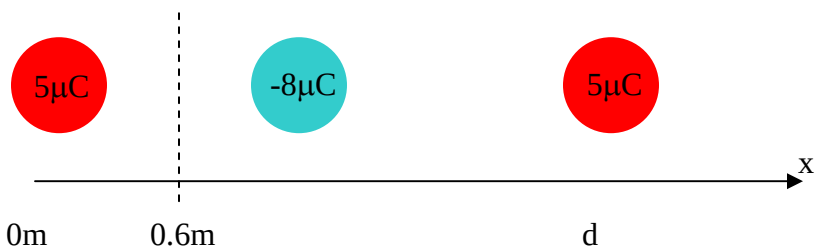
# Physics 111
# Fall 2007
# Electrostatic Forces and the Electric Field - Solutions

1. Two point charges, 5 μC and -8 μC are 1.2 m apart. Where should a third charge, equal to 5 μC, be placed to make the electric field at the mid-point between the first two charges equal to zero?

   From the diagram below we see that the charge has to be to the right of the -5μC charge located at some distance *d*. Measuring distances from the leftmost charge, here the 5μC charge, we'll place the additional 5μC charge at a distance *d* away. At the midpoint between the first two charges the electric field has to vanish, so we have $\vec{E}_{net} = 0 = k\left[\frac{5\times10^{-6}C}{(0.6m)^2} + \frac{8\times10^{-6}C}{(0.6m)^2} - \frac{5\times10^{-6}C}{(d-0.6m)^2}\right]\hat{i}$. Solving for *d* we find, *d* = *0.97m* from the leftmost charge.

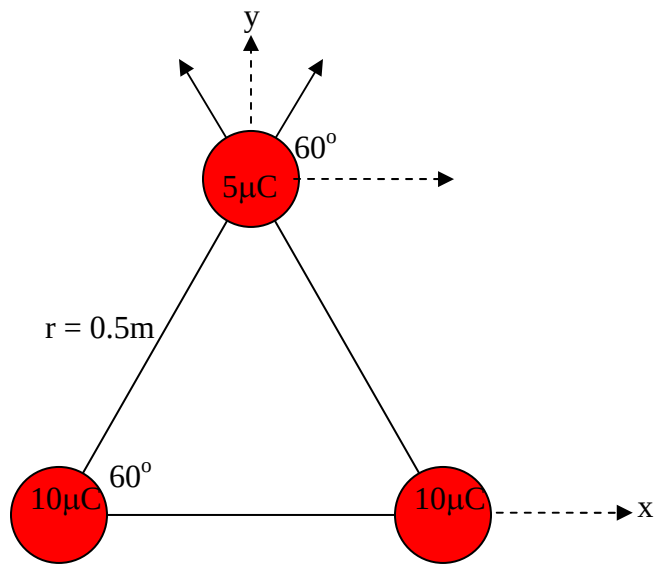



2. The electric field inside biological membranes is extremely high, roughly 1 x $10^7$ N/m. If this electric field generated the only force on a sodium ion, what would its acceleration be?

   Here we need the mass of a sodium ion. The atomic mass of sodium is 22 amu, so assuming that the proton and neutron have the same mass ($1.67\times10^{-27}$ kg) we have $3.67\times10^{-26}$ kg for the mass of the sodium ion. The charge on the ion is $1e^- = 1.6\times10^{-19}$ C. Thus the acceleration from Newton's 2nd law is $a = \frac{q}{m}E = \frac{1.6\times10^{-19}C\times1\times10^{7}\frac{N}{m}}{3.67\times10^{-26}kg} = 4.35\times10^{13}\frac{m}{s^2}$.

3. Find the force on a 5 μC point charge located at a vertex on an equilateral triangle of 0.5 m sides if 10 μC point charges are located at the other two vertices.

   From the diagram below and the symmetry in the problem, we see that the horizontal components of the force cancel while the vertical components of the force add. Thus




$$F_x = 0N$$

$$F_y = 2k\frac{Q_1Q_2}{r^2}\sin 60 = 2\times 9\times 10^9\ \tfrac{Nm^2}{C^2}\left(\frac{5\times 10^{-6}C\times 10\times 10^{-6}C}{(0.5m)^2}\right)\sin 60 = 3.12N$$

$$\vec{F}_{net} = 3.12N\hat{j} = 3.12N \quad \text{in the positive y-direction}$$

4. How close must two electrons be if the electric force between them is equal to the weight of either at the Earth's surface?

