Physics 110
Spring 2006
2-D Motion Problems: Projectile Motion – Their Solutions

1. A place-kicker must kick a football from a point 36 m (about 40 yards) from the goal, and half the crowd hopes the ball will clear the crossbar, which is 3.1m high. When kicked the all leaves the ground with a speed of 20 m/s at an angle of 53^0 to the horizontal.
   a. Does the ball clear or fall short of the crossbar?
   b. Does the ball approach the crossbar while still rising or while falling?

(a) From our equations of motion, the horizontal velocity is constant. This gives us the flight time for any horizontal distance starting with initial x velocity $v_i\cos\theta$. Thus the vertical height of the trajectory is given as $y = x \tan\theta_i - \dfrac{gx^2}{2v_i^2\cos^2\theta_i}$. With $x = 36.0$ m, $v_i = 20.0$ m/s, and $\theta_i =$ 53.0°, we find $y = (36.0\text{ m})(\tan 53.0°) - \dfrac{(9.80\text{ m/s}^2)(36.0\text{ m})^2}{(2)(20.0\text{ m/s})^2\cos^2 53.0°} = 3.94$ m. The ball clears the bar by $(3.94 - 3.05)$ m $= \boxed{0.889\text{ m}}$.

(b) The time the ball takes to reach the maximum height is $t_1 = \dfrac{v_i \sin q_i}{g} = \dfrac{(20.0\text{ m/s})(\sin 53.0°)}{9.80\text{ m/s}^2} = 1.63$ s. The time to travel 36.0 m horizontally is $t_2 = \dfrac{x}{v_{ix}}$, which gives $t_2 = \dfrac{36.0\text{ m}}{(20.0\text{ m/s})(\cos 53.0°)} = 2.99$ s.

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

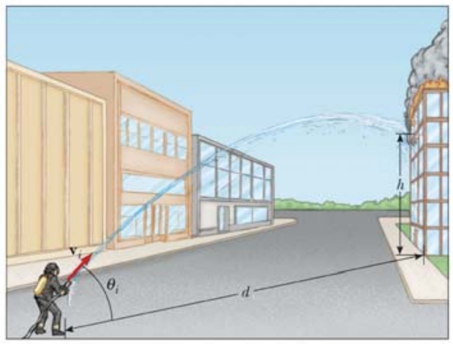
2. A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8m/s at an angle of 20^0 below the horizontal, where it strikes the ground 3 seconds later.
   a. How far horizontally from the base of the building does the ball strike the ground?
   b. At what height was the ball thrown?
   c. How long does it take the ball to reach a point 10m below the level of launching?

(a) $x = v_{xi}t = (8.00\cos 20.0°)(3.00) = \boxed{22.6\text{ m}}$

(b) Taking y positive downwards, $y = v_{yi}t + \frac{1}{2}gt^2 = 8.00(\cos 20.0°)3.00 + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}$

(c) $10.0 = 8.00 \cos 20.0° \, t + \frac{1}{2}(9.80)\, t^2$, which gives a quadratic in t. The solutions to $4.90t^2 + 2.74t - 10.0 = 0$ are given by the quadratic formula, $t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80}$ = $\boxed{1.18 \text{ s}}$

3. A firefighter 50m away from a burning building directs a stream of water from a fire hose at an angle of 30^0 above the horizontal as shown below. If the speed of the stream is 40m/s, at what height will the water strike the building?



From our equations of motion, the horizontal velocity is constant. This gives us the flight time for any horizontal distance starting with initial x velocity $v_i \cos\theta$.

