Final Exam Type Problems

Spring 2026

The problems that follow are meant to be representative of the major topics covered on the final exam that you should know from taking the course. These may or may not cover everything tested on the problems on the final exam but are designed to give you practice on multi-part questions of the types of things that you could expect to see on the final exam.

Final Exam Example #1

– 2D Projectile Motion, Equations of Motion & Vectors

A softball of mass $m_{sb} = 200g$ ($\sim 7\ ounces$) is hit by a bat that is horizontally swung $1m$ off the ground. The ball leaves the end of the bat with a speed of $21\frac{m}{s}$ ($\sim 47\frac{mi}{hr}$) at an angle $\theta = 36^0$, measured with respect to the horizontal.

- What is the time of flight of the softball from the height it was hit until it returns to the ground?
- In this time, what is the horizontal distance traveled by the softball and would the player have hit a homerun if the outfield wall is $225ft$ ($\sim 69m$) away and has a height $h = 2.0m$ measured from ground level, will the player have hit a homerun?
- If the player hits a home run with what velocity (magnitude and direction) will the ball clear the outfield wall and will the ball be rising or falling when it does?
- At what height above the wall will the softball clear the outfield wall if the player hits a home run.
- If the player does not hit a home run, by how much does the player miss the outfield wall, measured horizontally across the ground?
- Independent of whether the player hits a homerun, what is the impact velocity of the softball with the ground?
- Independent of whether the player hits a homerun, what's the highest point the softball reaches above the ground?
- If the player didn't hit a homerun, at what initial speed v_i would the ball need to be hit if θ remains unchanged to hit a homerun?

Final Exam Example #1 - Solutions

- What is the time of flight of the softball from the height it was hit until it returns to the ground?

$$v_{ix} = v_i \cos\theta = 21\frac{m}{s}\cos 36 = 17\frac{m}{s}$$

$$v_{iy} = v_i \sin\theta = 21\frac{m}{s}\sin 36 = 12.3\frac{m}{2}$$

$$y_f = y_i + v_{1y}t - \frac{1}{2}gt^2 \rightarrow 0 = 1 + t - 4.9t^2 \rightarrow \begin{cases} -0.1s \\ 2.6s \end{cases}$$

- In this time, what is the horizontal distance traveled by the softball and would the player have hit a homerun if the outfield wall is $225ft$ ($\sim 69m$) away and has a height $h = 2.0m$ measured from ground level, will the player have hit a homerun?

$$x_f = x_i + v_{1x}t + \frac{1}{2}a_x t^2 \rightarrow x_f = v_{ix}t = 17\frac{m}{s} \cdot 2.6s = 44.2m$$

- If the player does not hit a home run, by how much does the player miss the outfield wall, measured horizontally across the ground?

$$\Delta x = 69m - 44.2m = 24.8m$$

Final Exam Example #1 - Solutions

- Independent of whether the player hits a homerun, what is the impact velocity of the softball with the ground?

$$v_{fx} = v_{ix} + a_x t \rightarrow v_{fx} = v_{ix} = 17\frac{m}{s}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(17\frac{m}{s}\right)^2 + \left(-13.2\frac{m}{s}\right)^2} = 21.5\frac{m}{s}$$

$$v_{fy} = v_{iy} + a_y t = 12.3\frac{m}{s} - \left(9.8\frac{m}{s^2} \cdot 2.6s\right) = -13.2\frac{m}{s}$$

$$\tan\phi = \frac{v_{fy}}{v_{fx}} \rightarrow \phi = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{-13.2\frac{m}{s}}{17\frac{m}{s}}\right) = -37.8^0$$

- Independent of whether the player hits a homerun, what's the highest point the softball reaches above the ground?

Method 1

$$v_{fy} = 0 = v_{iy} - gt \rightarrow t = \frac{v_{iy}}{g} = \frac{12.3\frac{m}{s}}{9.8\frac{m}{s^2}} = 1.26s$$

$$y_f = y_i + v_{1y}t - \frac{1}{2}gt^2 = 1m + \left(12.3\frac{m}{s} \cdot 1.26s\right) - \frac{1}{2}\left(9.8\frac{m}{s^2}\right)(1.26s)^2 = 8.7m$$

Method 2

$$v_{fy}^2 = 0 = v_{iy}^2 - 2g\Delta y \rightarrow \Delta y = y_f - y_i = \frac{v_{iy}^2}{2g} \rightarrow y_f = y_i + \frac{v_{iy}^2}{2g} = 1m + \frac{\left(12.3\frac{m}{s}\right)^2}{2 \cdot 9.8\frac{m}{s^2}} = 8.7m$$

Final Exam Example #1 - Solutions

- If the player didn't hit a homerun, at what initial speed v_i would the ball need to be hit if θ remains unchanged to hit a homerun?

$$x_f = x_i + v_{1x}t \rightarrow t = \frac{x_f}{v_{ix}} = \frac{x_f}{v_i \cos\theta}$$

