Union College Winter 2022

Physics 120, Lab #5:

Hooke's Law with Spring Force and Work Done in Breaking a Board

Part I: Springs

Springs provide a simple but accurate model of how structural forces respond to deformations. The expression for the spring force is given by Hooke's Law, $F = -k_s x$, where k_s is the spring's stiffness parameter and x is the amount of stretch of the spring from its natural length. In this part of the lab, you will observe how well Hooke's Law describes the restoring force of a spring, obtain a representative plot of F vs. x, and determine the stiffness parameter for two springs.

1. Open Excel. Make one column for mass, one for amount of stretch, x, of the spring, and another column for the magnitude of the spring force.
2. Hang a spring from the hook. Measure the position of the end of the spring and record this as the spring's *relaxed position*. Be sure to include headers in your table. Remember to indicate the units, either in the headers or in a separate row.
3. Hang a small mass on the end of the spring, allow the mass to come to rest, and measure and record the new position of the end of the spring (*not the end of the mass*). Since the mass is stationary, the spring force must exactly cancel the weight of the mass. Use this to calculate and record the spring force.
4. Hang a slightly larger mass on the spring, measure the new position of the end of the spring, and calculate the force of the spring.
5. Repeat for a number of masses (use at least five masses, total).

Analysis:

6. For each spring, plot the spring force vs. the stretch of the spring. Does your spring obey Hooke's Law? What aspects of the plot supports your claim? Consider the expression of Hooke's law and whether your plot fits a proportionality (a linear relation through the origin).
7. Fit a straight line to your curve and use the fit to calculate the spring stiffness parameter. Should you demand that the straight line go through the origin?
8. Do a "regression" analysis to get the uncertainties in the fit parameters. Does a zero y-intercept agree with your fit?
9. Record your spring stiffness parameters, k_s, with uncertainty.
10. Repeat steps 3-9 for a different spring.

Part II: Restoring Force of Wood and the Work Done in Breaking A Board

When a force is applied to a wood board, the board must exert a force in return to hold itself together. But there is a limit; if the external force does enough work on the board, it will break.

1. Measure and record in a *new Excel table* the mass of five different bricks and calculate the average. Calculate the standard error in your inferred mass of an individual brick.
2. Measure the mass of the apparatus' tray and record in the Excel table.
3. In the Excel table, create columns for: # of bricks; gauge reading in mm; deflection of the board in m; total mass in kg; total force in N; and Work done on the board.

4. Carefully place a piece of wood on the cross bars and place the gauge's needle at the center of the board. Read the initial setting of the gauge. Record this number in a row above the columns and label as the relaxed position for the board. (Record your estimated uncertainty in this number also.)
5. Carefully hang the tray on the board and then and record the new gauge reading. Input the equations to calculate the total force hanging on the board due to the tray and the deflection of board from the relaxed position. The gauge's reading will *decrease* as the deflection of the board increases and the gauge will go around a few times; it will be helpful, then, to add 900 to your initial readings (and decrease the hundreds place each time around).
6. Carefully add one brick onto the middle of the tray (and enter '1' for number of bricks). In the total force cell, enter the equation to calculate the total force, which includes the number of bricks as well as the tray. Read and record the new gauge setting in the appropriate box.
7. Continue adding bricks, and entering the data until the board breaks. (Note: be careful to keep your feet away from the area below tray, in case the board breaks while you're there.)

Analysis

9. Plot F_{board} vs. x. Judging from the shape of the curve, does the wood board's restoring force obey Hooke's Law? Is Hooke's Law appropriate for modeling structural forces?
10. Use Excel to determine the stiffness parameter, with uncertainty, for the board?
11. What is the work done by the force of gravity with the addition of each brick? In the column for work done on the board enter an equation that correctly calculatets the amount of work done in each individual step—the force, F, with that many bricks, moves the center of the board a small step in x.
12. At the bottom, have Excel calculate the *total* work done in breaking the board.
13. Since $W = \int \vec{F} \cdot d\vec{r}$, or $\int F dx$ for the one-dimensional case here, work should also equal the area under the curve of force vs. distance. Note the shape of this plot and calculate the area under the curve. Do you get the same answer as in step 12?

