1. Pressure in a fluid varies linearly with depth.
   a. What is the absolute pressure at a depth of 1000m in the ocean if the density of seawater is 1024 kg/m^3?
   b. At this depth what force must the frame around a circular submarine porthole having a diameter of 30cm exert to counterbalance the force exerted by the water?

a. $P_d = P_{air} + \rho gh = 1.013\times10^5 \frac{N}{m^2} + \left(1024\frac{kg}{m^3}\right)\left(9.8\frac{m}{s^2}\right)(1000m) = 1.0137\times10^7 \frac{N}{m^2}$

b. $F = PA = 1.0137\times10^7 \frac{N}{m^2}\times\left(\pi(0.30m)^2\right) = 2.87\times10^6\,N$

2. A light balloon can be filled with either 400m^3 of helium (ρ_{He} = 0.179 kg/m^3) or 400m^3 of hydrogen (ρ_H = 0.0899 kg/m^3) and floated to carry a payload.
   a. If the balloon is nearly in equilibrium (meaning all accelerations are zero), what mass of a payload could a balloon filled with helium support at 0^oC?
   b. If the balloon is nearly in equilibrium (meaning all accelerations are zero), what mass of a payload could a balloon filled with hydrogen support at 0^oC?

$$\sum F_y: \quad F_{b,air} - m_{gas}g - m_{hanging}g = ma_y = 0$$

$$\rightarrow m_{hanging} = \frac{F_{b,air} - m_{gas}g}{g} = \rho_{air}V_{air} - \rho_{gas}V_{gas} = (\rho_{air} - \rho_{gas})V_{gas}$$

a. for He, we have $m_{hanging} = 444.4kg$

b. for H, we have $m_{hanging} = 480.0kg$

where $\rho = \frac{M}{V}$ and volume of air displaced by gas is the same as the volume of gas.

3. The U.S. Navy has the largest warships in the world, aircraft carriers of the *Nimitz* class (for example, the *USS Ronald Regan* shown on the right.) Suppose that 50, 29,000kg airplanes take off from the flight deck and the ship bobs up to float 11cm higher in the water, in an area where g = 9.78m/s^2. What is the horizontal area enclosed by the waterline of the ship? Compare this to the deck of an aircraft which has an area 18,000 m^2.


www.aerospaceweb.org/ question/history/q0226.shtml

$M_{total,planes} = 50 \times 29,ooo kg = 1.45 \times 10^6 kg$

The weight of the planes is equal to the weight of a slice of water

11cm thick with cross sectional area bounded by the waterline of the ship.

$M_{water} = \rho V = \rho_{water} dA \rightarrow 1.45 \times 10^6 kg = \left(1030 \frac{kg}{m^3}\right)(0.11m)A \rightarrow A = 1.28 \times 10^4 m^2$

$\therefore ratio = \frac{A_{waterline}}{A_{flightdeck}} = 0.71$

4. A helium filled balloon is tied to a 2.0m long string where the mass of the string is 0.05kg. The balloon is spherical with radius of 0.40m. When released the balloon lifts a length of string h and then remains in equilibrium. What length of the string is lifted if the balloon has a mass of 0.250kg?

From a free body diagram we find

$F_{air} - F_{g,balloon} - F_{g,He} - F_{g,string} = ma_y = 0 \rightarrow$

$0 = \rho_{air} gV - m_{balloon} g - \rho_{He} gV - m_{string} g \frac{h}{L}$

$\therefore h = \frac{(\rho_{air} - \rho_{He})V - m_{balloon}}{m_{string}} L = 1.91m$

5. What is the pressure in the ocean at a depth of 2000m assuming that salt water has a constant density of 1002 kg/m^3 and that salt water is incompressible?