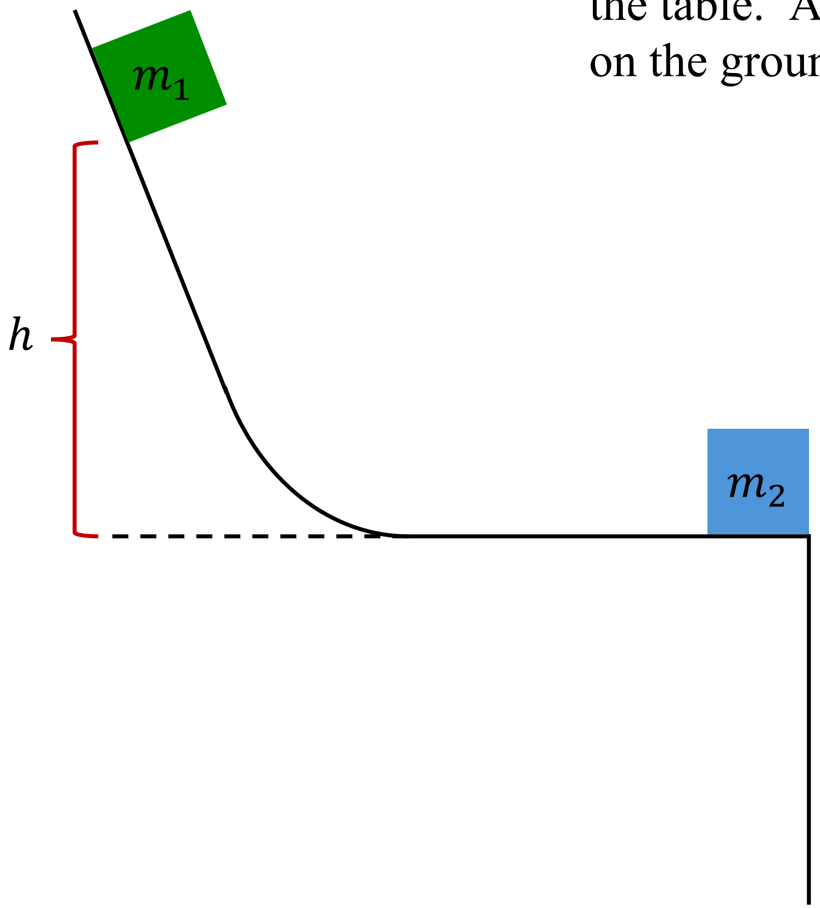
$$y_f = y_i + v_{1y}t - \tfrac{1}{2}gt^2 = y_i + v_i \sin\theta \left(\frac{x_f}{v_i \cos\theta}\right) - \tfrac{1}{2}g\left(\frac{x_f}{v_i \cos\theta}\right)^2 \rightarrow y_f = y_i + x_f \tan\theta - \frac{g x_f^2}{2v_i^2 \cos^2\theta}$$

$$\frac{g x_f^2}{2v_i^2 \cos^2\theta} = y_i - y_f + x_f \tan\theta \rightarrow v_i^2 = \frac{g x_f^2}{2\cos^2\theta\left(y_i - y_f + x_f \tan\theta\right)}$$

$$v_i^2 = \frac{g x_f^2}{2\cos^2\theta\left(y_i - y_f + x_f \tan\theta\right)} = \frac{9.8\frac{m}{s^2}(69m)^2}{2\cos^2 36\,(1m - 2m + 69m\tan 36)} \rightarrow v_i = 25.9\frac{m}{2}$$

Final Exam Example #2

– Conservation of Energy, Collisions, 2D Projectile Motion, Equations of Motion & Vectors



A block of mass $m_1 = 0.5kg$ is released from rest at a height $h = 1.0m$ above the top of a table. The block of mass m_1 collides with a block of mass $m_2 = 2kg$ at rest on the edge of the table. After the collision the block of mass m_2 is kicked off of the table and is seen to land on the ground a horizontal distance $d = 0.5m$ from where it was launched.

- What is the speed of the block of mass m_1 just before it collided with the block of mass m_2?
- If the block of mass m_2 falls through a vertical height of $0.75m$, with what speed was it launched from the top of the table?
- What was the time of flight of the block of mass m_2 to the ground?
- What was the impact velocity (magnitude and direction) that the block of mass m_2 made with the ground?
- What was the speed of the block of mass m_1 after the collision?
- Was the collision elastic or inelastic?

Final Exam Example #2 - Solution

- What is the speed of the block of mass m_1 just before it collided with the block of mass m_2?

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_T + \Delta U_g = \left(\frac{1}{2}m_1 v_{1f}^2 - \frac{1}{2}m_1 v_{1i}^2\right) + \left(m_1 g y_{if} - m_1 g y_{1i}\right)$$

$$\frac{1}{2}m_1 v_{1f}^2 = m_1 g y_{1i} \rightarrow v_{1f} = v_{1,horizontal} = \sqrt{2gh} = \sqrt{2 \cdot 9.8\frac{m}{s^2} \cdot 1m} = 4.4\frac{m}{s}$$

- If the block of mass m_2 falls through a vertical height of $0.75m$, with what speed was it launched from the top of the table?

$$x_f = x_i + v_{1x}t \rightarrow t = \frac{x_f}{v_{ix}} = \frac{d}{v_{2,after\ collision}}$$

$$y_f = y_i + v_{1y}t - \frac{1}{2}gt^2 \rightarrow -y = -\frac{1}{2}gt^2 = -\frac{1}{2}g\left(\frac{d}{v_{2,after\ collision}}\right)^2$$

$$\rightarrow v_{2,after\ collision} = \sqrt{\frac{gd^2}{2y}} = \sqrt{\frac{9.8\frac{m}{s^2}(0.5m)^2}{2 \cdot 0.75m}} = 1.27\frac{m}{s}$$

Final Exam Example #2 - Solution

- What was the time of flight of the block of mass m_2 to the ground?

$$-y = -\frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \cdot 0.75m}{9.8\frac{m}{s^2}}} = 0.39s$$

- What was the impact velocity (magnitude and direction) that the block of mass m_2 made with the ground?