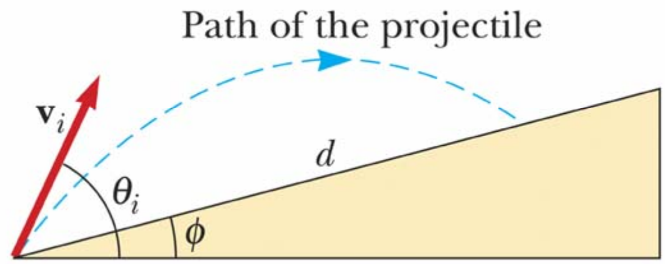
Thus the vertical height of the trajectory is given as $y = x \tan \theta_i - \frac{gx^2}{2v_i^2 \cos^2 \theta_i}$.

Substituting the known quantities, we find h = 18.7m.

4. As some molten metal splashes off of the ground, one droplet flies off to the east with an initial speed v_i at an angle θ_i above the horizontal while the other drop flies off to the west with the same speed and at the same angle above the horizontal. In terms of v_i and θi, find the distance between the droplets as a function of time.

At any time t, the two drops have identical y-coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore, $d = 2|x(t)| = 2(v_{xi}t) = 2(v_i \cos \theta_i)t = \boxed{2v_i t \cos \theta_i}$.

5. A projectile is fired up an incline (incline angle ϕ) with an initial speed v_i at an angle θ_i with respect to the horizontal ($\theta_i > \phi$) as shown below.



a. Show that the projectile travels a distance d up the incline, where d is given as $d = \dfrac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}$.

(a) $y = \tan(\theta_i)\, x - \dfrac{g}{2v_i^2 \cos^2(\theta_i)} x^2$. Setting $x = d\cos(\phi)$, and $y = d\sin(\phi)$, we have

$d\sin(\phi) = \tan(\theta_i) d\cos(\phi) - \dfrac{g}{2v_i^2 \cos^2(\theta_i)} (d\cos(\phi))^2$. Solving for d yields,

$$d = \frac{2v_i^2 \cos(\theta_i)\,[\sin(\theta_i)\cos(\phi) - \sin(\phi)\cos(\theta_i)]}{g\cos^2(\phi)} \quad \text{or} \quad d = \boxed{\frac{2v_i^2 \cos(\theta_i)\sin(\theta_i - \phi)}{g\cos^2(\phi)}}.$$

6. A student decides to measure the muzzle velocity of the pellets from his BB gun, which is pointed horizontally. The shots hit the target a vertical distance y below the gun.
   a. Show that the vertical displacement component of the pellets when traveling through the air is given by $y = Ax^2$, where A is a constant.
   b. Express the constant A in terms of the initial velocity of the projectiles and the acceleration due to gravity.
   c. If x = 3.000m, and y = 0.210m, what is the initial speed of the pellets?

(a)(b) Since the shot leaves the gun horizontally, the time it takes to reach the target is $t = \dfrac{x}{v_i}$.

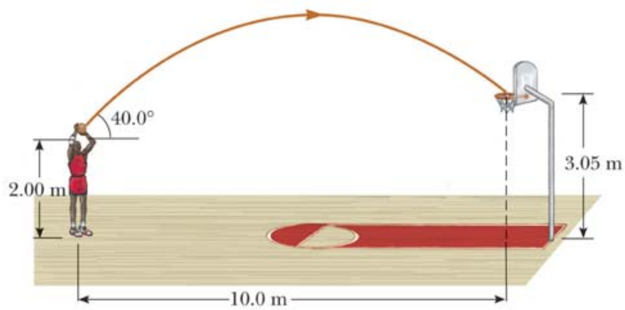
The vertical distance traveled in this time is $y = -\dfrac{1}{2} g t^2 = -\dfrac{g}{2}\left(\dfrac{x}{v_i}\right)^2 = Ax^2$ where

$$\boxed{A = -\frac{g}{2v_i^2}}$$

(c) If $x = 3.00$ m, $y = -0.210$ m, then $A = \dfrac{-0.210}{9.00} = -2.33 \times 10^{-2}$, so that

$$v_i = \sqrt{\frac{-g}{2A}} = \sqrt{\frac{-9.80}{-4.66 \infty 10^{-2}}} \text{ m/s} = \boxed{14.5 \text{ m/s}}$$

