# Physics 110
# Spring 2006
# 1-D Motion Problems and their Solutions

1. Jules Verne in 1865 proposed sending people to the moon by firing a space capsule from a 220-m long cannon with a final velocity of 10.97 km/s. What is the acceleration of this space capsule? Express your answer in g's, where 1 g = 9.8 m/s$^2$.

$$v_f^2 = v_i^2 + 2a\Delta x \rightarrow \left(10.97\times10^3 \tfrac{m}{s}\right)^2 = 2a(220m) \rightarrow a = 2.74\times10^5 \tfrac{m}{s^2} = 2.79\times10^4 g's$$

2. The minimum distance required to stop a car moving at 35 mi/hr is 40ft. What is the minimum stopping distance for the same car moving at 70 mi/hr assuming the same rate of deceleration?

$$v_f^2 = v_i^2 + 2a\Delta x \rightarrow 0 = \left(35\tfrac{mi}{hr}\times\frac{5280\,ft}{1mi}\right)^2 + 2a(40\,ft) \rightarrow a = -4.27\times10^8 \tfrac{ft}{hr^2} = 10\tfrac{m}{s^2}$$

$$v_f^2 = v_i^2 + 2a\Delta x \rightarrow 0 = \left(70\tfrac{mi}{hr}\times\frac{5280\,ft}{1mi}\right)^2 + 2\left(-4.27\times10^8 \tfrac{ft}{hr^2}\right)\Delta x \rightarrow \Delta x = 160\,ft$$

3. A large jet plane lands with a speed of 100 m/s and can decelerate at a maximum rate of 5 m/s$^2$ as it comes to rest.
   a. From the instant the plane touches down on the runway, what is the minimum time needed before the plane will come to rest?
   b. Could this plane land on a runway that is 2539 feet long?

$$v_f = v_i + at \rightarrow 0 = 100\tfrac{m}{s} - 5\tfrac{m}{s^2}t \rightarrow t = 20s$$

$$x_f = x_i + v_i t + \tfrac{1}{2}at^2 = 0 + \left(100\tfrac{m}{s}\times20s\right) - \tfrac{1}{2}\left(5\tfrac{m}{s^2}\right)(20s)^2 = 1000m$$

$1000m = 3333\,ft$, so no it could not stop in this distance.

4. You are driving at 30 m/s when you enter a one-lane tunnel. As soon as you enter you notice a slow moving van 155m ahead traveling at 5 m/s. If you apply the brakes, but can only decelerate at a 2 m/s$^2$ because the road is wet, will you collide with the van? If so, how far from the tunnels entrance will the collision occur? If there will be no collision, what is the closest distance you come to the van?

Take the original point to be when you notice the van. Choose the origin of the *x*-axis at your car. Thus we have

$x_{is} = 0$ $v_{is} = 30.0$ m/s $a_s = -2.00$ m/s$^2$ so her position is given by

$x_s(t) = x_{is} + v_{is}\,t + \frac{1}{2}\,a_s t^2 = (30.0 \text{ m/s})t + \frac{1}{2}(-2.00 \text{ m/s}^2)\,t^2$

For the van, $x_{iv} = 155$ m $v_{iv} = 5.00$ m/s $a_v = 0$ and $x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}\,a_v t^2$ $= 155 \text{ m} + (5.00 \text{ m/s})t + 0$

To test for a collision, we look for an instant $t_c$ when both are at the same place:

$30.0t_c - t_c^2 = 155 + 5.00t_c$; $0 = t_c^2 - 25.0t_c + 155$ From the quadratic formula $t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6 \text{ s or } \boxed{11.4 \text{ s}}$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position $155 \text{ m} + (5.00 \text{ m/s})(11.4 \text{ s}) = \boxed{212 \text{ m}}$ .

5. A student throws a set of keys vertically upward to her sorority sister, who is in a window 4m above. The keys are caught 1.5 seconds later by the sister's outstretched hand.
    a. With what initial velocity were the keys thrown?
    b. What was the velocity of the keys just before they were caught?

(a) $y = v_i t + \frac{1}{2}\,at^2$; $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $\boxed{v_i = 10.0 \text{ m/s upward}}$

(b) $v = v_i + at = 10.0 - (9.80)(1.50) = -4.68$ m/s; $v = \boxed{4.68 \text{ m/s downward}}$

6. A commuter train travels between two downtown stations. Since the stations are only 1 km apart the train never reaches the maximum possible cruising speed. The engineer minimizes the time *t* between the two stations by accelerating at a rate of 0.1 m/s$^2$ for a time $t_1$ and then by braking with an acceleration of -0.5 m/s$^2$ for a time $t_2$. What is the minimum time of travel *t* and the time $t_1$?