$$v_{fx} = v_{ix} + a_x t \rightarrow v_{fx} = v_{ix} = 1.27\frac{m}{s}$$

$$v_{fy} = v_{iy} + a_y t = -\left(9.8\frac{m}{s^2} \cdot 0.39ss\right) = -3.8\frac{m}{s}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(1.27\frac{m}{s}\right)^2 + \left(-3.8\frac{m}{s}\right)^2} = 4\frac{m}{s}$$

$$\tan\phi = \frac{v_{fy}}{v_{fx}} \rightarrow \phi = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{-3.8\frac{m}{s}}{1.27}\right) = -71.5^0$$

Final Exam Example #2 - Solution

- What was the speed of the block of mass m_1 after the collision?

$$\Delta p_x = 0 = p_{fx} - p_{ix} \rightarrow p_{ix} = p_{fx}$$

$$m_1 v_{1,horizontal} = m_1 v_{1,after,collision} + m_2 v_{2,after,collision}$$

$$\rightarrow v_{1,after,collision} = \frac{m_1 v_{1,horizontal} - m_2 v_{2,after\ collision}}{m_1} = \frac{\left(0.5kg \cdot 4.4\frac{m}{s}\right) - \left(2kg \cdot 1.27\frac{m}{2}\right)}{0.5kg} = -0.68\frac{m}{s}$$

- Was the collision elastic or inelastic?

$$\Delta K = K_f - K_i = \left(\tfrac{1}{2}m_1 v_{1ac}^2 + \tfrac{1}{2}m_2 v_{2ac}^2\right) - \tfrac{1}{2}m_1 v_{1h}^2$$

$$\Delta K = \left(\tfrac{1}{2}(0.5kg)\left(-0.68\tfrac{m}{s}\right)^2 + \tfrac{1}{2}(2kg)\left(1.27\tfrac{m}{2}\right)^2\right) - \tfrac{1}{2}(0.5kg)\left(4.4\tfrac{m}{s}\right)^2 = -3.11J$$

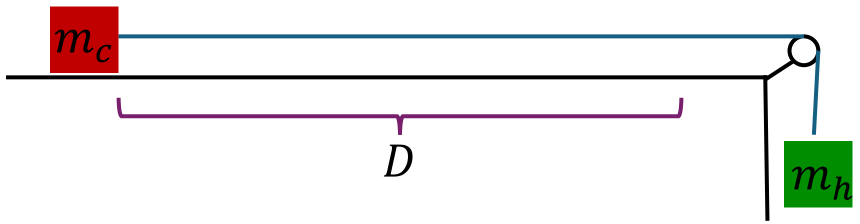
Since this is not zero, the collision is inelastic.

Final Exam Example #3

– Motion in 1D, constant Forces, Constat Acceleration, Equations of Motion, Data Analysis, and Uncertainty

Suppose that for physics lab you were tasked with experimentally determining the mass of a low-friction cart m_c used for a laboratory experiment. To do this, you set up the apparatus shown below. A light, non-stretchable string is attached to the cart, passed over a pulley of negligible mass and friction, and attached to a hanger, where various hanging masses m_h could be added.

The cart is released from rest at a marked starting line, and a motion sensor records the time t it takes for the cart to travel a fixed distance D along the horizontal track. The track is leveled so that friction between the cart and the track is negligible.



- From a carefully labeled free-body (force) diagram, what is the translational acceleration of the low-friction cart in terms of m_c, m_h, and g?

Final Exam Example #3
– Motion in 1D, constant Forces, Constant Acceleration, Equations of Motion, Data Analysis, and Uncertainty

- Suppose that you wanted to determine the mass of the low-friction cart m_c, by taking data on the time t it took to travel the horizontal distance D, and the translational acceleration of the system a. Derive an equation that would allow you to accomplish this and explain how you would determine the mass of the low-friction cart from your equation.

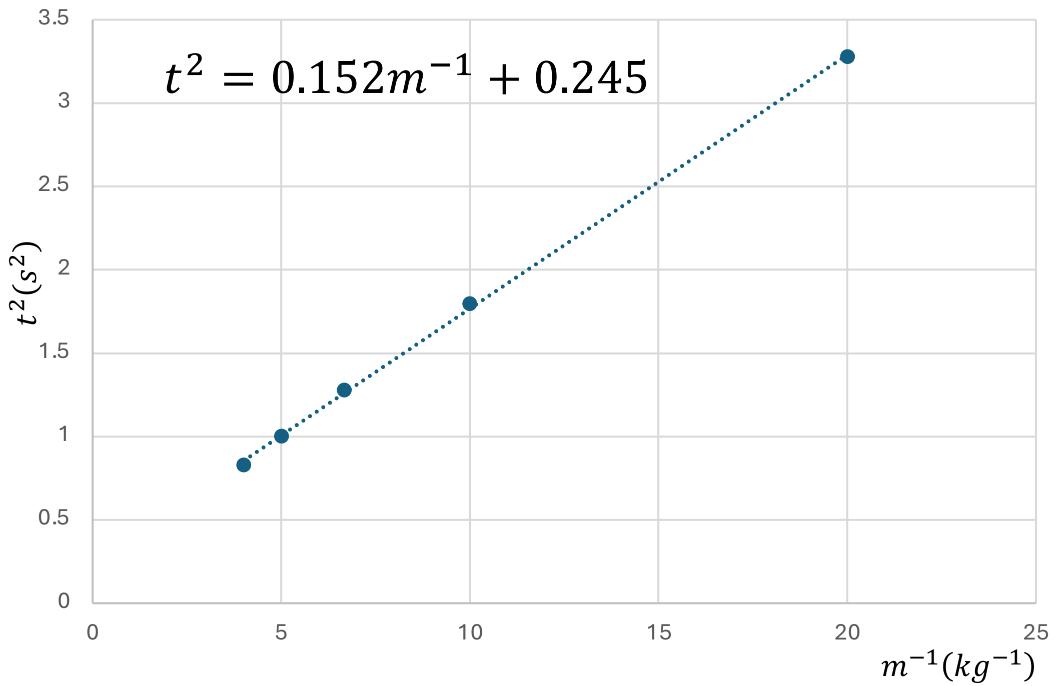
- Suppose that you took the following data on the masses m hung on the holder and the time t it took the low-friction cart to cover the constant horizontal distance $D = 1.2m$ from rest. What would you have to plot in order to determine the mass of the low-friction cart?

