Physics 111
Fall 2007
Electric Potential Solutions

1. A lightning flash transfers 4.0 C of charge and 4.2 MJ of energy to the Earth. (*a*) Across what potential difference did it travel? (*b*) How much water could this boil and vaporize, starting from room temperature?

(*a*) The energy is related to the charge and the potential difference by Eq. 17-3.

$$\Delta\mathrm{PE} = q\Delta V \quad \rightarrow \quad \Delta V = \frac{\Delta\mathrm{PE}}{q} = \frac{4.2\times10^{6}\,\mathrm{J}}{4.0\,\mathrm{C}} = 1.05\times10^{6}\,\mathrm{V} \approx \boxed{1.1\times10^{6}\,\mathrm{V}}$$

(*b*) The energy (as heat energy) is used to raise the temperature of the water and boil it. Assume that room temperature is 20°C.

$$Q = mc\Delta T + mL_{\mathrm{f}} \quad \rightarrow$$

$$m = \frac{Q}{c\Delta T + L_{\mathrm{f}}} = \frac{4.2\times10^{6}\,\mathrm{J}}{\left(4186\,\frac{\mathrm{J}}{\mathrm{kg}\cdot{}^{\mathrm{o}}\mathrm{C}}\right)\left(80\,\mathrm{C}^{\mathrm{o}}\right)+\left(22.6\times10^{5}\,\frac{\mathrm{J}}{\mathrm{kg}}\right)} = \boxed{1.6\,\mathrm{kg}}$$

2. In a television picture tube, electrons are accelerated by thousands of volts through a vacuum. If a television set were laid on its back, would electrons be able to move upward against the force of gravity? What potential difference, acting over a distance of 3.0 cm, would be needed to balance the downward force of gravity so that an electron would remain stationary? Assume that the electric field is uniform.

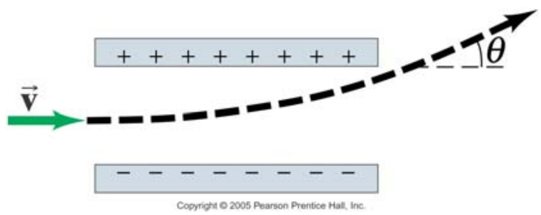
The electric force on the electron must be the same magnitude as the weight of the electron. The magnitude of the electric force is the charge on the electron times the magnitude of the electric field. The electric field is the potential difference per meter: $E = V/d$.

$$F_E = mg \quad \rightarrow \quad eV/d = mg \quad \rightarrow$$

$$V = \frac{mgd}{e} = \frac{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\left(3.0\times10^{-2}\,\mathrm{m}\right)}{1.60\times10^{-19}\,\mathrm{C}} = \boxed{1.7\times10^{-12}\,\mathrm{V}}$$

Since it takes such a tiny voltage to balance gravity, the thousands of volts in a television set are more than enough (by many orders of magnitude) to move electrons upward against the force of gravity.

3. An electron is accelerated horizontally from rest in a television picture tube by a potential difference of 5500 V. It then passes between two horizontal plates 6.5 cm long and 1.3 cm apart that have a potential difference of 250 V, as shown below. At what angle θ will the electron be traveling after it passes between the plates?



To find the angle, the horizontal and vertical components of the velocity are needed. The horizontal component can be found using conservation of energy for the initial acceleration of the electron. That component is not changed as the electron passes through the plates. The vertical component can be found using the vertical acceleration due to the potential difference of the plates, and the time the electron spends between the plates.