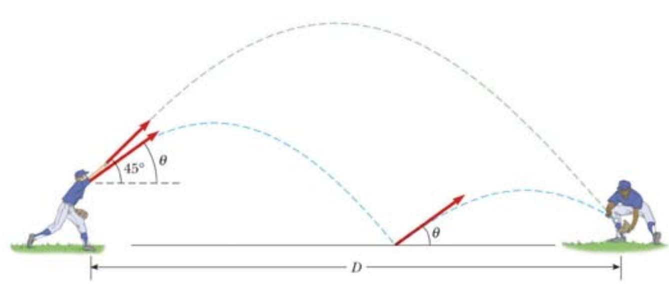
7. A basketball player who is 2m tall is standing on the floor 10m from the basket. If the ball is shot at a 40° angle with the horizontal, at what initial speed must the ball be thrown so it goes through the hoop without striking the backboard? Assume that the basket height is 3.05m



$x = v_{ix}t = v_i t \cos 40.0°$ Thus, when $x = 10.0$ m, $t = \dfrac{10.0 \text{ m}}{v_i \cos 40.0°}$. At this time, y should be 3.05 m – 2.00 m = 1.05 m. Thus, 1.05 m = $\dfrac{(v_i \sin 40.0°)\, 10.0 \text{ m}}{v_i \cos 40.0°}$ +

$\frac{1}{2}(9.80 m/s^2)\left[\dfrac{10.0m}{v_i \cos 40.0^o}\right]^2$. From this, v_i = $\boxed{10.7 \text{ m/s}}$

8. When baseball players throw the ball in from the outfield, they usually allow it to take one bounce before it reaches the infielder on the theory that the ball arrives sooner. Suppose that the angle at which the bounced ball leaves the ground is the same as the angle at which the outfielder launched it but that the ball's speed after the bounce is one half of what it was before the bounce.
   a. Assuming the ball is thrown at the same initial speed, at what angle θ, should the ball be thrown in order to go the same distance D with one bounce (blue path) as the ball thrown upward at 45° with no bounce (green path)?
   b. Determine the ratio of the times for the one-bounce and no bounce throws.



a. Assuming that the projectile starts and ends at the same height (which is not strictly true) we can use the equations for the range of the projectile being thrown at 45°, the green curve, to find $R_{45} = \dfrac{v_i^2}{g}\sin 90$. For the ball that bounces one time, the blue curve we have $R_{bounce} = R_1 + R_2 = \frac{v_i^2}{g}\sin 2\theta + \frac{\left(v_i/2\right)^2}{g}\sin 2\theta$. Here we require the two equations for the range to be equal, and this determines the angle q for the blue path. Thus we have,

$$R_{45} = R_{bounce} \to \frac{v_i^2}{g}\sin 90 = \frac{v_i^2}{g}\sin 2\theta + \frac{\left(v_i/2\right)^2}{g}\sin 2\theta \to \sin 2\theta = \frac{4}{5} \to \theta = 26.6^o.$$

b. The time for any symmetric parabolic flight is given as $y_f = y_i + (v_i \sin\theta)t - \frac{1}{2}gt^2 \to 0 = ((v_i \sin\theta) - \frac{1}{2}gt)t$ where the solution t = 0 is the time the ball is thrown and the time $t = \frac{2v_i \sin\theta}{g}$ is the time to land. At 45^{o}, we find the landing time to be $t_{45} = \frac{2v_i \sin 45}{g}$ and for the ball bouncing at 26.6^{o}, the total landing time is given as

$$t_{total\ bouncing} = t_1 + t_2 = \frac{2v_i \sin 26.6}{g} + \frac{2\left(v_i/2\right)\sin 26.6}{g} = \frac{3v_i \sin 26.6}{g}.$$

Thus the ratio of the time for bouncing to the time for no bounce is $\frac{t_{total\ bouncing}}{t_{no\ bounce}} = \frac{3v_i g \sin 26.6}{2v_i g \sin 45} = \frac{1.34}{1.41} = 0.949$. This gives the time for bouncing very slightly less than the time with no bouncing.