Hanging mass m (kg)	Time $t(s)$ to cover distance D
0.05	1.81
0.10	1.34
0.15	1.13
0.20	1.00
0.25	0.91

Final Exam Example #3

– Motion in 1D, constant Forces, Constant Acceleration, Equations of Motion, Data Analysis, and Uncertainty

- Here's the plot and from the plot, what is the mass m_c of the low-friction cart?



- Suppose you later discover that the string was not perfectly level, but was actually tilted slightly upward toward the pulley. Would the acceleration of the cart be **greater than**, **less than**, or **equal to** the acceleration on a perfectly level track? Explain your reasoning.

- How would this tilt affect the mass of the low-friction cart? Would it be an overestimate, underestimate, or would it not affect the result at all? Explain your reasoning.

Final Exam Example #3 - Solutions

- Suppose that you wanted to determine the mass of the low-friction cart m_c, by taking data on the time t it took to travel the horizontal distance D, and the translational acceleration of the system a. Derive an equation that would allow you to accomplish this and explain how you would determine the mass of the low-friction cart from your equation.

For the cart parallel to the horizontal surface (to the right is the positive x-direction):

$$F_T = m_c a$$

For the hanging mass (vertically up is the positive y-direction)

$$F_T - m_h g = -m_h a$$

$$\rightarrow m_c a - m_h g = -m_h a$$

$$\rightarrow a = \left(\frac{m_h}{m_h + m_c}\right) g$$

Final Exam Example #3 - Solutions

The horizontal distance D traveled in time t is given by the cart:

$$x_f = x_i + v_{1x}t + \tfrac{1}{2}a_x t^2 \rightarrow D = \tfrac{1}{2}\left(\frac{m_h}{m_h + m_c}\right) g t^2$$

$$\rightarrow t^2 = \frac{2D}{m_h g}(m_h + m_c) = \frac{2D}{g} + \left(\frac{2Dm_c}{g}\right)\frac{1}{m_h}$$

I would make a plot of t^2 versus $\frac{1}{m_h}$ and from this plot, the slope of the line could be used to calculate the mass of the cart.

$$slope = \frac{2Dm_c}{g} \rightarrow m_c = \frac{slope \cdot g}{2D} = \frac{0.152kg \cdot s^2 \cdot 9.8\frac{m}{s^2}}{2 \cdot 1.2m} = 0.62kg$$

We could also evaluate the y-intercept and in doing so, we see that it is the same value as is given on the graph.

$$y - intercept = \frac{2D}{g} = \frac{2 \cdot 1.2m}{9.8\frac{m}{s^2}} = 0.245s^2$$

Final Exam Example #3 - Solutions

- Suppose you later discover that the string was not perfectly level, but was actually tilted slightly upward toward the pulley. Would the acceleration of the cart be **greater than**, **less than**, or **equal to** the acceleration on a perfectly level track? Explain your reasoning.

$$F_T \cos\theta = m_c a \rightarrow (m_h g - m_h a)\cos\theta = m_c a \rightarrow a = \left(\frac{m_h \cos\theta}{m_h \cos\theta + m_c}\right) g$$

$$F_T - m_h g = -m_h a \rightarrow F_T = m_h g - m_h a$$

To see what happens to the acceleration, let's try some extremes. For $\theta = 0$ this reduces to our original equation. For $\theta \rightarrow 90$, this gives $a \rightarrow 0$. So, the acceleration of the cart would be less than that if the string were level. We could also argue this from $F_T \cos\theta$. As $\theta \uparrow$, $F_T \downarrow$ and thus $a \downarrow$.

- How would this tilt affect the mass of the low-friction cart? Would it be an overestimate, underestimate, or would it not affect the result at all? Explain your reasoning.

Since the acceleration is smaller, it would take longer to cover the same distance for each given hanging mass. Thus, the slope would be larger and thus the mass would be larger. This would be an overestimation of the mass of the cart.

$$slope = \frac{2Dm_c}{g} \rightarrow slope \uparrow, m_c \uparrow$$

Final Exam Example #4

- Conservation of Energy, Circular Motion and Forces, Work, Frictional Work, Springs and Elastic Potential Energy, Work-Kinetic Energy Theorem

A sled of mass $m = 100kg$ is at rest at the top of a tall snowy hill of height $h = 35m$. The sled starts to slide down the hill from rest.

- What is the speed of the sled at the bottom of the hill?
- The hill is part of an obstacle course and when the sled gets to the bottom of the hill it encounters a loop-the-loop that makes up part of the track. If the bottom of the loop-the-loop is at ground level, what is the magnitude of the normal force on the sled at the top of the loop-the-loop if the loop-the-loop is a circle of radius $R = 10m$?
- How much work was done by gravity on the sled from the bottom to the top of the loop-the-loop?
- After the sled exits the loop-the-loop it slides across the horizontal ground until it encounters a spring of stiffness k and under the spring a patch of friction exists. What is the stiffness of the spring if the cart needs to be brought to rest over a distance of $7m$? Assume the coefficient of kinetic friction is $\mu_k = 0.75$ and the spring is initially at equilibrium.
- How much work was done by the spring in bringing the sled to rest? How much work was done by friction brining the sled to rest?
- How does the total work done on the sled by the spring and friction compare to the change in kinetic energy of the sled? Is it as it should be? Explain.