So we have from integrating the hydrostatic condition for an incompressible fluid, $p_2 - p_1 = -\rho g (z_2 - z_1)$ and taking sea level as datum point 1, then:
$p_1 = 0$ Nm^{-2}, $\rho = 1002$ kgm^{-3}, $g = 9.81$ ms^{-2}, $z_2 = -2000$ m and $z_1 = 0$ m.
$p_2 - 0 = -1002 \times 9.81 \times (-2000 – 0)$, gives $p_2 = 19.66$ MNm^{-2}

6. What depth of oil, with specific gravity 0.8, will produce a pressure of 120 kN/m^2? What would be the corresponding depth of water?

a. Specific gravity of oil $= \frac{\rho_{oil}}{\rho_{water}} = 0.8 \rightarrow \rho_{oil} = 0.8 \times 1000 \frac{kg}{m^3} = 800 \frac{kg}{m^3}$
$p = \rho g h$, where $p = 120 \times 10^3$ Nm^{-2}, $\rho = 800$ kgm^{-3},
$g = 9.81$ ms^{-2}, therefore, $120 \times 10^3 = 800 \times 9.81 \times h$, and **h = 15.3 m**
b. As before, $p = 120 \times 10^3$Nm^{-2} and $g = 9.81$ ms^{-2}, however, $\rho = 1000$ kgm^{-3}.
$p = \rho g h$ and thus $120 \times 10^3 = 1000 \times 9.81 \times h$ $\therefore$ **h = 12.2 m**

7. A hydraulic press has a diameter ratio between the two pistons of 8:1. The diameter of the larger piston is 600mm and it is required to support a mass of 3500kg. The piston is filed with hydraulic fluid of specific gravity 0.8.
    a. What force is required on the smaller piston to provide the required force on the large piston when the two pistons are level?

b. What force is required on the smaller piston to provide the required force on the large piston when the smaller piston is 2.6m below the larger piston?

a. Area of large piston = πr^2 where r = 0.3 m and the area of the large piston is A_l = 0.283 m^2. The area of small piston = πr^2 where r =0.3 x 8= 0.0375 m, since the ratio is 8:1. Therefore the area of the small piston is A_s = 4.418 x$10^{-3}$$m^2$. If the mass supported by the larger piston is 3500 kg, then the force on the larger area is given by : force = mg = 3500x 9.81 = 34335 N. Since the pressure = force x area, the pressure = 34335 x 0.283 = 121.33 kNm^{-2}
This is transmitted to the smaller piston with no difference in height, hence p is Unchanged, therefore the force = p x A_s = 121.33 x 10^3 x 4.418x10^{-3} and the Force = 536 N.

b. If the smaller piston is 2.6 m below the larger piston, then the additional pressure on the smaller area is given by : p =ρgh. Here, we know spec. gravity $= \frac{\rho_{oil}}{\rho_{water}} = 0.8 \rightarrow \rho_{oil} = 0.8 \times 1000 \frac{kg}{m^3} = 800 \frac{kg}{m^3}$. Now g = 9.81 ms^{-2} and h = 2.6 m, thus p = 800 x 9.81 x 2.6 = 20.4 kNm^{-2}. Since this is in addition to the 121.33 kNm^{-2}, the total pressure is p_T =121.33 + 20.4 kNm^{-2} = 141.73 kNm^{-2}. This pressure is applied over the area A_s, hence the force can be found from: force = p_T x A_s = 141.73 x103 x4.418 x10^{-3}, and the **Force = 626.2 N**

8. Show that the ratio of the pressures (p_2/p_1) and densities (ρ_2/ρ_1) for altitudes h_2 and h_1 in an isothermal atmosphere is given by

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} = e^{-\frac{g(h_2-h_1)}{RT}}$$

See class notes for the pressures. And for the density, consider that p = ρRT

9. From the results of question 8, what increase in altitude is necessary in the stratosphere to halve the pressure? Assume a constant temperature -56.5°C and the gas constant is R = 287 J/kgK.

In the stratosphere, the above equation applies and if the pressure is halved with altitude, then $\frac{p_2}{p_1} = \frac{1}{2} = e^{-\frac{g}{RT}(h_2-h_1)} = e^{-\frac{9.8\frac{m}{s^2}}{287\frac{J}{kg\times K}\times 216.5K}(h_2-h_1)} \rightarrow (h_2 - h_1) = 4390m$.