$a_1 = 0.100$ m/s$^2$, $a_2 = -0.500$ m/s$^2$, thus $x = 1000 \text{ m} = \frac{1}{2}\,a_1 t_1^2 + v_1 t_2 + \frac{1}{2}\,a_2 t_2^2$

The total travel time is given as $t = t_1 + t_2$ and $v_1 = a_1 t_1 = -a_2 t_2$ therefore, $1000 = \frac{1}{2}\,a_1 t_1^2 + a_1 t_1\left(-\frac{a_1 t_1}{a_2}\right) + \frac{1}{2}\,a_2\left(\frac{a_1 t_1}{a_2}\right)^2$ which gives $1000 = \frac{1}{2}\,a_1\left(1 - \frac{a_1}{a_2}\right)t_1^2$

$t_1 = \sqrt{\frac{20{,}000}{1.20}} = \boxed{129 \text{ s}}$ and $t_2 = \frac{a_1 t_1}{-a_2} = \frac{12.9}{0.500} \approx 26$ s, therefore the total time $= t = \boxed{155 \text{ s}}$

7. A rock is dropped from rest into a well.
    a. If the splash is heard 2.4s later, how far below the top of the well is the water's surface? (The speed of sound in air is 343 m/s.)
    b. If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

(a) $d = \frac{1}{2}(9.80)\,t_1^2$ $d = 336t_2$, where $t_1 + t_2 = 2.40$s. Therefore, $336t_2 = 4.90(2.40 - t_2)^2$, or expanding $4.90t_2^2 - 359.5t_2 + 28.22 = 0$

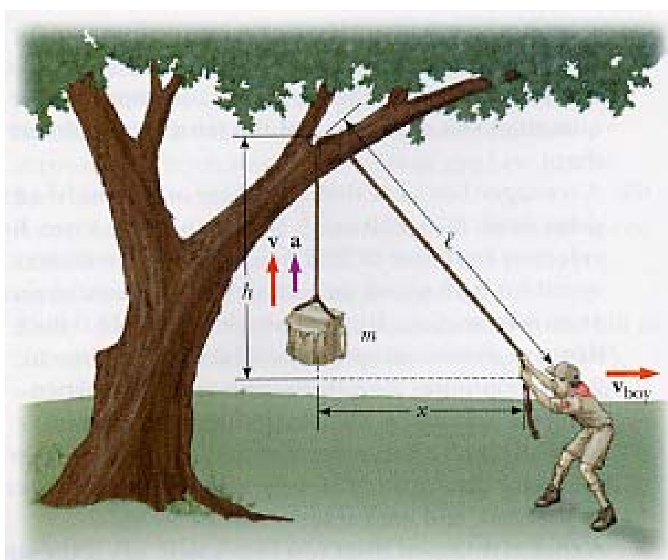
Using the quadratic formula, $t_2 = \dfrac{359.5 \pm \sqrt{(359.5)^2 - 4(4.90)(28.22)}}{9.80} =$

$\dfrac{359.5 \pm 358.75}{9.80} = 0.0765\text{ s}$ $\therefore d = 336t_2 = \boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of

$\boxed{6.82\%}$ .

8. To protect his food from hungry bears, a boy scout raises his food pack with a rope that is thrown over a tree limb at a height *h* above his hands. He walks away from the vertical rope with a constant velocity $v_{boy}$, holding the free end of the rope in his hands as shown below.
   a. Show that the speed v of the food pack is given as $v = v_{boy}\dfrac{x}{\sqrt{x^2 + h^2}}$.
   b. Show that the acceleration of the food pack is $a = v_{boy}^2\dfrac{h^2}{\left(x^2 + h^2\right)^{\frac{3}{2}}}$. (Hint: This part uses calculus, do it for fun if you'd like!!)
   c. What values do the acceleration and velocity have shortly after the boy leaves the point under he pack, at x = 0?
   d. What values do the pack's velocity and acceleration approach as the distance x continues to increases?