Final Exam Example #4 - Solution

- What is the speed of the sled at the bottom of the hill?

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_T + \Delta U_g = \left(\tfrac{1}{2}mv_{bottom}^2 - \tfrac{1}{2}mv_{top}^2\right) + \left(mgy_{bottom} - mgy_{top}\right)$$

$$\tfrac{1}{2}mv_{bottom}^2 = mgy_{top} \rightarrow v_{1,bottom} = \sqrt{2gh} = \sqrt{2 \cdot 9.8\tfrac{m}{s^2} \cdot 35m} = 26.2\tfrac{m}{s}$$

- The hill is part of an obstacle course and when the sled gets to the bottom of the hill it encounters a loop-the-loop that makes up part of the track. If the bottom of the loop-the-loop is at ground level, what is the magnitude of the normal force on the sled at the top of the loop-the-loop if the loop-the-loop is a circle of radius $R = 10m$?

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_T + \Delta U_g = \left(\tfrac{1}{2}mv_{top}^2 - \tfrac{1}{2}mv_{bottom}^2\right) + \left(mgy_{top} - mgy_{bottom}\right)$$

$$v_{top}^2 = v_{bottom}^2 - 2gy_{top} = \left(26.2\tfrac{m}{s}\right)^2 - 2 \cdot 9.8\tfrac{m}{s^2} \cdot (2 \cdot 10m) = 294.4\tfrac{m^2}{s^2} \rightarrow v_{top} = 17.2\tfrac{m}{s}$$

$$-F_N - mg = -m\frac{v_{top}^2}{R} \rightarrow F_N = m\frac{v_{top}^2}{R} - mg = 100kg\left(\frac{294.4\tfrac{m^2}{s^2}}{10m} - 9.8\tfrac{m}{s^2}\right) = 1964.4N$$

Final Exam Example #4 - Solution

- How much work was done by gravity on the sled from the bottom to the top of the loop-the-loop?

Method 1: $W = F\Delta y \cos\theta = mg(2R)\cos 180 = -2Rmg = -2 \cdot 10m \cdot 100kg \cdot 9.8\frac{m}{s^2} = -19600J$

Method 2: $W = -\Delta U_g = -(mgy_{top} = mgy_{bottom}) = -2Rmg = -19600J$

Method 3: $W = \Delta K = \frac{1}{2}mv_{top}^2 - \frac{1}{2}mv_{bottom}^2 = \frac{1}{2} \cdot 100kg\left(\left(17.2\frac{m}{s}\right)^2 - \left(26.2\frac{m}{s}\right)^2\right) = -19530J$

- After the sled exits the loop-the-loop it slides across the horizontal ground until it encounters a spring of stiffness k and under the spring a patch of friction exists. What is the stiffness of the spring if the cart needs to be brought to rest over a distance of $7m$? Assume the coefficient of kinetic friction is $\mu_k = 0.75$ and the spring is initially at equilibrium.

$$\Delta E = W_{fr} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s \rightarrow W_{fr} = \Delta K_T + \Delta U_s \rightarrow F_{fr}\Delta x\cos\theta = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_{bottom}^2\right) + \left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right)$$

$$\mu F_N x_f \cos 180 = -\mu mgx_f = -\frac{1}{2}mv_{bottom}^2 + \frac{1}{2}kx_f^2$$

$$k = \frac{mv_{bottom}^2 - 2\mu mgx_f}{x_f^2} = \frac{100kg\left(\left(26.2\frac{m}{s}\right)^2 - 2 \cdot 0.75 \cdot 9.8\frac{m}{s^2} \cdot 7m\right)}{(7m)^2} = 1190.9\frac{N}{m}$$

Final Exam Example #4 - Solution

- How much work was done by the spring in bringing the sled to rest? How much work was done by friction brining the sled to rest?

$$W_{fr} = -\mu m g x_f = -0.75 \cdot 100kg \cdot 9.8\frac{m}{s^2} \cdot 7m = -5145J$$

$$W_s = -\Delta U_s = -\frac{1}{2}kx_f^2 = \frac{1}{2} \cdot 1190.9\frac{N}{m} \cdot (7m)^2 = -29177J$$

- How does the total work done on the sled by the spring and friction compare to the change in kinetic energy of the sled? Is it as it should be? Explain.

$$W_{net} = W_{fr} + W_s = W_{fr} - \Delta U_s = \Delta K$$

$$W_{net} = W_{fr} + W_s = -5145J - 29177J = -34322J$$

$$W_{net} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_{bottom}^2 = -\frac{1}{2} \cdot 100kg \cdot \left(26.2\frac{m}{s}\right)^2 = -34322J$$

The net work done is equal to the change in kinetic energy as it should be.

Final Exam Example #5

- Forces, Torques, Equations of Motion for Constant Acceleration, & Conservation of Energy

A box of mass m_1 is at rest on a horizontal surface. This box is connected to a second box of mass m_2 by a light string that passes over a pulley of mass m_p and radius r_p. The box of mass m_2 hangs vertically at rest.

