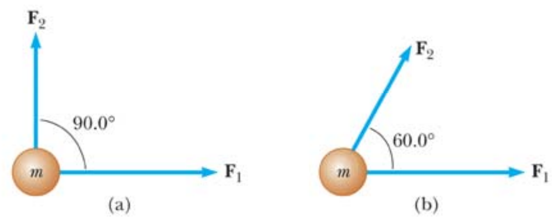
Physics 110
Spring 2006
Forces in 1- and 2-Dimensions – Their Solutions

1. Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ acts on a 5kg mass. If the magnitudes of $\mathbf{F_1}$ and $\mathbf{F_2}$ are 20N and 15N respectively what are the accelerations of each of the masses below?



$$a.\ \ F_{net}=\sqrt{F_{net,x}^2+F_{net,y}^2}\ @\ \theta=\tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right)=ma_{net}$$

$$\to a_{net}=\frac{F_{net}}{m}=\frac{\sqrt{F_{net,x}^2+F_{net,y}^2}}{m}\ @\ \theta=\tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right)$$

$$F_{net,x}=20N;\ \ F_{net,y}=15N$$

$$a_{net}=\frac{\sqrt{(20N)^2+(15N)^2}}{5kg}\ @\ \theta=\tan^{-1}\left(\frac{15N}{20N}\right)=5\tfrac{m}{s^2}\ @\ \theta=31^0$$

$$b.\ \ F_{net}=\sqrt{F_{net,x}^2+F_{net,y}^2}\ @\ \theta=\tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right)=ma_{net}$$

$$\to a_{net}=\frac{F_{net}}{m}=\frac{\sqrt{F_{net,x}^2+F_{net,y}^2}}{m}\ @\ \theta=\tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right)$$

$$F_{net,x}=F_1+F_2\cos\theta=20N+15N\cos 60=27.5N$$

$$F_{net,y}=F_2\sin\theta=16\sin 60=13N$$

$$a_{net}=\frac{\sqrt{(27.5N)^2+(13N)^2}}{5kg}\ @\ \theta=\tan^{-1}\left(\frac{13N}{27.5N}\right)=6.08\tfrac{m}{s^2}\ @\ \theta=25.3^0$$

2. You stand on the seat of a chair and then hop off.

a. During the time you are in flight down to the floor, the Earth is moving up toward you with an acceleration of what order of magnitude?

b. The Earth moves up through a distance of what order of magnitude?

a. Both you and the Earth exert equal and opposite forces on each other.

Thus, $F_{Earth} = F_{you} \rightarrow m_{Earth}a_{Earth} = m_{you}a_{you} \therefore a_{Earth} = \dfrac{m_{you}}{m_{Earth}} a_{you}$

$$a_{Earth} = \frac{70kg}{6.0\times10^{24}kg}\times 9.8\tfrac{m}{s^2} = 1.2\times10^{-22}\tfrac{m}{s^2}$$

b. For a chair that is say, 50cm high, you both move for equal times.

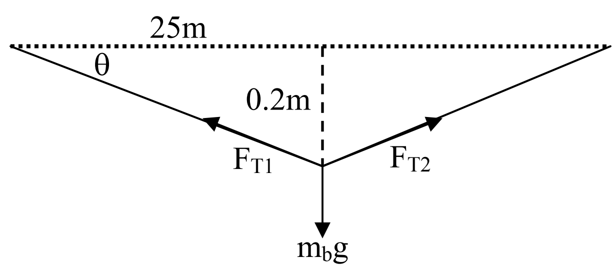
$$x_{Earth} = \tfrac{1}{2}a_{Earth}t^2 \rightarrow t^2 = \frac{2x_{Earth}}{a_{Earth}} = \frac{2x_{you}}{a_{you}}$$

$$\rightarrow x_{Earth} = x_{you}\times\frac{a_{Earth}}{a_{you}} = 0.50m\times\frac{1.2\times10^{-22}\tfrac{m}{s^2}}{9.8\tfrac{m}{s^2}} = 5.8\times10^{-24}m$$

3. The distance between two telephone poles is 50m. When a 1kg bird lands on the telephone wire halfway between the poles, the wire sags 0.2m.

a. Draw a free body diagram of the bird.

a.



b. How much tension does the bird produce in the wire?