9. A Northrop B-2 Stealth bomber is flying horizontally over level ground, with a speed of 275m/s at an altitude of 3000m. Neglect air resistance in the following problems.
   a. How far will a bomb travel horizontally between its release and its impact on the ground?
   b. If the plane maintains its original course and speed,
      where will it be when the bomb hits the ground?
   c. At what angle from the vertical should the bombsight be set so that the bomb will hit the target seen in the sight at the time of release?

http://es.geocities.com/scuadrafotos/B-2-6.jpg

(a) $\Delta y = -\frac{1}{2}gt^2$; $\Delta x = v_i t$. Combine the equations eliminating t: $\Delta y = -\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2$ from this $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right)v_i^2$. Thus $\Delta x = v_i\sqrt{\frac{-2\,\Delta y}{g}} = 275\sqrt{\frac{-2(-300)}{9.80}} = 6.80 \infty 10^3$ m = $\boxed{6.80\text{ km}}$

(b) The plane has the same velocity as the bomb in the x direction. Therefore, the plane will be $\boxed{\text{3000 m directly above the bomb}}$ when it hits the ground.

(c) When θ is measured from the vertical, $\tan\theta = \frac{\Delta x}{\Delta y}$; therefore,

$$\theta = \tan^{-1}\frac{\Delta x}{\Delta y} = \tan^{-1}\left(\frac{6800}{3000}\right) = \boxed{66.2}□$$

10. A person standing at the top of a hemispherical rock of radius R kicks a ball (initially at rest) to give it a horizontal velocity v_i.
    a. What must be the rock's minimum speed if the ball is never to hit the rock after it is kicked?
    b. With this initial speed, how far from the base of the rock does the ball hit the ground?

Measure heights above the level ground. The elevation y_b of the ball follows

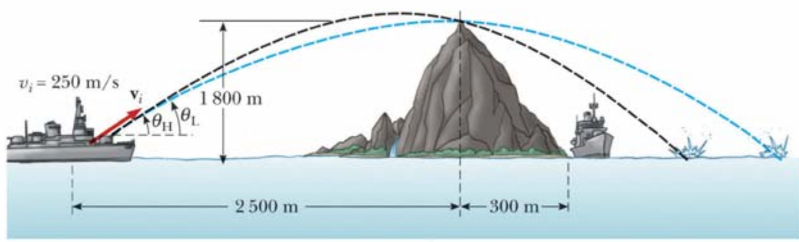
$y_b = R + 0 - \frac{1}{2}gt^2$ with $x = v_i t$ so $y_b = R - gx^2/2v_i^2$

(a) The elevation y_r of points on the rock is described by $y_r^2 + x^2 = R^2$. We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

Thus, $\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$, or $R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$ and finally, $\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$. We get the strictest requirement for x approaching zero. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1 > \frac{gR}{v_i^2}$, or when $\boxed{v_i > \sqrt{gR}}$.

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$ or $x = \sqrt{2}\,R$. The distance from the rock's base is $x - R = \boxed{(\sqrt{2} - 1)R}$

11. An enemy ship is on the western side of a mountain island. The enemy ship can maneuver to within 2500 m of the 1800 m high mountain peak and can shoot projectiles with an initial speed of 250m/s. If the eastern shore line is horizontally 300m from the peak, what are the distances from the eastern shore at which a shop can be safe from the bombardment of the enemy ship?