- If the system is released from rest, what is the translational acceleration of the system of masses? Use forces/torques and assume friction (with coefficient of kinetic friction μ_k) exists on the horizontal surface that mass m_1 slides.
- From the translational acceleration you determined from forces/torques, what is the speed of mass m_1 (and mass m_2 also) after the mass m_1 has slid a distance d from rest?
- How long does it take mass m_1 to slide distance d from rest?
- Using the translational acceleration of the system, what is the speed of mass m_1 after time if the mass $m_1$were released from rest ? How does this compare to the result calculated above?
- Using energies, what is the speed of mass m_1 after it has slid a distance d from rest?
- What coefficient of kinetic friction would be needed so the box of mass m_2 falls at a constant speed?

Final Exam Example #5 - Solution

- If the system is released from rest, what is the translational acceleration of the system of masses? Use forces/torques and assume friction (with coefficient of kinetic friction μ_k) exists on the horizontal surface that mass m_1 slides.

For mass m_1

$$F_{net,x} = F_{TL} - F_{fr} = m_1 a \rightarrow F_{TL} = m_1 a + F_{fr} = m_1 a + \mu F_N = m_1 a + \mu m_1 g$$

$$F_{net,y} = F_N - F_W = m_1 a_y = 0 \rightarrow F_N = F_w = m_2 g$$

For mass m_2

$$F_{net,y} = F_{TR} - F_{W2} = -m_2 a_y \rightarrow F_{TR} = F_{W2} - m_2 a = m_2 g - m_2 a$$

For the pulley

$$\tau_{net} = \tau_{F_{TL}} - \tau_{F_{TR}} = I\alpha \rightarrow r_p F_{TL} \sin 90 - r_p F_{TR} \sin 90 = -\tfrac{1}{2} m_p r_p^2 \left(\frac{a}{r_p}\right)$$

$$F_{TL} - F_{TR} = m_1 a + \mu m_1 g - m_2 g + m_2 a = -\tfrac{1}{2} m_p r_p^2 \left(\frac{a}{r_p}\right)$$

$$a = \left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2} m_p}\right) g$$

Final Exam Example #5 - Solution

- From the translational acceleration you determined from forces/torques, what is the speed of mass m_1 (and mass m_2 also) after the mass m_1 has slid a distance d from rest?

$$v_{fx}^2 = v_{ix}^2 + 2a\Delta x \rightarrow v_{fx} = \sqrt{2ad} = \sqrt{2\left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}m_p}\right)gd}$$

- How long does it take mass m_1 to slide distance d from rest?

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 \rightarrow d = \frac{1}{2}\left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}m_p}\right)gt^2 \rightarrow t = \sqrt{\frac{2d}{g}\left(\frac{m_1 + m_2 + \frac{1}{2}m_p}{m_2 - \mu m_1}\right)}$$

- Using the translational acceleration of the system, what is the speed of mass m_1 after time if the mass m_1 were released from rest ? How does this compare to the result calculated above?

$$v_{fx} = v_{ix} + a_x t = \left[\left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}m_p}\right)g\right]\sqrt{\frac{2d}{g}\left(\frac{m_1 + m_2 + \frac{1}{2}m_p}{m_2 - \mu m_1}\right)} = \sqrt{2\left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}m_p}\right)gd}$$

Final Exam Example #5 - Solution

- Using energies, what is the speed of mass m_1 after it has slid a distance d from rest?

$$\Delta E = W_{fr} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_{T1} + \Delta K_{T2} + \Delta K_{Rp} + \Delta U_{g2}$$

$$\mu F_N d \cos 180 = -\mu m_1 g d = \left(\tfrac{1}{2}m_1 v_{1f}^2 - \tfrac{1}{2}m_1 v_{1i}^2\right) + \left(\tfrac{1}{2}m_2 v_{2f}^2 - \tfrac{1}{2}m_2 v_{2i}^2\right) + \left(\tfrac{1}{2}I\omega_{pf}^2 - \tfrac{1}{2}I\omega_{pi}^2\right) + \left(m_2 g y_{2f} - m_2 g y_{2i}\right)$$

$$-\mu m_1 g d = \tfrac{1}{2}(m_1 + m_2)v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m_p r_p^2\right)\left(\frac{v_f}{r_p}\right)^2 - m_2 g d$$

$$-\mu m_1 g d = \tfrac{1}{2}\left(m_1 + m_2 + \tfrac{1}{2}m_p\right)v_f^2 - m_2 g d$$

$$v_f = \sqrt{2\left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \tfrac{1}{2}m_p}\right) g d}$$

Final Exam Example #5 - Solution

- What coefficient of kinetic friction would be needed so the box of mass m_2 falls at a constant speed?

For mass m_1

$$F_{net,x} = F_{TL} - F_{fr} = m_1 a = 0 \rightarrow F_{TL} = F_{fr} = \mu m_1 g$$

For mass m_2

$$F_{net,y} = F_{TR} - F_{W2} = -m_2 a_y = 0 \rightarrow F_{TR} = F_{W2} = m_2 g$$

For the pulley

$$\tau_{net} = \tau_{F_{TL}} - \tau_{F_{TR}} = I\alpha = 0 \rightarrow r_p F_{TL} \sin 90 = r_p F_{TR} \sin 90 \rightarrow F_{TL} = F_{TR}$$