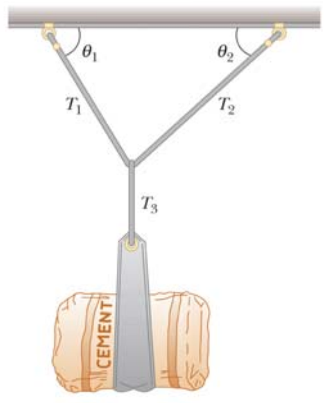
$$b.\ \tan\theta = \frac{0.2m}{25m} = 0.008 \rightarrow \theta = 0.458^o$$

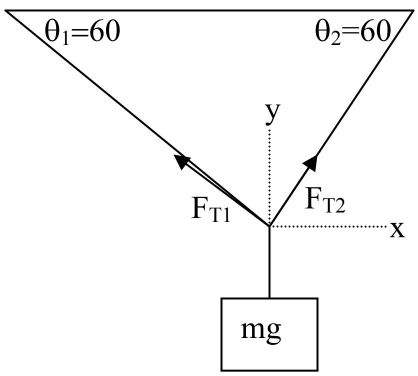
$$\sum F_x : F_{T_1}\cos\theta - F_{T_2}\cos\theta = ma_x = 0 \rightarrow F_{T_1} = F_{T_2}$$

$$\sum F_y : 2F_{T_1}\sin\theta - m_b g = ma_y = 0 \rightarrow 2F_{T_1}\sin\theta = m_b g$$

$$F_{T_1} = \frac{m_b g}{\sin\theta} = \frac{1kg\times 9.8\tfrac{m}{s^2}}{\sin(0.458)} = 612.5N$$

4. A bag of cement of weight 325N hangs from three wires as shown below. Two of the wires make angles $\theta_1 = 60^o$ and $\theta_2 = 25^o$ with respect to the horizontal. If the system is in equilibrium, what are the three tension forces in the wire?





From the free body diagram,

$F_{T3} = F_g = 325\text{N}$ (1)

ΣF_x: $F_{T1} \cos\theta_1 - F_{T2} \cos\theta_2 = ma_x = 0$ (2)

ΣF_x: $F_{T1} \sin\theta_1 + F_{T2} \sin\theta_2 = F_g$ (3)

From (2) we have $F_{T_2} = F_{T_1} \dfrac{\cos\theta_1}{\cos\theta_2}$. Substituting this into (3) we have $F_{T_1} \dfrac{\cos\theta_1}{\cos\theta_2} \sin\theta_2 + F_{T_1} \sin\theta_1 = 325N \rightarrow F_{T_1} = 295.7$.

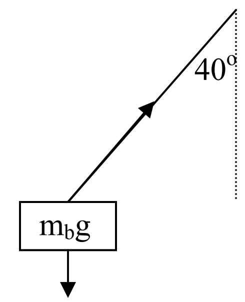
Therefore, F_{T2}=163.1N

5. A fire/rescue helicopter caries a 620kg bucket of water at the end of a 20m long cable. As the aircraft flies back from a fire at constant speed of 40m/s, the cable makes an angle of 40^o with respect to the vertical.

a. What is the force due to air resistance on the bucket?
b. After filling the bucket with water, the pilot returns to the fire at the same speed with the bucket now making a 7^o angle with the vertical. What is the mass of the water in the bucket?

http://www.fs.fed.us/r4/sc/fire/2005/photos/fallscrk/helicopter.jpg

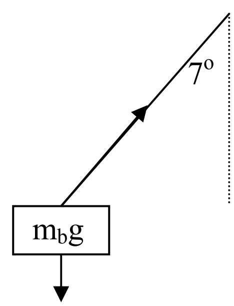
a. From the free body diagram we have



$$\sum F_x : F_T \cos 50 - F_{air} = ma_x = 0 \qquad (1)$$
$$\sum F_y : F_T \sin 50 - m_b g = ma_y = 0 \qquad (2)$$

From (2) we have that $F_T = \dfrac{m_b g}{\sin 50} = \dfrac{620kg \times 9.8\frac{m}{s^2}}{\sin 50} = 7931.7N$