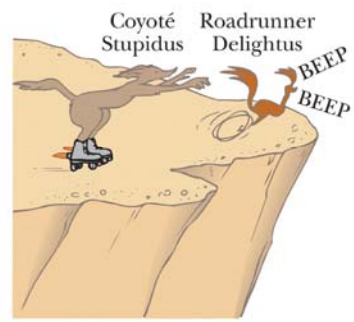
For this problem, we need θ_{high} and θ_{low}. From the diagram, θ_{high} will set the highest elevation that will clear the mountain and given the closest range, while θ_{low} will set the lowest elevation that will clear the mountain and given the farthest range. To find these angles we use our displacement equations in the horizontal and vertical directions. Thus we have $x = (v_i \cos\theta)t$ and $y = (v_i \sin\theta)t - \frac{1}{2}gt^2$. Solving the horizontal equation of motion for t and substituting this into the vertical equation of motion produces $y = x\tan\theta - \frac{gx^2}{2v_i^2} \times \frac{1}{\cos^2\theta}$. Since $\sin^2\theta + \cos^2\theta = 1$, dividing by $\cos^2\theta$ produces $\tan^2\theta + 1 = \frac{1}{\cos^2\theta}$ Therefore we have a quadratic in tan θ that will determine θ_{high} and $\theta_{low.}$ For the values of x = 2500m and y = 1800m, with v_i = 250 m/s and g = 9.8m/s^2, we find tan θ = {1.197, 3.906}. Using the first number we have θ_{low} = 50.1° and the second number θ_{high} = 75.6°.

The range at θ_{high}: $R_{\theta_{high}} = \frac{v_i^2 \sin 2\theta_{high}}{g} = \frac{(250\,\frac{m}{s})^2 \sin(151.2)}{9.8\,\frac{m}{s^2}} = 3.07\times10^3\,m.$ Thus the closest distance to the shore is 3.07x10^{3}m – 300m – 2500m = 270m from shore.

The range at θ_{low}: $R_{\theta_{low}} = \frac{v_i^2 \sin 2\theta_{high}}{g} = \frac{(250\,\frac{m}{s})^2 \sin(100.2)}{9.8\,\frac{m}{s^2}} = 6.28\times10^3\,m.$ Thus the closest distance to the shore is 6.28x10^{3}m – 300m – 2500m = 3.48 x10^{3}m from shore.

So the safe distances are less than 270m or greater than 3480m from shore.

12. The determined coyote is out once more to try to capture the elusive roadrunner. The coyote wears a pair of Acme jet-powered roller skates, which provide a constant horizontal acceleration of 15 m/s^2. The coyote starts off at rest 70m from the edge of a cliff at the instant the roadrunner zips past him in the direction of the cliff.
    a. If the roadrunner moves with constant speed, determine the minimum speed he must have to reach the cliff before the coyote.
    b. At the brink of the cliff the roadrunner escapes by making a sudden turn, while the coyote continues straight off of the cliff. If the cliff is 100m above the floor of a canyon, where does the coyote land, assuming that his skates remain horizontal and continue to work while in flight?
    c. What are the components of the coyote's impact velocity?



(a) Coyote: $\Delta x = \frac{1}{2}at^2$; $70.0 = \frac{1}{2}(15.0)t^2$: Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$. Solving the above, we get $v_i = \boxed{22.9 \text{ m/s}}$ and $t = 3.06$ s

(b) At the edge of the cliff $v_{xi} = at = (15.0)(3.06) = 45.8$ m/s, $\Delta y = \frac{1}{2}a_y t^2$.

Substituting we find $-100 = \frac{1}{2}(-9.80)t^2$ and $\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 = (45.8)t + \frac{1}{2}(15.0)t^2$.

Solving the above gives $\Delta x = \boxed{360 \text{ m}}$ $t = 4.52$ s

(c) For the Coyote's motion through the air $v_{xf} = v_{xi} + a_x t = 45.8 + 15(4.52) = \boxed{114 \text{ m/s}}$

And $v_{yf} = v_{yi} + a_y t = 0 - 9.80(4.52) = \boxed{-44.3 \text{ m/s}}$