Physics 120 Homework
Problem 2.P.58 & 2.P.59
SL

2.P.58

a. From the momentum principle, the velocity is given by
$\vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}}{m}\Delta t = \langle -11,16,-6\rangle \tfrac{m}{s} + \left(\langle 0,-9.8,0\rangle \times 0.5\right)\tfrac{m}{s} = \langle -11,11.1,-6\rangle \tfrac{m}{s}$.
b. The arithmetic average would give the best estimate of the location of the ball over this time interval because the force is constant.
c. The average velocity is
$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\langle -11,16,-6\rangle \tfrac{m}{s} + \langle -11,11.1,-6\rangle \tfrac{m}{s}}{2} = \langle -11,13.6,-6\rangle \tfrac{m}{s}$.
d. The location of the ball is given by the position-update. We have
$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \langle 9,0,-6\rangle m + \left(\langle -11,13.6,-6\rangle \tfrac{m}{s} \times 0.5s\right) = \langle 3.5,6.8,-9\rangle m$.
e. When the ball reaches its maximum vertical height, the y-component of its velocity is zero.
f. From the momentum principle we have $v_{fy} = v_{iy} + \frac{F_{net,y}}{m}\Delta t \Rightarrow 0\tfrac{m}{s} = 16\tfrac{m}{s} - 9.8\tfrac{m}{s^2}\Delta t$.
g. Using part f, we can solve to the time to reach maximum height and we get 1.63s.
h. The y-component of the average velocity is given as
$v_{avg,y} = \frac{v_{iy} + v_{fy}}{2} = \frac{v_{iy}}{2} = \frac{16\tfrac{m}{s}}{2} = 8\tfrac{m}{s}$. Then the maximum height is determined from the position-update and we have $y_f = y_i + v_{avg,y}\Delta t = 0 + \left(8\tfrac{m}{s} \times 1.63s\right) = 13.1m$.

2.P.59

a. The velocity vector when the projectile is at maximum height is given from
$\vec{v}_f = \langle v_{fx}, v_{fy}, v_{fz}\rangle = \langle 5,0,0\rangle \tfrac{m}{s}$.
b. The vertical component of the velocity is given by
$v_{fy} = v_{iy} - \frac{F_E}{m}\Delta t \rightarrow 0 = 8\tfrac{m}{s} - 9.8\tfrac{m}{s^2}\Delta t_{rise} \rightarrow \Delta t_{rise} = \frac{8\tfrac{m}{s}}{9.8\tfrac{m}{s^2}} = 0.82s$.
c. The average velocity is given by $\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\langle 5,8,0\rangle \tfrac{m}{s} + \langle 5,0,0\rangle \tfrac{m}{s}}{2} = \langle 5,4,0\rangle \tfrac{m}{s}$.
d. The position of the ball is given from the position update formula:
$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \langle 0,0,0\rangle m + \langle 5,4,0\rangle \tfrac{m}{s} \times 0.82s = \langle 4.1,3.3,0\rangle m$.
e. Since the ball returns to the same height at which it was launched, the time of flight is twice the rise time, 1.64s. Another way that this can be calculated is from the position update:
$\vec{r}_f = \langle x_f, y_f, z_f\rangle = \left\langle (x_i + v_{ix}\Delta t), \left(y_i + v_{iy}\Delta t - \frac{1}{2}g\Delta t^2\right), (z_i + v_{iz}\Delta t)\right\rangle$. The vertical -

motion will give us the time of flight. Therefore

$$y_f = y_i + v_{iy}\Delta t - \frac{1}{2}g\Delta t^2 \rightarrow 0 = 0 + 8\Delta t - 4.9\Delta t^2 \rightarrow \left\{\begin{matrix} 0s \\ 1.63s \end{matrix}\right\}.$$

f. The position update will also determine the horizontal range. Therefore $x_f = x_i + v_{ix}\Delta t = 0 + \left(5\tfrac{m}{s} \times 1.63s\right) = 8.2m$.

g. The vertical component of the velocity at maximum horizontal range is given by:

$$v_{fy} = v_{iy} - \frac{F_E}{m}\Delta t_{tof} = 8\tfrac{m}{s} - 9.8\tfrac{m}{s^2} \times 1.63s = -7.97\tfrac{m}{s} \sim -8\tfrac{m}{s}$$

h. The launch angle is given by: $\tan\theta = \frac{v_{iy}}{v_{ix}} \rightarrow \theta_{launch} = \tan^{-1}\left(\frac{v_{iy}}{v_{ix}}\right) = \tan^{-1}\left(\frac{8}{5}\right) = 58^0$.

i. The impact angle is given by: $\tan\theta = \frac{v_{iy}}{v_{ix}} \rightarrow \theta_{impact} = \tan^{-1}\left(\frac{v_{iy}}{v_{ix}}\right) = \tan^{-1}\left(\frac{-8}{5}\right) = -58^0$ or 58^o below the +x-axis.