From (1) we have that $F_{air} = 7931.7N \cos 50 = 5098.4N$



b. From the free body diagram we have

$$\sum F_x : F_T \cos 83 - F_{air} = ma_x = 0 \qquad (1)$$
$$\sum F_y : F_T \sin 83 - (m_b + m_w)g = ma_y = 0 \qquad (2)$$

From (1) we have that, since F_{air} is a constant, $F_T = \dfrac{F_{air}}{\cos 83} = \dfrac{5098.4N}{\cos 83} = 41835N$

From (2) we have solving for m_w : $m_w = \dfrac{F_T \sin 83 - m_b g}{g} = 3617.1kg$

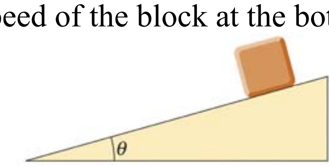
6. A block slides down a frictionless plane having an inclination of $\theta = 15^o$.

a. If the block starts from rest, what is the acceleration of the block down the incline?

b. What is the speed of the block at the bottom of the incline if it travels a distance of 2m down the incline?

c. Suppose now that friction does exist between the block and incline. What is the new acceleration if the coefficient of kinetic friction, $\mu_k = 0.25$ (NOTE μ_K HAS BEEN CHANGED FROM THE ORIGINAL ASSIGNEMENT.)?

d. What is the new speed of the block at the bottom of the incline?



a. From class we've seen, that for a coordinate system with the y-axis perpendicular to the plane and the x-axis down the plane we have the acceleration $a = g\sin\theta = 9.8\frac{\mu m}{s^2}\sin 15 = 2.54\frac{m}{s^2}$.

b. If the block travels 2m then the final speed is given by $v_f = \sqrt{2a\Delta x} = \sqrt{2\times 2.54\frac{m}{s^2}\times 2m} = 10.15\frac{m}{s}$.

c. When we included friction, as in class, the acceleration is lower and is given by $a = g\sin\theta - \mu_k g\cos\theta = 9.8\frac{\mu m}{s^2}\sin 15 - 0.25\times 9.8\frac{\mu m}{s^2}\cos 15 = 0.17\frac{m}{s^2}$.

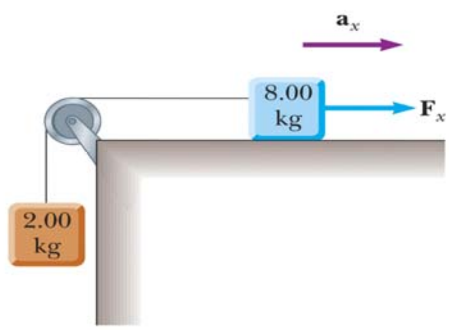
d. The final speed is given as $v_f = \sqrt{2a\Delta x} = \sqrt{2\times 0.174\frac{m}{s^2}\times 2m} = 0.68\frac{m}{s}$

7. In the system shown below a horizontal force F_x acts on the 8kg mass. Consider two cases, one where the horizontal surface is frictionless and one where there is friction and the coefficient of kinetic friction is $\mu_k = 0.75$.

a. For what values of F_x does the 2kg mass accelerate upwards?

b. For what values of F_x is the tension in the cord zero?

c. Plot the acceleration of the 8kg mass versus F_x. Include values of F_x from -100N to 100N.

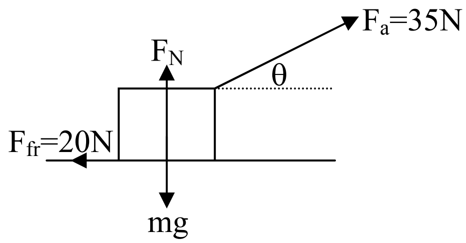


(Question #7 will not be on exam #1! Solution will be posted after the exam.)

