Force, Mass, and Motion

*A vulgar Mechanik can practice what he has been taught or seen done, but if he is in an error he knows not how to find it out and correct it, and if you put him out of his road, he is at a stand; whereas he that is able to reason nimbly and judiciously about figure, force and motion, is never at rest til he gets over every rub.*

Isaac Newton (1694)

**OVERVIEW**

In this lab you will investigate the quantitative relationship between the magnitude and direction of the force, and the magnitude and direction of the resulting acceleration. The force you will be using will not be constant, and you will be looking for a relationship between the force at any instant of time and the acceleration of the object at that same instant.

**Comparing Force and Motion**

In this section you will use your hand to exert a time varying force on a low-friction cart, and you will use a motion sensor to record the resulting motion.

**Preliminary**

*Open lab file* FORCE *(icon on the desktop)*

1. Make a sketch of the experimental setup. Make it a simple line drawing with a rough scale included and with labels for the important elements.

2. Press the *tare button* on the side of the force sensor without touching the hook. In addition, when pushing or pulling the cart, be sure to grasp the hook only. If your hand touches any other part of the cart, the force sensor will not record the actual amount of force.

3. Place the cart on the track with the hook end of the force sensor away from the motion sensor. Hold the cart stationary with one hand and first push and hold and then pull and hold the hook while taking data. The
direction away from the motion sensor should be set up as the positive
direction. Make sure that the sign of the force recorded in your graph
corresponds to the direction of the force you exert on the cart. Record
briefly in words what you have done and what the result was.

Force, Velocity and Acceleration

**Using “Alt –“ delete the data from the screen**

4a) Now open graph 2. This graph shows the force and the velocity of the cart
and the acceleration of the cart. Grip the hook again and record data as
you push and pull the cart back and forth along the track smoothly while
allowing it to move. You should try to make some slow and some fast
oscillations as well as some fairly irregular movements, but nothing too
abrupt. Repeat this process until you have some smooth reliable data
showing the force you exerted on the cart as a function of time. Print out
your graph. Your graph should have a single curve on each of the three
graphs.

b) How well do the velocity and acceleration graphs compare with the force
graph in overall shape? Do you see a strong connection between force and
acceleration? Try a couple more runs if you want more convincing set of
graphs.

Are Force and Acceleration Proportional to Each Other? The fact that the force
and acceleration graphs have the same general shape suggests that the two might
be proportional to each other. If so, a graph of force vs. acceleration would be a
straight line through the origin and the equation relating them would be \( F = ma \),
where \( F \) is the force, \( a \) the acceleration, and \( m \) (the constant of proportionality) is
the slope of the graph. Since you now have a graph of values of force and
acceleration, you can use the data from it to test the proportionality of \( F \) and \( a \).

**Delete the velocity plot.**

5a) Prepare a table like the one shown at right. Use
the smart cursor to read data on force and
acceleration from your graphs. Begin by
recording values of the *time variable* when the
force has one of its largest positive value, one of
its largest negative value, a time when it is near
zero, and a few times when the force is in-
between. Then go back and use the smart cursor
to record the values of force and acceleration at
those times.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Force (N)</th>
<th>Accel. (m / s²)</th>
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b) Prepare a graph with Force on the vertical axis and acceleration on the horizontal axis. Label the axes, include units, as usual, and title your graph.

c) Graphs of experimentally determined relationships are never perfect – there is always some scatter in the data due to uncertainties. Taking this into account, does the relationship between force and acceleration appear to be a proportion? Why or why not? Refer to the figure in your comments.

If the relationship is a proportion, draw in the best fit straight line. What is the value of the constant of proportionality? Show your work. Include units.

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**The Effect of Mass on Acceleration**

The proportionality of force and acceleration is almost, but not quite, one of Newton’s laws of motion. To get the law in its final form, we will need to explore how the acceleration is affected when similar forces are exerted on different bodies. This will give rise to the concept of mass.

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**Defining and Measuring Gravitational Mass**

Probably your most immediate notion of an object’s mass has to do with how hard it is to lift. Objects which have more “stuff” seem to be harder to lift. The idea that mass measures “stuff” gives us an immediate way to compare the masses of objects of the same composition, like different size pieces of iron. We could define a certain piece of iron to be a standard kilogram (We will use our double iron bar as a standard kilogram.), then two of the bars would be called 2 kg, etc. But how could we compare the masses of objects with different compositions. One time-honored way is to use an equal arm balance. If two objects balance, then it seems they are subject to the same gravitational pull, and would be equally hard to lift. We say they have the same gravitational mass.

There is another, rather different way of approaching the idea of mass. Suppose a good friend has called you in a panic. His battery is dead, and he needs to have you come and push his car to get it started. You answer that you will be right over. Then you remember that your friend owns two vehicles – a large delivery van and a small sports car. Since you are feeling a little weak today, you hope that your friend wants to push the easier of the two cars.

Since we can exert the same force on both cars, there must be something about them that causes one to have a greater acceleration than the other. We can identify this with mass; an object has less acceleration for the same force if it has more mass. We can make this quantitative simply by defining mass to be the ratio
of force to acceleration for an object, in other words we can define mass to be the constant of proportionality in the formula \( F = ma \). (It is just luck that \( m \) is a good symbol for mass, remember we started out thinking of it as slope.) To distinguish this mass from the gravitational mass, we call it inertial mass. Now let us compare the gravitational and inertial masses of objects.

Return to the computer, delete the current data with “Alt – “ and open graph 3.

6a) Repeat the pushing and pulling exercise that you conducted earlier and examine the resulting force vs. time graph. Now change the horizontal axis from time to acceleration. The graph which now appears on the screen shows force on the cart vs. acceleration of the cart. You should see a proportion. Use the Fit feature of the software to determine the constant of proportionality. Print out your graph.

b) Delete the data with “Alt – ,” remove the double bar from the tray and repeat. Record the constant of proportionality and print out the new graph.

c) Delete the data and repeat with two double bar masses in the tray. Record the constant of proportionality and print out the graph.

d) Weigh the double bars on the electronic balance.

e) The slope of the force vs acceleration graph is called the inertial mass. Do the gravitational and inertial masses seem to compare well? Does the formula \( F = ma \) seem to describe the data well? Comment.

It is not obvious that the two definitions of mass – gravitational and inertial should be the same. After all, one has to do with gravity the other does not. The equivalence of the two is, however, a part of both Newton’s and Einstein’s theories of gravity and the equivalence has been measured with great precision. In fact, sophisticated experiments have shown that within the limits of experimental uncertainty, there is no difference between the two to within one part in \( 10^{11} \).