Horizontal:

$$\mathrm{PE}_{\text{inital}} = \mathrm{KE}_{\text{final}} \rightarrow qV = \tfrac{1}{2}mv_x^2 \qquad t = \frac{\Delta x}{v_x}$$

Vertical:

$$F_{\mathrm{E}} = qE_y = ma = m\frac{\left(v_y - v_{y0}\right)}{t} \rightarrow v_y = \frac{qE_y t}{m} = \frac{qE_y \Delta x}{mv_x}$$

Combined:

$$\tan\theta = \frac{v_y}{v_x} = \frac{\dfrac{qE_y\Delta x}{mv_x}}{v_x} = \frac{qE_y\Delta x}{mv_x^2} = \frac{qE_y\Delta x}{2qV} = \frac{E_y\Delta x}{2V} = \frac{\left(\dfrac{250\,\mathrm{V}}{0.013\,\mathrm{m}}\right)\left(0.065\,\mathrm{m}\right)}{2\left(5{,}500\,\mathrm{V}\right)} = 0.1136$$

$$\theta = \tan^{-1} 0.1136 = \boxed{6.5^{\circ}}$$

4. In lightning storms, the potential difference between the Earth and the bottom of the thunderclouds can be as high as 35,000,000 V. The bottoms of the thunderclouds are typically 1500 m above the Earth, and can have an area of $110\ \mathrm{km}^2$. Modeling the Earth-cloud system as a huge capacitor, calculate (*a*) the capacitance of the Earth-cloud system, (*b*) the charge stored in the "capacitor," and (*c*) the energy stored in the "capacitor."

(a) $$C=\frac{\varepsilon_0 A}{d}=\frac{\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(110\times10^{6}\,\mathrm{m^2}\right)}{\left(1500\,\mathrm{m}\right)}=6.49\times10^{-7}\,\mathrm{F}\approx\boxed{6.5\times10^{-7}\,\mathrm{F}}$$

(b) $$Q=CV=\left(6.49\times10^{-7}\,\mathrm{F}\right)\left(3.5\times10^{7}\,\mathrm{V}\right)=22.715\,\mathrm{C}\approx\boxed{23\,\mathrm{C}}$$

(c) $$\mathrm{PE}=\tfrac{1}{2}QV=\tfrac{1}{2}\left(22.715\,\mathrm{C}\right)\left(3.5\times10^{7}\,\mathrm{V}\right)=\boxed{4.0\times10^{8}\,\mathrm{J}}$$

5. Imagine assembling four equal charges one at a time and putting them at the corners of a square. Find the total work done to assemble these if the charges are each 5 μC and the square has 25 cm sides.

To bring in the first 5 μC charge and place it at the origin costs no work since no other charges are present. Thus $W_1 = q\Delta V = 0J$.

To place the second 5 μC charge at a distance of 0.25m away (say on the x- or y-axis) takes work. The work is $W_2 = q\Delta V = 5\times10^{-6}C\times\left(0V-\left(9\times10^{9}\,\tfrac{Nm^2}{C^2}\times\frac{5\times10^{-6}C}{0.25m}\right)\right)=-0.9V$.

To place the third 5 μC charge at a distance of 0.25m away (on the other axis from part b) takes work. This work is $W_3 = q\Delta V = 5\times10^{-6}C\times\left[\left(0V-\left(9\times10^{9}\,\tfrac{Nm^2}{C^2}\times\frac{5\times10^{-6}C}{0.25m}\right)\right)+\left(0V-\left(9\times10^{9}\,\tfrac{Nm^2}{C^2}\times\frac{5\times10^{-6}C}{0.25m}\right)\right)\right]=-1.8V$.

To place the last charge at the final corner of the square (at a distance along the diagonal from the first charge takes work and this work is

$$W_4 = q\Delta V = 5\times10^{-6}C\times\left[\left(0V-\left(9\times10^{9}\,\tfrac{Nm^2}{C^2}\times\frac{5\times10^{-6}C}{0.25m}\right)\right)+\left(0V-\left(9\times10^{9}\,\tfrac{Nm^2}{C^2}\times\frac{5\times10^{-6}C}{0.25m}\right)\right)+\left(0V-\left(9\times10^{9}\,\tfrac{Nm^2}{C^2}\times\frac{5\times10^{-6}C}{0.35m}\right)\right)\right]$$
$$=-2.44V$$

The net work done is the sum of all of the individual works needed to assemble the system, or -5.14V.