Set the magnitude of the electric force equal to the magnitude of the force of gravity and solve for the distance.

$$F_E = F_G \quad \rightarrow \quad k\frac{e^2}{r^2} = mg \quad \rightarrow$$

$$r = e\sqrt{\frac{k}{mg}} = \left(1.602\times 10^{-19}\,\text{C}\right)\sqrt{\frac{\left(8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(9.11\times 10^{-31}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}} = \boxed{5.08\,\text{m}}$$

5. A proton $(m = 1.67\times 10^{-27}\ \text{kg})$ is suspended at rest in a uniform electric field $\vec{\mathbf{E}}$. Take into account gravity at the Earth's surface, and determine $\vec{\mathbf{E}}$.

Since the gravity force is downward, the electric force must be upward. Since the charge is positive, the electric field must also be upward. Equate the magnitudes of the two forces and solve for the electric field.

$$F_E = F_G \quad \rightarrow \quad qE = mg \quad \rightarrow \quad E = \frac{mg}{q} = \frac{\left(1.67\times 10^{-27}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.602\times 10^{-19}\,\text{C}\right)} = \boxed{1.02\times 10^{-7}\ \text{N/C, up}}$$

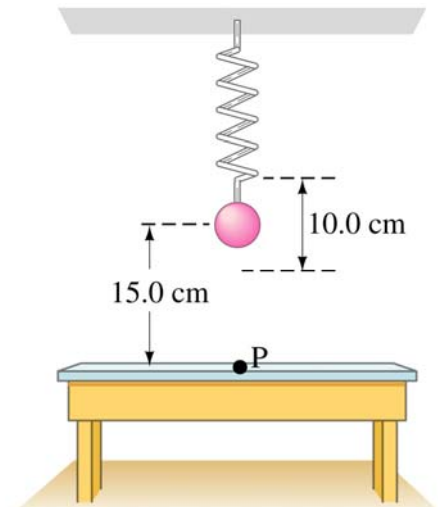
6. In a simple model of the hydrogen atom, the electron revolves in a circular orbit around the proton with a speed of $1.1\times10^{6}\ \text{m/s}$. Determine the radius of the electron's orbit. [*Hint*: what do you recall about circular motion?]

The electric force must be a radial force in order for the electron to move in a circular orbit.

$$F_{\text{E}} = F_{\text{radial}} \quad \rightarrow \quad k\frac{Q^2}{r_{\text{orbit}}^2} = \frac{mv^2}{r_{\text{orbit}}} \quad \rightarrow$$

$$r_{\text{orbit}} = k\frac{Q^2}{mv^2} = \left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\,\text{C}\right)^2}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.1\times10^{6}\ \text{m/s}\right)^2} = \boxed{2.1\times10^{-10}\,\text{m}}$$

7. A small lead sphere is encased in insulating plastic and suspended vertically from an ideal spring $(k = 126\ \text{N/m})$ above a lab table, shown below. The total mass of the coated sphere is 0.800 kg, and its center lies 15.0 cm above the tabletop when in equilibrium. The sphere is pulled down 5.00 cm below equilibrium, an electric charge $Q = -3.00\times10^{-6}\ \text{C}$ is deposited on it and then it is released. Using what you know about harmonic oscillation, write an expression for the electric field strength as a function of time that would be measured at the point on the tabletop (P) directly below the sphere.





The sphere will oscillate sinusoidally about the equilibrium point, with an amplitude of 5.0 cm. The angular frequency of the sphere is given by $\omega = \sqrt{k/m} = \sqrt{126\,\text{N/m}/0.800\,\text{kg}} = 12.5\,\text{rad/s}$. The distance of the sphere from the table is given by $r = \left[0.150 - 0.050\cos\left(12.5t\right)\right]\text{m}$. Use this distance and the charge to give the electric field value at the tabletop. That electric field will point upwards at all times, towards the negative sphere.

$$E = k\frac{|Q|}{r^2} = \frac{\left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times10^{-6}\,\text{C}\right)}{\left[0.150-0.050\cos\left(12.5t\right)\right]^2\ \text{m}^2} = \frac{2.70\times10^{4}}{\left[0.150-0.050\cos\left(12.5t\right)\right]^2}\ \text{N/C}$$

$$= \boxed{\frac{1.08\times10^{7}}{\left[3.00-\cos\left(12.5t\right)\right]^2}\ \text{N/C, upwards}}$$

8. Given the two charges shown below, at what position(s) $x$ is the electric field zero? Is the field zero at any other points, not on the $x$ axis?

