to slow me down) so that my time is positive.

$$x_{total} = x_{react} + x_{stop} = v_{ix} t_{react} + v_{ix} t_{stop} + \frac{1}{2} a_x t_{stop}^2$$

The total distance traveled by you right to the deer is the distance traveled in the time it takes you to react (assuming your car travels at a constant speed during this reaction time) plus the distance it takes you to come to rest after you put on the brakes which will total 35m.

$$x_{total} = v_{ix} t_{react} + v_{ix} \left(\frac{v_{ix}}{a_x} \right) - \frac{1}{2} a_x \left(\frac{v_{ix}}{a_x} \right)^2 = v_x t_{react} + \frac{v_{ix}^2}{2a_x}$$

This gives me a quadratic in square of the initial velocity. By the quadratic formula we determine v_{ix} .

$$\longrightarrow 0 = -35 + \frac{1}{4} v_{ix} + \frac{v_{ix}^2}{2 \cdot 10} \rightarrow v_{ix} = \begin{cases} 24.1 \frac{m}{s} \\ -29.1 \frac{m}{s} \end{cases} \longrightarrow v_{ix} = 24.1 \frac{m}{s}$$

The maximum speed I could be traveling at and not hit the deer is $24.1 \frac{m}{s}$.