6. Equal and opposite ± 10 μC charges lie along the x-axis with the + charge at x = 0.1 m and the - charge at x = -0.1 m. Find a) the electric potential at the origin; b) the electric field at the origin; c) the work required to bring a third +10 μC charge from far away to the origin. Repeat all three parts if now all charges are +10 μC.

a. The electric potential at the origin due to the two charges is

$$V_{origin} = V_{+10} + V_{-10} = k\left(\frac{10\times10^{-6}C}{0.1m} - \frac{10\times10^{-6}C}{0.1m}\right) = 0V$$

b. The electric field at the origin is

$$\vec{E}_{origin} = \vec{E}_{+10} + \vec{E}_{-10} = k\left(-\frac{10\times10^{-6}C}{(0.1m)^2} - \frac{10\times10^{-6}C}{(0.1m)^2}\right) = -1.8\times10^{7}\,\tfrac{N}{C}\,\hat{i}\ .$$

c. The work required to bring a third charge in from infinity along an equipotentail line is 0J.

d. The electric potential at the origin due to the two charges is

$$V_{origin} = V_{+10} + V_{-10} = k\left(\frac{10\times10^{-6}C}{0.1m} + \frac{10\times10^{-6}C}{0.1m}\right) = 1.8\times10^{6}V\ .$$

The electric field at the origin is

$$\vec{E}_{origin} = \vec{E}_{+10} + \vec{E}_{-10} = k\left(+\frac{10\times10^{-6}C}{(0.1m)^2} - \frac{10\times10^{-6}C}{(0.1m)^2}\right) = 0\,\tfrac{N}{C}\,\hat{i}$$

The work required to bring a third charge in from infinity along an equipotentail line is

$$W = q\Delta V = 10\times10^{-6}C\times\left(0V - 1.8\times10^{6}V\right) = -18J\ .$$

7. Suppose a biological membrane with a specific capacitance of 1 μF/cm^2 has a resting surface charge density of 0.1 μC/cm^2. Also suppose there are 50 sodium channels per μm^2 and that when each opens for 1 ms 1000 Na^+ ions flow through the channel. Find the membrane voltage 1 ms after 10% of these channels open, assuming no other changes occur during this time.

Here we need to calculate the actual charge that flows per unit area. To calculate this we need

$$\frac{Q_{total}}{1\mu m^2} = \left(\frac{50_{Na^+ channels}}{1\mu m^2}\right) \times \left(\frac{1000_{Na^+ ions}}{channel}\right) \times \left(\frac{1.6\times10^{-19} C}{Na^+ ion}\right) \times 10\% = \frac{8\times10^{-16} C}{1\mu m^2}.$$

Then, the membrane voltage is simply

$$\Delta V = \frac{Q_{total}}{Area} \times \frac{Area}{C} = \frac{8\times10^{-16} C}{\mu m^2} \times \frac{1\times10^{8}\,\mu m^2}{1\times10^{-6} F} = 0.08V = 80mV$$

8. In a 100 μm^2 area of a muscle membrane having a density of sodium channels of 50 per μm^2 of surface area, when the sodium channels open there is a rapid flow of 1000 ions per channel across the membrane. Assuming a 100 mV resting potential, all the channels opening at once and a membrane capacitance of 1 μF/cm^2, find the voltage change across this area of membrane due solely to the sodium ion flow.

The potential across the membrane is given as the total charge that flows divided by the capacitance of the muscle membrane. The total charge that flows is given from $Q_{total} = \left(\frac{50_{Na^+ channels}}{1\mu m^2} \times 100\mu m^2\right) \times \left(\frac{1000_{Na^+ ions}}{channel}\right) \times \left(\frac{1.6\times10^{-19} C}{Na^+ ion}\right) = 8\times10^{-13} C$.

Second the capacitance is product of the specific capacitance and the muscle area. This gives the capacitance of the muscle as 1x10^{-12}F. Thus the potential difference across the membrane is $\Delta V = \frac{Q_{total}}{C} = \frac{8\times10^{-13} C}{1\times10^{-12} F} = 0.8V = 800mV$ or a change of 700mV.