(a) In walking a distance $\Delta x$, in a time $\Delta t$, the length of rope $\ell$ is only increased by $\Delta x \sin\theta$. $\therefore$ The pack lifts at a rate $\dfrac{\Delta x}{\Delta t}\sin\theta$.

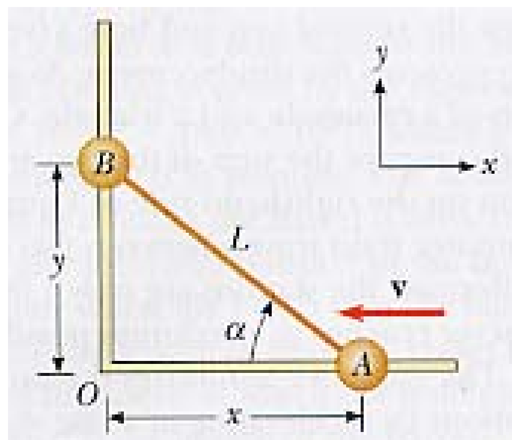
$$v = \frac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\frac{x}{\ell} = \boxed{v_{\text{boy}}\frac{x}{\sqrt{x^2 + h^2}}}$$

(b) $a = \frac{dv}{dt} = \frac{v_{boy}}{\ell}\frac{dx}{dt} + v_{boy}x\frac{d}{dt}\left(\frac{1}{\ell}\right) = v_{boy}\frac{v_{boy}}{\ell} - \frac{v_{boy}x}{\ell^2}\frac{d\ell}{dt}$, but $\frac{d\ell}{dt} = v \therefore a =$
$\frac{v_{boy}^2}{\ell}\left(1 - \frac{x^2}{\ell^2}\right) = \frac{v_{boy}^2}{\ell}\frac{h^2}{\ell^2} = \boxed{\frac{h^2 v_{boy}^2}{(x^2 + h^2)^{3/2}}}$

(c) $\frac{v_{boy}^2}{h}$, 0

(d) $v_{boy}$, 0

9. Two objects $A$ and $B$ are connected by a rigid rod that has a length $L$. The objects like along perpendicular guide rails as shown below. If $A$ slides to the left with constant speed $v$, find the speed of $B$ when $\alpha = 60^o$. (HINT: This problem uses Calculus – do it for fun if you'd like!!)




The distance $x$ and $y$ are always related by $x^2 + y^2 = L^2$. Differentiating this equation with respect to time, we have $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

Now $\frac{dy}{dt}$ is $v_B$, the unknown velocity of $B$; and $\frac{dx}{dt} = -v$.

From the equation resulting from differentiation, we have $\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v)$

But $\frac{y}{x} = \tan\alpha$ so $v_B = \left(\frac{1}{\tan\alpha}\right)v$. When $\alpha = 60.0^o$, $v_B = \frac{v}{\tan 60.0^o} = \frac{v\sqrt{3}}{3} =$ $\boxed{0.577v}$

10. Astronauts on a distant planet toss a rock into the air. With the aid of a camera that takes pictures at a steady rate, they record the height of the rock as a function of time. The values are given in the table below.
    a. What is the average velocity of the rock in the time intervals between each measurement and the next?

b. Using these average velocities to approximate the instantaneous velocities at the midpoints of the time intervals, make a graph of velocity versus time. Does the rock move with a constant acceleration?

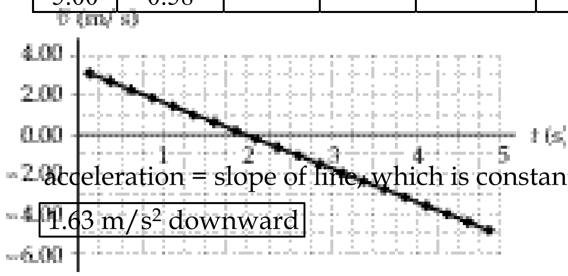
c. If the answer to *b* is yes, what is the acceleration of the rock?

d. What is the acceleration of the rock in g's?

| Time (s) | Height (m) | Time (s) | Height (m) |
|---|---|---|---|
| 0.00 | 5.00 | 2.75 | 7.62 |
| 0.25 | 5.75 | 3.00 | 7.25 |
| 0.50 | 6.40 | 3.25 | 6.77 |
| 0.75 | 6.94 | 3.50 | 6.20 |
| 1.00 | 7.38 | 3.75 | 5.52 |
| 1.25 | 7.72 | 4.00 | 4.73 |
| 1.50 | 7.96 | 4.25 | 3.85 |
| 1.75 | 8.10 | 4.50 | 2.86 |
| 2.00 | 8.13 | 4.75 | 1.77 |
| 2.25 | 8.07 | 5.00 | 0.58 |
| 2.50 | 7.90 | | |