Error Analysis:

14. Your final results are the stiffness parameters of the 2 spring plus that of the board, and the work done to break the board. You already have uncertainties in the stiffness parameters (from using regression analysis), but you also need an uncertainty in the work done on the board, and you have only one calculation. Therefore, you now need to do "propagation of uncertainty" calculation. Your calculation of work depends on the mass of each brick, the number of bricks, and the deflections of the board. You should already have uncertainties in the mass of each brick and the deflection of the board. You do, also, have an uncertainty in the number of bricks, because you had to add one whole brick at a time – perhaps the board would have broken with the addition of half a brick at the end. So, consider your uncertainty in the number of bricks as 1. You now need to propagate these 3 uncertainties to get an uncertainty in the total work. Use your equation in step 13 to do this calculation.

Futher Contemplation.

1. The work that the weight of the bricks put into the board did not change the board's kinetic energy. Where did this work, or energy, go?
2. Consider breaking the board by dropping a hard object onto the board. As the object falls, gravity does work to give it kinetic energy. When the object hits the board, the force of the board does negative work on the object to bring it to a stop. Use Work-Energy considerations to calculate the minimum height from which a 1-kg mass must be dropped in order for it to gain enough kinetic energy to break the board. Use propagation of uncertainty to get an uncertainty in this number.
3. Determine a method to calculate the mass of a human fist, with uncertainty. Starting from the definition of work, calculate the speed that your hand must moving just before it strikes the board in order to break a board. Propagate your uncertainties again. Do you think this can be accomplished?

The following questions and results will be collected on Tuesday, February 18, 2020 in class. Please type answers to the questions on a separate piece of paper and show all calucations. You may hand write the calculations or type them, whichever is easier for you.

- Fully labeled graphs for your two springs and answers to the following: Does your spring obey Hooke's Law? What aspects of the plot supports your claim? Consider the expression of Hooke's law and whether your plot fits a proportionality (a linear relation through the origin).

- The spring stiffness parameters, k_s, with uncertainty (show the calculation) for each spring.

- Fully labeled graph for your board and answers to the following: The spring stiffness parameter, k_B, with uncertainty (show the calculation) for your board. Judging from the shape of the curve, does the wood board's restoring force obey Hooke's Law? Is Hooke's Law appropriate for modeling structural forces?

- The *total* work done (with uncertainties) in breaking the board based on calculating the work done in Excel from each mass added to the cradle.

- Starting from the definition of work, calculate the area under the force versus stretch curve and compare this to the *total* work (with uncertainties) done in the previous question.

- The work that the weight of the bricks put into the board did not change the board's kinetic energy. Where did this work, or energy, go?

- Using the definition of work, derive an expression for the height that a mass *m* would have to be dropped in order to break the board. Then, from this expression, calculate the height (with uncertainty).

- Write a procedure for determining the mass of a human fist and then from this procedure, calculate the mass of your fist with its uncertainty.

- From the definition of work, derive an expression for the speed (with an uncertainty) at which your hand would have to be moving just before it strikes the board in order to break the board. Then from this expression, evaluate your expression for the speed of the hand with its uncertainty. Does this seem reasonable. Explain why or why not.

Worksheet for Board Breaking Experiment

Specify uncertainties and appropriate units for all quantities.

1) Cradle mass ____________________+/-_________________

2) Avg brick mass ____________________+/-_________________

3) Number of bricks that broke the board ______+/-______

4) Spring constant of board __________________+/-_________________

5) Total work done on board by summing over the steps______________

6) Total Work done on board by calculating area under the curve _____________

7) Uncertainty in Total Work _________________

8) Height for 1kg mass to break board ___________________+/- _______________________
Calculation (including final formula in terms of variables):

9) Mass of your hand ________________+/-__________________________

10) Speed necessary to break board with fist ____________________+/-____________________
Calculation (including final equation in terms of variables).