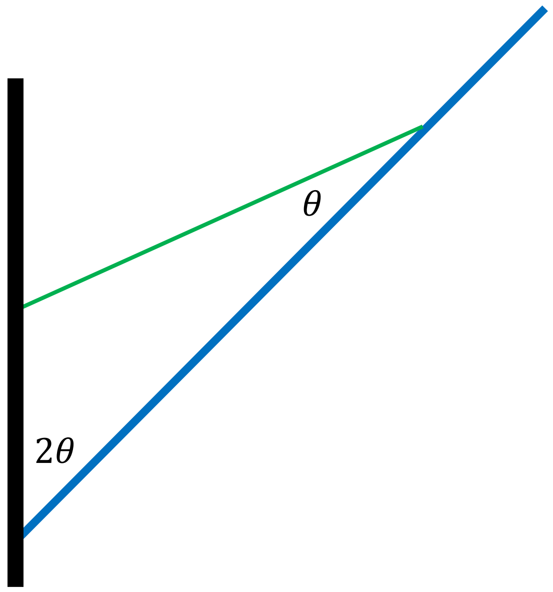
$$F_{TL} = F_{TR} \rightarrow F_{fr} = F_{W2} \rightarrow m_2 g = \mu m_1 g$$

$$\mu = \frac{m_2}{m_1}$$

Final Exam Example #6

- Rotational and Translational Statics, Forces, Torques, Rotational Motion, Rotational Kinetic Energy, & Conservation of Energy

A bar of mass $m_b = 50kg$ and length $L = 2m$ is attached to the side of a building by a cable as shown below.



- What is the tension in the cable if $\theta = 15^0$ and the cable is attached at a point 75% of the length of the bar?
- What is the reaction force (magnitude and direction) on the attachment point at the building?
- Suppose the cable breaks, what is the initial angular acceleration of the system? (Hint: The moment of inertia of a thin rod spun about one end is $\frac{1}{3}ml^2$.)
- When the bar is horizontal what is its rotational speed? Hint: Use energy ideas.
- When the bar is horizontal, what is the translational speed of the center-of-mass of the bar?

Final Exam Example #6 - Solution

- What is the tension in the cable if $\theta = 15^0$ and the cable is attached at a point 75% of the length of the bar?

$$F_{net,x} = F_{Rx} - F_{Tx} = F_{Rx} - F_T \cos(90 - 3\theta) = 0 \rightarrow F_{Rx} = F_T \cos 45$$

$$F_{net,y} = F_{Ry} - F_w - F_{Ty} = F_{Ry} - F_W - F_T \sin(90 - 3\theta) = 0 \rightarrow F_{Ry} = F_w + F_T \sin 45$$

$$\tau_{net} = -\tau_w + \tau_{F_T} = -r_w F_w \sin 2\theta + r_{F_T} F_T \sin\theta = 0$$

$$\rightarrow F_T = \frac{r_w F_w \sin 2\theta}{r_{F_T} \sin\theta} = \frac{\frac{L}{2} mg \sin 30}{\frac{3}{4} L \sin 15} = \frac{\frac{2}{3} \cdot 50kg \cdot 9.8\frac{m}{s^2} \sin 30}{\sin 15} = 631.1N$$

$$\rightarrow F_{Rx} = F_T \cos 75 = 631.1N \cdot \cos 45 = 446.3N$$

$$\rightarrow F_{Ry} = F_w + F_T \sin 45 = 50kg \cdot 9.8\frac{m}{s^2} + 631.1N \cdot \sin 45 = 936.2N$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(446.3N)^2 + (936.2N)^2} = 1037.2N$$

$$\tan\phi = \frac{F_{Ry}}{F_{Rx}} \rightarrow \phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{936.2N}{446.3N}\right) = 64.5^0$$

Final Exam Example #6 - Solution

- Suppose the cable breaks, what is the initial angular acceleration of the system? (Hint: The moment of inertia of a thin rod spun about one end is $\frac{1}{3}ml^2$.)

$$\tau_{net} = -\tau_w = I\alpha = \frac{1}{3}ml^2\alpha \rightarrow \alpha = -\frac{3\tau_w}{ml^2} = -\frac{3\left(\frac{l}{2}mg\sin 2\theta\right)}{ml^2} = -\frac{3g}{2l}\sin 2\theta = -\frac{3\cdot 9.8\frac{m}{s^2}}{2\cdot 2m}\sin 30 = -3.68\frac{rad}{s^2}$$

- When the bar is horizontal what is its rotational speed?

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_T + \Delta U_g = \left(\tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2\right) + \left(mgy_{bottom} - mgy_{top}\right)$$

$$\omega_f = \sqrt{\frac{2mgy_{top}}{I}} = \sqrt{\frac{2mg\frac{l}{2}\cos 2\theta}{\frac{1}{3}ml^2}} = \sqrt{\frac{3g\cos 2\theta}{l}} = \sqrt{\frac{3\cdot 9.8\frac{m}{s^2}}{2m}\cos 30} = 3.57\frac{rad}{s}$$

- When the bar is horizontal, what is the translational speed of the center-of-mass of the bar?

$$v_{com} = r\omega = \frac{l}{2}\omega = \frac{2m}{2}\cdot 3.57\frac{rad}{s} = 3.57\frac{m}{s}$$

Just a comment: You either have to solve this problem by torques (which is an integral problem) or by using energy, which we did. You can't use the rotational equations of motion because they are for constant angular acceleration. Here the angular acceleration changes with angle and is not constant.

Final Exam Example #7

- Fluids, Fluid Pressure, Equation of Continuity, Bernoulli's Equation, Conservation of Energy, Kinetic and Gravitational Potential Energies, and Graphs

A large, cylindrical water tower of radius $r_c = 4m$ sits on top of a hill at the edge of a cliff which drops off vertically by a distance $y = 20m$ to a valley below. The tower is open at the top and filled with water $\left(\rho_w = 1000\frac{kg}{m^3}\right)$ to a height $H = 9.0m$. A small, solid spherical plastic buoy with a mass $m_b = 3.0kg$ and diameter $d_b = 60cm$ is held entirely submerged at the bottom of the tank by a light string (of length $10cm$ attached to the tank floor. At some point in time, someone punctures the side of the tank such that a small circular hole of radius $r = 1.0cm$ is opened on the side of the tank at a depth $h = 4m$ below the tank's upper surface.