9. An air-spaced parallel-plate capacitor has an initial charge of 0.05 μC after being connected to a 10 V battery.
   a) What is the total energy stored between the plates of the capacitor?
   b) If the battery is disconnected and the plate separation is tripled to 0.3 mm, what is the electric field before and after the plate separation change?
   c) What is the final voltage across the plates and the final energy stored between the plates?
   d) Calculate the work done in pulling the plates apart. Does this fully account for the energy change in part (b)?

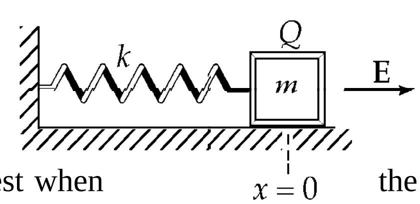
a. The energy is given by $E = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV = \frac{1}{2}\times 0.05\times 10^{-6}C\times 10V = 2.5\times 10^{-7}J$.

b. The electric field is given through $E = \frac{\Delta V}{\Delta d}$. The initial electric field is thus 10V / 0.1mm = 100 kN/m and the electric field after the separation is depends on the potential across the plates. To calculate this potential we know that the charge remains fixed (the battery has been disconnected) and the capacitance decreases by a factor of 3. Thus the new potential difference is give $\Delta V_{new} = \frac{\Delta Q}{C_{new}} = \frac{0.05\times 10^{-6}C}{5\times 10^{-9}F/3} = 30V$ and the electric field is 30V / 0.3mm = 100kN/m

c. The final potential is the same as the battery or 30V and the energy stored in the capacitor is $E = \frac{1}{2}\frac{Q^2}{C_{new}} = \frac{1}{2}\times\frac{\left(0.05\times 10^{-6}C\right)^2}{5\times 10^{-9}C/3} = 7.5\times 10^{-7}J$.

d. The work done is the change in energy which is $5\text{x}10^{-7}$J.

10. A 4 kg block carrying a charge of Q = 50 μC is connected to a spring for which k = 100 N/m. The block lies on a friction less horizontal track and the system is immersed in a uniform electric field of magnitude $5\text{x}10^5$ V/m directed as shown. If the block is released from rest when the spring is unstretched at x = 0m,



a) by what amount does the spring stretch?

b) What is the equilibrium position of the block?

c) Is the motion of the block simple harmonic?

d) Repeat part a) if the coefficient of kinetic friction between the block and the surface is 0.2.

(a) Arbitrarily choose $V = 0$ at 0. Then at other points

$V=-Ex$ and $U_e=QV=-QEx$.

Between the endpoints of the motion,

$$(K+U_s+U_e)_i=(K+U_s+U_e)_f$$

$$0+0+0=0+\frac{1}{2}kx_{\max}^2-QEx_{\max} \text{ so } x_{\max}=\boxed{\frac{2QE}{k}}.$$

(b) At equilibrium,

$$\sum F_x=-F_s+F_e=0 \text{ or } kx=QE.$$

So the equilibrium position is at $x=\boxed{\frac{QE}{k}}$.

(c) The block's equation of motion is $\sum F_x=-kx+QE=m\frac{d^2x}{dt^2}$.

Let $x'=x-\frac{QE}{k}$, or $x=x'+\frac{QE}{k}$,

so the equation of motion becomes:

$$-k\left(x'+\frac{QE}{k}\right)+QE=m\frac{d^2(x+QE/k)}{dt^2}, \text{ or } \frac{d^2x'}{dt^2}=-\left(\frac{k}{m}\right)x'.$$

This is the equation for simple harmonic motion $a_{x'}=-\omega^2x'$

with $\omega=\sqrt{\frac{k}{m}}$.

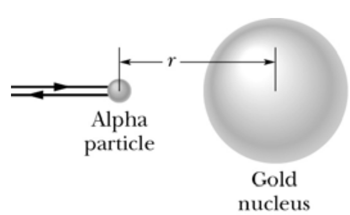
The period of the motion is then $T=\frac{2\pi}{\omega}=\boxed{2\pi\sqrt{\frac{m}{k}}}$.