8. A woman at the airport is towing her 20kg suitcase at constant speed by pulling on a strap at an angle of θ above the horizontal. She pulls the strap with a force of 35N and the frictional force on the suitcase is 20N.

a. What is the angle θ that the strap makes with the horizontal?

b. What normal force does the ground exert on the suitcase?



$$\sum F_x: \quad F_a \cos\theta - 20N = ma_x \quad (1)$$
$$\sum F_y: \quad F_N + F_a \sin\theta - mg = ma_y = 0 \; (2)$$

From (1) we get the angle: $\theta = \cos^{-1}\left(\frac{20N}{35N}\right) = 55.2^o$

From 2 we the normal force: $F_N = mg - F_a \sin\theta = 167.3N$

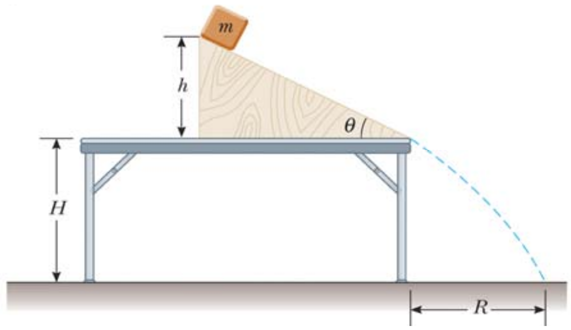
9. A boy drags his 60N sled at constant speed up a 15° hill. He does so by pulling with a 25N force on a rope attached to the sled. If the rope is inclined at 35° to the horizontal,
   a. What is the coefficient of kinetic friction between the sled and the snow?
   b. At the top of the hill he jumps on the sled and slides don the hill. What is the magnitude of the acceleration of the boy and sled?

   a. From the free body diagram we see that the x and y forces are
   $$\sum F_x: \quad 25N\cos 20 - F_{Fr} - 60N\sin 15 = ma_x = 0 \quad (1)$$
   $$\sum F_y: \quad F_N + 25N\sin 20 - 60N\cos 15 = ma_y = 0 \quad (2)$$
   From (2) we get the normal force: $F_N = 49.43N.$
   Thus from (1) we get the coefficient of kinetic friction since $F_{Fr} = \mu_K F_N$.
   $\mu_K = 0.16$

   b. From question 6c, the acceleration is given by $a = g\sin\theta - \mu_k g\cos\theta = 9.8\frac{\mu m}{s^2}\sin 15 - 0.16 \times 9.8\frac{\mu m}{s^2}\cos 15 = 1.02\frac{m}{s^2}$

10. A block of mass m = 2kg is released from rest h = 0.5m from the surface of a table, at the top of a θ = 30° incline. The frictionless incline is fixed on a table of height H = 2m.
   a. What is the acceleration of the block as it slides down the incline?
   b. What is the velocity of the block as it leaves the incline?
   c. How far from the table will the block land?
   d. How much time has elapsed between the block was released and when it hits the floor?
   e. Does the mass of the block affect any of the above calculations?

f. Redo the above calculations including a frictional force between the block and the incline with a coefficient of kinetic friction of $\mu_k = 0.57$.



a. From class, the acceleration down the incline is given as $a = g\sin\theta = 9.8\,\frac{\mu m}{s^2}\sin 30 = 4.9\,\frac{m}{s^2}$.

b. The velocity is given as $v_f = \sqrt{2a\Delta x} = \sqrt{2\times 4.9\,\frac{m}{s^2}\times 1m} = 3.13\,\frac{m}{s}$, where, sin θ = h/d so d = h/sinθ = 1m.

c.

$$v_{ix} = 3.13\,\tfrac{m}{s}\cos 30 = 2.71\,\tfrac{m}{s}$$
$$v_{iy} = -3.13\,\tfrac{m}{s}\sin 30 = -1.57\,\tfrac{m}{s}$$
$$R = 2.71\,\tfrac{m}{s}\times t$$

To calculate R, we need to know the time.

This we get from the vertical motion.

$\mathrm{H} = 2 = -1.57\,\frac{\mathrm{m}}{\mathrm{s}}t - 4.9\,\frac{m}{s^2}t^2$ which gives by the quadratic formula,

$\mathrm{t} = \{0.499\mathrm{s}, -0.819\mathrm{s}\}$, and we choose $\mathrm{t} = 0.499\mathrm{s}$.