- Before the hole is made in the water tower, what is the pressure at the buoy depth and what is the tension in the string connecting the buoy to the bottom of the water tower?

- Using energy ideas, what is the speed of the water that exits the hole after the water tower has been punctured?

- What is the time of flight of the water to the valley floor?

- What is the horizontal distance at which the water hits the valley floor? Assume the speed of the fluid exiting the hole does not change. It does. How does the speed change and what would happen to the horizontal distance if the speed changes?

- What is the impact velocity of the water with the valley floor? Assume the speed of the fluid exiting the hole does not change.

Final Exam Example #7

- Fluids, Fluid Pressure, Equation of Continuity, Bernoulli's Equation, Conservation of Energy, Kinetic and Gravitational Potential Energies, and Graphs

- Suppose that at before the hole is made inside of the water tower, the string connecting the buoy to the water tower's bottom is cut. What is the acceleration of the buoy toward the surface of the water tower? Assume fluid drag is negligible. It is not, but that's a very hard problem to solve.

- What is the kinetic energy change as a function of time if there is no fluid drag? What if there was viscous drag?

- What is the potential energy as a function of time? How does it relate to the change in kinetic energy?

Final Exam Example #7 - Solution

- Before the hole is made in the water tower, what is the pressure at the buoy depth and what is the tension in the string connecting the buoy to the bottom of the water tower?

$$P_d = P_{air} + \rho g d = 1.01\times10^5 \frac{N}{m^2} + 1000\frac{kg}{m^3}\cdot 9.8\frac{m}{s^2}\cdot(9m - 0.1m) = 1.88\times10^5\frac{N}{m^2}$$

$$F_{net,y} = -F_T - F_W + F_B = 0 \rightarrow F_T = F_B - F_W = \rho g V - mg = 1000\frac{kg}{m^3}\cdot 9.8\frac{m}{s^2}\cdot\left(\frac{4}{3}\pi(0.3m)^3\right) - \left(3kg\cdot 9.8\frac{m}{s^2}\right) = 323.4N$$

- Using energy ideas, what is the speed of the water that exits the hole after the water tower has been punctured?

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_T + \Delta U_g = \left(\frac{1}{2}mv_{side}^2 - \frac{1}{2}mv_{top}^2\right) + \left(mgy_{bottom} - mgy_{top}\right)$$

$$v_{side} = \sqrt{2g\left(y_{top} - y_{bottom}\right)} = \sqrt{2\cdot 9.8\frac{m}{s^2}\cdot(9m - 4m)} = 9.9\frac{m}{s}$$

- What is the time of flight of the water to the valley floor?

$$y_f = y_i + v_{iy}t - \frac{1}{2}gt^2 \rightarrow -y = -\frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\cdot(20m + 5m)}{9.8\frac{m}{s^2}}} = 2.26s$$

Final Exam Example #7 - Solution

- What is the horizontal distance at which the water hits the valley floor? Assume the speed of the fluid exiting the hole does not change. It does. How does the speed change and what would happen to the horizontal distance if the speed changes?

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = v_{ix}t = 9.9\frac{m}{s} \cdot 2.26s = 22.4m$$

As the heigh of the water decreases in the tower, the speed of the exiting fluid decreases by conservation of energy. Thus, the horizontal distance will decrease but the time of flight will remain the since that is determined by the vertical fall.

- What is the impact velocity of the water with the valley floor? Assume the speed of the fluid exiting the hole does not change.

$$v_{fx} = v_{ix} + a_x t \rightarrow v_{fx} = v_{ix} = 9.9\frac{m}{s}$$

$$v_{fy} = v_{iy} + a_y t = -\left(9.8\frac{m}{s^2} \cdot 2.26s\right) = -22.2\frac{m}{s}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(9.9\frac{m}{s}\right)^2 + \left(-22.2\frac{m}{s}\right)^2} = 24.3\frac{m}{s}$$

$$\tan\phi = \frac{v_{fy}}{v_{fx}} \rightarrow \phi = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{-22.2\frac{m}{s}}{9.9\frac{m}{s}}\right) = -56.3^0$$

Final Exam Example #7 - Solution

- Suppose that at before the hole is made inside of the water tower, the string connecting the buoy to the water tower's bottom is cut. What is the acceleration of the buoy toward the surface of the water tower? Assume fluid drag is negligible. It is not, but that's a very hard problem to solve.

$$F_{net} = F_B - F_w = ma_y \rightarrow a_y = \frac{F_B - F_w}{m} = \frac{1000\frac{kg}{m^3} \cdot 9.8\frac{m}{s^2} \cdot \left(\frac{4}{3}\pi(0.3m)^3\right) - 3kg \cdot 9.8\frac{m}{s^2}}{3kg} = 107.8\frac{m}{s^2}$$

- What is the kinetic energy change as a function of time if there is no fluid drag? What if there was viscous drag?

  Since the buoy accelerates upwards from rest, its kinetic energy increases. If there were viscous drag, there may be a point at which the upward buoyant force will equal the downward weight. If this happens before the buoy reaches the surface, it will stop accelerating and move upward with a constant velocity.

- What is the potential energy as a function of time? How does it relate to the change in kinetic energy?

  Since the energy is constant and the kinetic energy increases, the potential energy must decrease. This seems counter intuitive, but as the buoy rises, the fluid that it displaces falls. This leads to a decreases in potential energy in the system.