Physics 111
Fall 2007
Electric Currents and DC Circuits

1. What is the average current when all the sodium channels on a 100 μm^2 patch of muscle membrane open together for 1 ms? Assume a density of 50 sodium channels per μm^2 of surface and a flow rate of 1000 ions per ms through each channel.

2. The immediate cause of many deaths is ventricular fibrillation, an uncoordinated quivering of the heart as opposed to proper beating. An electric shock to the chest can cause momentary paralysis of the heart muscle, after which the heart will sometimes start organized beating again. A *defibrillator* is a device that applies a strong electric shock to the chest over a time interval of a few milliseconds. The device contains a capacitor of several microfarads, charged to several thousand volts. Electrodes called paddles, about 8 cm across and coated with conducting paste, are held against the chest on both sides of the heart. Their handles are insulated to prevent injury to the operator, who calls "Clear!" and pushes a button on one paddle to discharge the capacitor through the patient's chest. Assume that an energy of 300 J is to be delivered from a 30.0-μF capacitor.

   a. To what potential difference must it be charged?

   Consider the following, a *defibrillator* connected to a 32 μF capacitor and a 47 kΩ resistor in a RC circuit. The circuitry in this system applies 5000 V to the RC circuit to charge it.

   b. What is the time constant of this circuit?

   c. What is the maximum charge on the capacitor?

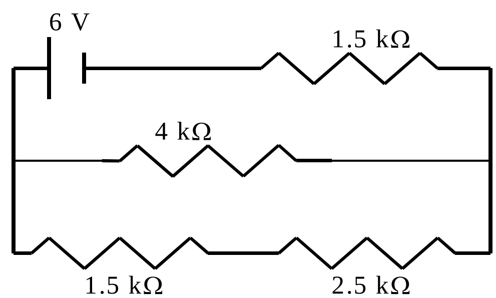
   d. What is the maximum current in the circuit during the charging process?

   e. What are the charge and current as functions of time?

   f. How much energy is stored in the capacitor when it is fully charged?

3. An immersible heater coil is to be designed to heat an insulated container with 4 liters of distilled water from 20^o to 50^o C in less than 30 minutes.
   a. How much energy must be input to heat the water to this temperature?
   b. To heat the water, what minimum power must be supplied?
   c. If a 12 V power supply is to be used, what minimum current must flow in the heating coil?
   d. What must be the total resistance of the heating coil? Is this a maximum or minimum resistance to heat the water in 30 minutes or less?

4. Analyze the circuit shown below to find the currents flowing through and the power generated in each resistor.



5. *RC* time constants can be easily estimated by measuring the time (known as the half-time) for the capacitor voltage to decrease to half of some arbitrary starting value when discharging through a resistor. Further, we know that the voltage across the capacitor will vary as $V(t) = V_o e^{-\frac{t}{RC}}$. Show how a single measurement of the half-time can be used to determine the *RC* time constant. (Hint: substitute $V(t) = V_o/2$)

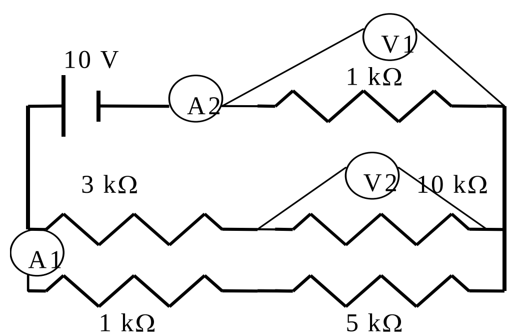
6. A simple *RC* series circuit has a 100 μF capacitor.
   a. If the time constant is 50 s, what is the value of the resistor?
   b. Suppose that a second identical resistor is inserted in series with the first. What is the new time constant of the circuit?
   c. Suppose the second identical resistor is placed in parallel with the first resistor, still connected to the capacitor. What is the new time constant in this case?

d. Suppose that we use the single resistor from part a, but now a second identical capacitor is connected in series with the first. What is the new time constant of the circuit?

e. Suppose that we use the single resistor from part a, but now a second identical capacitor is connected in parallel with the first. What is the new time constant of the circuit?