Thus, $\mathrm{R} = 1.35\mathrm{m}$

d. The total time is the sum of the sliding motion and the falling motion. The time to fall is from part c and is 0.499s. The time to slide is found from $v_f = v_i + at \rightarrow t = \dfrac{3.13\,\frac{m}{s}}{4.9\,\frac{m}{s^2}} = 0.639s$. Therefore the total time from release to landing is 0.499s + 0.639s = 1.14s.

e. No, the mass does not affect any result since it does not appear in any of the above calculations.

f. Including friction, the acceleration is given by $a = g\sin\theta - \mu_k g\cos\theta = 9.8\,\frac{\mu m}{s^2}\sin 30 - 0.57\times 9.8\,\frac{\mu m}{s^2}\cos 30 = 0.062\,\frac{m}{s^2}$.

The velocity is given as $v_f = \sqrt{2a\Delta x} = \sqrt{2\times 0.062\,\frac{m}{s^2}\times 1m} = 0.35\,\frac{m}{s}$.

$v_{ix} = 0.35\,\frac{m}{s}\cos 30 = 0.303\,\frac{m}{s}$

$v_{iy} = -0.35\,\frac{m}{s}\sin 30 = -0.176\,\frac{m}{s}$

$R = 2.71\,\frac{m}{s} \times t$

To calculate R, we need to know the time.

This we get from the vertical motion.

H = 2 = $-.176\,\frac{m}{s}t - 4.9\,\frac{m}{s^2}t^2$ which gives by the quadratic formula,

t = $\{0.434\text{s}, -0.470\text{s}\}$, and we choose t = 0.434s.

Thus, R = 0.132m.

The total time is give from the sum of the fall time and the slide time.

The slide time is given as $v_f = v_i + at \rightarrow t = \dfrac{0.35\,\frac{m}{s}}{0.062\,\frac{m}{s^2}} = 5.65s$, thus the total time is 0.434s + 5.65s = 6.08s.

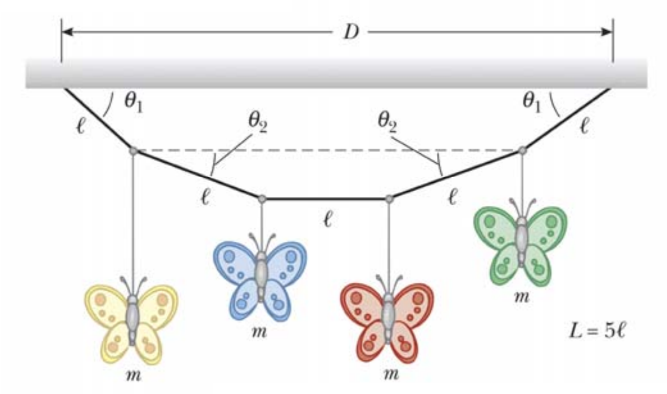
Again, mass has no effect on the result.

11. A mobile hangs in a child's room formed by supporting four metal butterflies of equal mass *m* from a string of length *L*. The points of support are evenly spaced a distance *l* apart. The string forms an angle θ_1 with the ceiling at each point. The center section of the string is horizontal.

a. What is the tension in each section of the string in terms of θ_1, m and g?

b. What is the angle θ_2, in terms of θ_1, that the sections of the string between the outside butterflies and the inside butterflies form with the horizontal.

c. *For a cold spring night*: Show that the distance D between the end points of the string is $D = \dfrac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\tfrac{1}{2}\tan\theta_1\right)\right] + 1\right\}$.



(Question #11 will not be on exam #1! Solution will be posted after the exam.)

12. Before 1960 it was believed that the maximum attainable coefficient of static friction for an automobile tire was less than 1. Then about 1962, three

companies independently developed racing tires with coefficients of 1.6. Since then, tires have improved. According to the 1990 Guinness book of Records, the fastest time in which a piston-engine car initially at rest has covered a distance of ¼ mile is 4.96s. This record was set by Shirley Muldowney in September 1989.

a. Assuming that the rear wheels lifted the front off the pavement, what minimum value of μ is necessary to achieve this record time?
b. Suppose Muldowney were able to double her engine power, keeping all other factors equal, how would this change the elapsed time?

(Question #12 will not be on exam #1! Solution will be posted after the exam.)