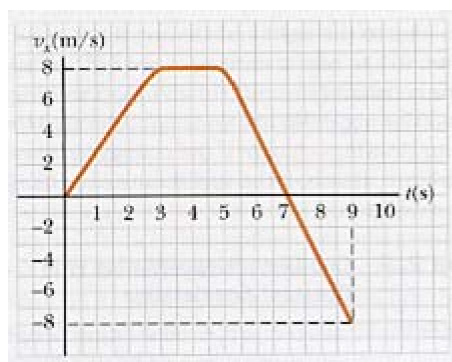
| Time $t$(s) | Height $h$(m) | $\Delta h$ (m) | $\Delta t$ (s) | $\bar{v}$ (m/s) | midpt time $t$ (s) |
|---|---|---|---|---|---|
| 0.00 | 5.00 | | | | |
| | | 0.75 | 0.25 | 3.00 | 0.13 |
| 0.25 | 5.75 | | | | |
| | | 0.65 | 0.25 | 2.60 | 0.38 |
| 0.50 | 6.40 | | | | |
| | | 0.54 | 0.25 | 2.16 | 0.63 |
| 0.75 | 6.94 | | | | |
| | | 0.44 | 0.25 | 1.76 | 0.88 |
| 1.00 | 7.38 | | | | |
| | | 0.34 | 0.25 | 1.36 | 1.13 |
| 1.25 | 7.72 | | | | |
| | | 0.24 | 0.25 | 0.96 | 1.38 |
| 1.50 | 7.96 | | | | |
| | | 0.14 | 0.25 | 0.56 | 1.63 |
| 1.75 | 8.10 | | | | |
| | | 0.03 | 0.25 | 0.12 | 1.88 |
| 2.00 | 8.13 | | | | |
| | | –0.06 | 0.25 | –0.24 | 2.13 |
| 2.25 | 8.07 | | | | |
| | | –0.17 | 0.25 | –0.68 | 2.38 |
| 2.50 | 7.90 | | | | |
| | | –0.28 | 0.25 | –1.12 | 2.63 |
| 2.75 | 7.62 | | | | |
| | | –0.37 | 0.25 | –1.48 | 2.88 |
| 3.00 | 7.25 | | | | |
| | | –0.48 | 0.25 | –1.92 | 3.13 |
| 3.25 | 6.77 | | | | |
| | | –0.57 | 0.25 | –2.28 | 3.38 |

| | | | | | |
|---|---|---|---|---|---|
| 3.50 | 6.20 | | | | |
| | | –0.68 | 0.25 | –2.72 | 3.63 |
| 3.75 | 5.52 | | | | |
| | | –0.79 | 0.25 | –3.16 | 3.88 |
| 4.00 | 4.73 | | | | |
| | | –0.88 | 0.25 | –3.52 | 4.13 |
| 4.25 | 3.85 | | | | |
| | | –0.99 | 0.25 | –3.96 | 4.38 |
| 4.50 | 2.86 | | | | |
| | | –1.09 | 0.25 | –4.36 | 4.63 |
| 4.75 | 1.77 | | | | |
| | | –1.19 | 0.25 | –4.76 | 4.88 |
| 5.00 | 0.58 | | | | |




acceleration = slope of line, which is constant, therefore, $\bar{a}$ = –1.63 m/s$^2$ = $\boxed{1.63 \text{ m/s}^2 \text{ downward}}$

11. A student drives a motorcycle along a straight road as described by the velocity-time graph shown below.
   a. Draw the position versus time graph corresponding to the motion of the motorcycle.
   b. Draw the acceleration versus time graph corresponding to the motion of the motorcycle.
   c. What is the acceleration at t = 6s?
   d. Find the position of the particle (relative to the starting point) at t = 6s.
   e. What is the motorcycle's final position at t = 9s?