(d) $$(K+U_s+U_e)_i+\Delta E_{\text{mech}}=(K+U_s+U_e)_f$$

$$0+0+0-\mu_k mgx_{\max}=0+\frac{1}{2}kx_{\max}^2-QEx_{\max}$$

$$x_{\max}=\boxed{\frac{2(QE-\mu_k mg)}{k}}$$

11. An alpha particle (which contains 2 protons and 2 neutrons) passes through the region of electron orbits in a gold atom, moving directly toward the gold nucleus, which has 79 protons and 118 neutrons. The alpha particle slows and then comes to a momentary rest, at a center-to-center separation r =



9.23×10^{-15}m before it begins to move back along its original path. (This technique is called *Rutherford Backscattering Spectroscopy* and the alpha particles are usually accelerated using a particle accelerator, like the one we have in the basement of Science and Engineering!)

a) What was the initial kinetic energy of the alpha particle when it was initially far away, external to the gold atom? (Hint: Assume that the gold atom does not move since it is much more massive than the alpha particle.)

b) Given the kinetic energy in part a), through what potential difference was the alpha particle accelerated?

c) How much work was done on the alpha particle in accelerating it through the potential difference in part b)?

d) The Union College Pelletron particle accelerator 1.1MV tandem electrostatic accelerator. When using this machine alpha particles are accelerated two times in succession (hence the tandem) and interact with the nucleus of a gold atom. Supposing that the alpha particles reach an energy of 3.3MeV using the Pelletron accelerator, what will the center-to-center separation of the alpha particle and the gold nucleus?

a. By conservation of energy we have the PE acquired by the alpha particle equal to the KE lost as the alpha particle is brought to rest. Thus

$$KE = PE = k\frac{Q_1 Q_2}{r} = 9\times 10^{9}\,\tfrac{Nm^2}{C^2}\left(\frac{2e\times 79e}{9.23\times 10^{-15}m}\right) = 3.94\times 10^{-12}\,J = 24.6MeV$$

b. By the work kinetic energy theorem, the work done accelerating the alpha particle is equal to its change in kinetic energy. Thus

$$KE = q\Delta V \rightarrow 3.94\times 10^{-12}\,J = (2e)\Delta V \rightarrow \Delta V = \frac{3.94\times 10^{-12}\,J}{2\times 1.6\times 10^{-19}C} = 1.23\times 10^{7}V = 12.3MV$$

c. From part b, the work done is the change in kinetic energy, $3.94\times 10^{-19}\,J$.

d. The distance of closest approach is given by the conversion of the alpha particle's kinetic energy into potential energy.

$$KE = PE \to 3.3MeV = 5.28\times10^{-13}J = k\frac{Q_1Q_2}{r}$$

$$r_{new} = k\frac{Q_1Q_2}{KE} = 9\times10^9 \tfrac{Nm^2}{C^2}\left(\frac{2e\times79e}{5.28\times10^{-13}J}\right) = 6.90\times10^{-14}m$$

This is about a factor of 8x farther away.

12. The nucleus of a hydrogen atom consists of a single proton, which can be treated as a point charge.

   a) With the electric potential equal to zero at an infinite distance, what is the electric potential due to the proton at the electron's position, $0.53\text{x}10^{-10}$m away?
   b) What is the electric potential energy in electron volts, of an electron at the given distance away from the proton?
   c) If the electron moves farther from the proton (to a higher orbit) does the electric potential energy increase or decrease?

a. With the electric potential equal to zero at an infinite distance away, and since we can treat the proton as a point particle, we have

$$V = k\frac{Q}{r} = 9\times10^9 \tfrac{Nm^2}{C^2}\times\frac{1.6\times10^{-19}C}{0.53\times10^{-10}m} = 27.2V\ .$$

b. The electric potential energy is the work done in assembling the hydrogen atom and is given as

$$W = q\Delta V = -1.6\times10^{-19}C\times27.2V = -4.35\times10^{-18}J = -27.2eV\ .$$

c. As the electron moves farther from the proton, the electric potential decreases, since *r* increases. Thus the value in part b decreases. Since the electron is negatively charged, the potential energy is getting less negative, or increasing.