7. A 100 μF capacitor wired in a simple series *RC* circuit is initially charged to 10 μC and then discharged through a 10 kΩ resistor.
   a. What is the time constant of the circuit?
   b. What is the initial current that flows?
   c. How much charge is left on the capacitor after 1 time constant?
   d. What is the current after 1 time constant?
   e. How much charge is left on the capacitor after 3 time constants have elapsed and what current is flowing then?

8. Find the reading that each of the (ideal) meters would have in the following circuit.



9. Two 100 m 14 gauge wires (1.63 mm diameters), one of copper and one of aluminum, are soldered together and the 200 m wire is then connected to a 6 V dc power supply with unlimited current.
   a. How much current flows in the wire?
   b. What is the potential across each 100 m section of wire?
   c. How much power is developed in each section of wire?

d. How much total power was delivered by the battery? Does this equal the power dissipated by the circuit? If not, where did the extra energy go?

10. The Earth's atmosphere is able to act as a capacitor, with one plate the ground and the other the clouds and in between the plates an air gap. Air, however, is not a perfect insulator and can be made to conduct, so that the separation of charges from the cloud to ground can be bridged. Such an event is called a lightning strike. For this question, we'll model the atmosphere as a spherical capacitor and try to calculate the number of lightning strikes that happen every day. First, let's assume that the clouds are distributed around the entire Earth at a distance of 5000 m above the ground of area $4pR^2_{Earth}$, where R_{Earth} = 6400km.

    a. What is the resistance of the air gap?

    Next we need to calculate the capacitance of the Earth-cloud capacitor. Here we will use the fact that the capacitance $C = \frac{Q}{\Delta V}$ and we'll calculate ΔV. Assuming that we have a spherical charge distribution $V = k\frac{Q}{r}$, so that ΔV is the difference in potential between the lower plate (the Earth's surface) and the upper plate (the clouds) and that in a typical day, $5x10^5$ C of charge is spread over the surface of the Earth.

    b. What is ΔV?

    c. What is the capacitance of the Earth-cloud capacitor?

    Since the accumulated charge will dissipate through the air, we have a simple RC circuit.

    d. What is the time constant for this discharge that is spread over the whole surface of the earth?

    Experimentally it is found that for each lightning strike about 25 C of negative charge is delivered to the ground.

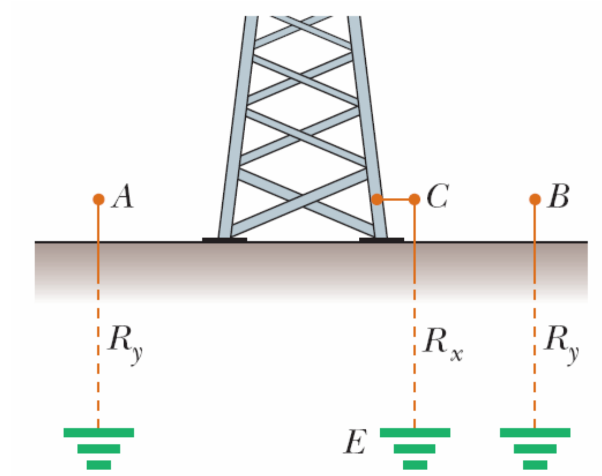
    e. What is the number of lightning for this amount of charge?

    f. Approximately how long would it take the Earth-cloud capacitor to discharge to 0.3% of its initial amount?

    So to answer the question of how many lightning strikes per day, we know that we get part e number of strokes in part f amount of time.

g. How many lightning strokes per day?

11. The student engineer of a campus radio station wishes to verify the effectiveness of the lightning rod on the antenna mast as shown below. The unknown resistance R_x is between points C and E. Point E is a true ground but is inaccessible for direct measurement because this stratum is several meters below the Earth's surface. Two identical rods are driven into the ground at A and B, introducing an unknown resistance R_y. The procedure is as follows. Measure resistance R_1 between points A and B, then connect A and B with a heavy conducting wire and measure resistance R_2 between points A and C.



(a) Derive an equation for R_x in terms of the observable resistances, R_1 and R_2.

(b) A satisfactory ground resistance would be $R_x < 2.00\ \mathbf{\Omega}$. Is the grounding of the station adequate if measurements give $R_1 = 13.0\ \mathbf{\Omega}$ and $R_2 = 6.00\ \mathbf{\Omega}$?