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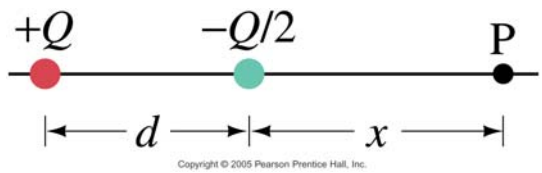


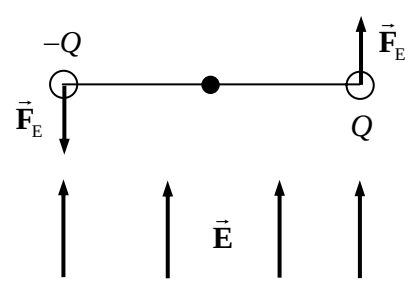

On the $x$-axis, the electric field can only be zero at a location closer to the smaller magnitude charge. Thus the field will never be zero to the left of the midpoint between the two charges. Also, in between the two charges, the field due to both charges will point to the left, and so the total field cannot be zero. Thus the only place on the x-axis where the field can be zero is to the right of the negative charge, and so $x$ must be positive. Calculate the field at point $P$ and set it equal to zero.

$$E = k\frac{(-Q/2)}{x^2} + k\frac{Q}{(x+d)^2} = 0 \;\rightarrow\; 2x^2 = (x+d)^2 \;\rightarrow\; x = \boxed{\frac{d}{\sqrt{2}-1} \approx 2.41d}$$

The field cannot be zero at any points off the $x$-axis. For any point off the $x$-axis, the electric fields due to the two charges will not be along the same line, and so they can never combine to give 0.

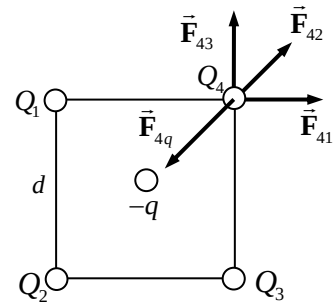
9. Two point charges, $+Q$ and $-Q$ of mass $m$, are placed on the ends of a massless rod of length $L$, which is fixed to a table by a pin through its center. If the apparatus is then subjected to a uniform electric field $E$ parallel to the table and perpendicular to the rod, find the net torque on the system of rod plus charges.

The electric field will put a force of magnitude $F_E = QE$ on each charge. The distance of each charge from the pivot point is $L/2$, and so the torque caused by each force is $\tau = F_E r_\perp = \frac{QEL}{2}$. Both torques will tend to make the rod rotate counterclockwise in the diagram, and so the net torque is $\tau_{net} = 2\left(\frac{QEL}{2}\right) = \boxed{QEL}$.



10. Four equal positive point charges, each of charge $8.0\,\mu\text{C}$, are at the corners of a square of side 9.2 cm. What charge should be placed at the center of the square so that all charges are at equilibrium? Is this a stable or unstable equilibrium in the plane?

A negative charge must be placed at the center of the square. Let $Q = 8.0\,\mu\text{C}$ be the charge at each corner, let -$q$ be the magnitude of negative charge in the center, and let $d = 9.2\,\text{cm}$ be the side length of the square. By the symmetry of the problem, if we make the net force on one of the corner charges be zero, the net force on each other corner charge will also be zero.




$$F_{41} = k\frac{Q^2}{d^2} \quad \rightarrow \quad F_{41x} = k\frac{Q^2}{d^2}\ ,\ F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \quad \rightarrow \quad F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}\ ,\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \quad \rightarrow \quad F_{43x} = 0\ ,\ F_{43y} = k\frac{Q^2}{d^2}$$

$$F_{4q} = k\frac{qQ}{d^2/2} \quad \rightarrow \quad F_{4qx} = -k\frac{2qQ}{d^2}\cos 45^\circ = -k\frac{\sqrt{2}qQ}{d^2} = F_{4qy}$$