(a)

Choose $x = 0$ at $t = 0$

At $t = 3$ s, $x = \frac{1}{2}(8 \text{ m/s})(3 \text{ s}) = 12$ m

At $t = 5$ s, $x = 12 \text{ m} + (8 \text{ m/s})(2 \text{ s}) = 28$ m

At $t = 7$ s, $x = 28 \text{ m} + \frac{1}{2}(8 \text{ m/s})(2 \text{ s}) = 36$ m

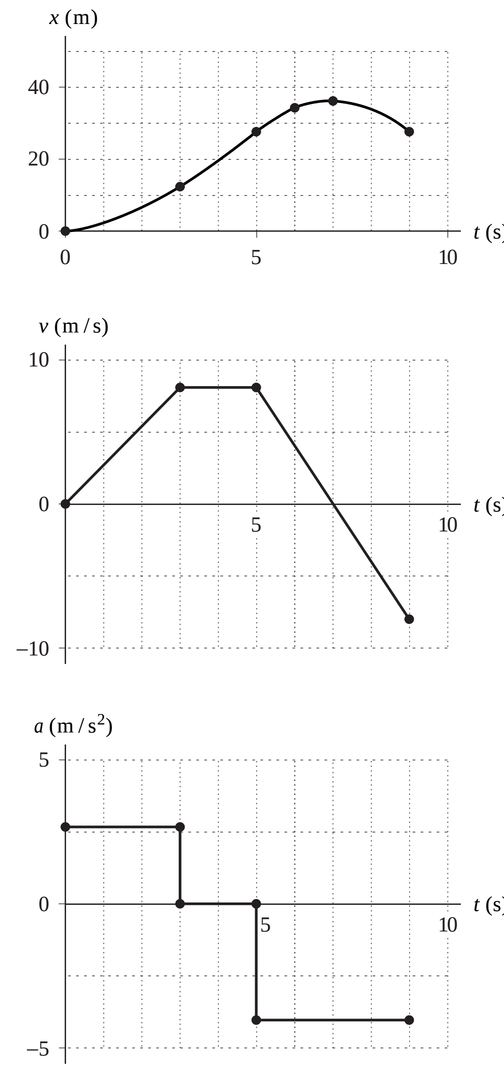
(b) For $0 < t < 3$ s, $a = (8 \text{ m/s})/3 \text{ s} = 2.67 \text{ m/s}^2$

For $3 < t < 5$ s, $a = 0$

(c) For $5 \text{ s} < t < 9$ s, $a = -(16 \text{ m/s})/4 \text{ s} =$ $\boxed{-4 \text{ m/s}^2}$

(d) At $t = 6$ s, $x = 28 \text{ m} + (6 \text{ m/s})(1 \text{ s}) =$ $\boxed{34 \text{ m}}$

(e) At $t = 9$ s, $x = 36 \text{ m} + \frac{1}{2}(-8 \text{ m/s})\ 2 \text{ s} =$ $\boxed{28 \text{ m}}$




12. A speeder moves at a constant 15 m/s in a school zone. A police car starts from rest just as the speeder passes it. The police car accelerates at 2 m/s$^2$ until it reaches a maximum velocity of 20 m/s. Where and when does the speeder get caught?

For the portion of the motion in which the cop accelerates we have: $v_{f,cop} = v_{i,cop} + a_{cop}t \rightarrow 20\frac{m}{s} = 2\frac{m}{s^2}t \rightarrow t = 10s.$. In this time, the cop has traveled a distance of $x_{f,cop} = x_{i,cop} + v_{i,cop}t + \frac{1}{2}a_{cop}t^2 = \frac{1}{2}\left(2\frac{m}{s^2}\right)(10s)^2 = 100m$ and the car has traveled $x_{f,car} = v_{i,car}t = 15\frac{m}{s} \times 10s = 150m.$

Now, let these positions be the initial positions of the cop and car and the cop will catch the speeder when they are at the same final location at the same time. To solve for the final position and time we use $x_{f,cop} = x_{i,cop} + v_{i,cop}t_{catch}$ and $x_{f,car} = x_{i,car} + v_{i,car}t_{catch}$. setting these two equations equal, we have

$$x_{i,cop} + v_{i,cop}t_{catch} = x_{i,car} + v_{i,car}t_{catch} \rightarrow 100m + 20\tfrac{m}{s}t_{catch} = 150m + 15\tfrac{m}{s}t_{catch} \rightarrow t_{catch} = 10s$$

. Therefore, the time it takes the cop to catch the speeder is 20 seconds (10s for the acceleration portion and 10s for the constant velocity portion of the motion) and the cop catches the speeder at 300m (=150m + 15m/s(10s)) from the cops starting position.