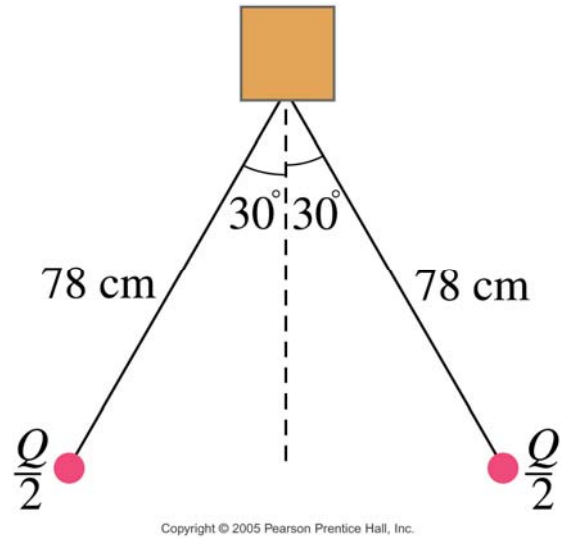
The net force in each direction should be zero.

$$\sum F_x = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 - k\frac{\sqrt{2}qQ}{d^2} = 0 \quad \rightarrow \quad q = Q\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = 7.66\times 10^{-6}\,\text{C}$$

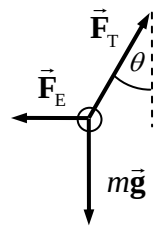
So the charge to be placed is $-q = \boxed{-7.66\times 10^{-6}\,\text{C}}$.

This is an $\boxed{\text{unstable equilibrium}}$. If the center charge were slightly displaced, say towards the right, then it would be closer to the right charges than the left, and would be attracted more to the right. Likewise the positive charges on the right side of the square would be closer to it and would be attracted more to it, moving from their corner positions. The system would not have a tendency to return to the symmetric shape, but rather would have a tendency to move away from it if disturbed.

11. A large electroscope is made with "leaves" that are 78-cm-long wires with tiny 24-g spheres at the ends. When charged, nearly all the charge resides on the spheres. If the wires each make a 30° angle with the vertical as shown below, what total charge $Q$ must have been applied to the electroscope? Ignore the mass of the wires.




The wires form two sides of an equilateral triangle, and so the two charges are separated by a distance $d = 78$ cm and are directly horizontal from each other. Thus the electric force on each charge is horizontal. From the free-body diagram for one of the spheres, write the net force in both the horizontal and vertical directions and solve for the electric force. Then write the electric force by Coulomb's law, and equate the two expressions for the electric force to find the charge.




$$\sum F_y = F_{\mathrm{T}} \cos\theta - mg = 0 \;\rightarrow\; F_{\mathrm{T}} = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_{\mathrm{T}} \sin\theta - F_{\mathrm{E}} = 0 \;\rightarrow\; F_{\mathrm{E}} = F_{\mathrm{T}} \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

$$F_{\mathrm{E}} = k\frac{(Q/2)^2}{d^2} = mg\tan\theta \;\rightarrow\; Q = 2d\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2\left(7.8\times10^{-1}\,\mathrm{m}\right)\sqrt{\frac{\left(24\times10^{-3}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\tan 30^\circ}{\left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)}} = 6.064\times10^{-6}\,\mathrm{C} \approx \boxed{6.1\times10^{-6}\,\mathrm{C}}$$

12. Suppose that electrical attraction, rather than gravity, were responsible for holding the Moon in orbit around the Earth. If equal and opposite charges $Q$ were placed on the Earth and the Moon, what should be the value of $Q$ to maintain the present orbit? Use these data: mass of $\text{Earth} = 5.98\times10^{24}$ kg, mass of $\text{Moon} = 7.35\times10^{22}$ kg, radius of $\text{orbit} = 3.84\times10^{8}$ m. Treat the Earth and Moon as point particles.

Set the Coulomb electrical force equal to the Newtonian gravitational force on one of the bodies (the Moon).

$$F_{\text{E}} = F_{\text{G}} \quad \rightarrow \quad k\frac{Q^2}{r_{\text{orbit}}^2} = G\frac{M_{\text{Moon}}M_{\text{Earth}}}{r_{\text{orbit}}^2} \quad \rightarrow$$

$$Q = \sqrt{\frac{GM_{\text{Moon}}M_{\text{Earth}}}{k}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}} = \boxed{5.71\times10^{13}\,\text{C}}$$