

Physics 111

Fall 2007

Light Solutions

1. The human eye is most sensitive to light having a wavelength of $5.50 \times 10^{-7} \text{ m}$, which is in the green–yellow region of the visible electromagnetic spectrum. What is the frequency of this light?

The frequency is given from $c = f\lambda \rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8 \frac{\text{m}}{\text{s}}}{5.5 \times 10^{-7} \text{ m}} = 5.5 \times 10^{14} \text{ s}^{-1}$.

2. A diathermy machine, used in physiotherapy, generates electromagnetic radiation that gives the effect of “deep heat” when absorbed in tissue. One assigned frequency for diathermy is 27.33 MHz. What is the wavelength of this radiation?

The wavelength is given from $c = f\lambda \rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8 \frac{\text{m}}{\text{s}}}{27.33 \times 10^6 \text{ s}^{-1}} = 10.98 \text{ m}$.

3. A neodymium–yttrium–aluminum garnet laser used in eye surgery emits a 3.00-mJ pulse in 1.00 ns, focused to a spot $30.0 \mu\text{m}$ in diameter on the retina.
- Find (in SI units) the power per unit area at the retina. (This quantity is called the *irradiance* in the optics industry.)
 - What energy is delivered to an area of molecular size, taken as a circular area 0.600 nm in diameter?

(a)
$$I = \frac{(3.00 \times 10^{-3} \text{ J})}{(1.00 \times 10^{-9} \text{ s}) \left[\pi (15.0 \times 10^{-6} \text{ m})^2 \right]} = \boxed{4.24 \times 10^{15} \text{ W/m}^2}$$

(b)
$$(3.00 \times 10^{-3} \text{ J}) \frac{(0.600 \times 10^{-9} \text{ m})^2}{(30.0 \times 10^{-6} \text{ m})^2} = \boxed{1.20 \times 10^{-12} \text{ J}} = 7.50 \text{ MeV}$$

4. A handheld cellular telephone operates in the 860- to 900-MHz band and has a power output of 0.600 W from an antenna 10.0 cm long.
- Find the average magnitude of the Poynting vector 4.00 cm from the antenna, at the location of a typical person’s head. Assume that the antenna emits energy with cylindrical wave fronts. (The actual radiation from antennas follows a more complicated pattern.)
 - The ANSI/IEEE C95.1-1991 maximum exposure standard is 0.57 mW/cm^2 for persons living near cellular telephone base stations, who would be continuously exposed to the radiation. Compare the answer to part (a) with this standard.

The area over which we model the antenna as radiating is the lateral surface of a cylinder,
 $A = 2\pi r\ell = 2\pi(4.00 \times 10^{-2} \text{ m})(0.100 \text{ m}) = 2.51 \times 10^{-2} \text{ m}^2$.

(a) The intensity is then: $S = \frac{P}{A} = \frac{0.600 \text{ W}}{2.51 \times 10^{-2} \text{ m}^2} = \boxed{23.9 \text{ W/m}^2}$.

(b) The standard is:
 $0.570 \text{ mW/cm}^2 = 0.570 \left(\text{mW/cm}^2 \right) \left(\frac{1.00 \times 10^{-3} \text{ W}}{1.00 \text{ mW}} \right) \left(\frac{1.00 \times 10^4 \text{ cm}^2}{1.00 \text{ m}^2} \right) = 5.70 \text{ W/m}^2$

While it is on, the telephone is over the standard by

$$\frac{23.9 \text{ W/m}^2}{5.70 \text{ W/m}^2} = \boxed{4.19 \text{ times}}.$$

5. A plane electromagnetic wave is traveling along the x-axis. If the electric field of the wave has a maximum value of $2 \times 10^{-4} \text{ N/C}$ and lies along the y-axis, find the wave's maximum magnetic field and its direction.

The magnetic field is given by $E = cB \rightarrow B = \frac{E}{c} = \frac{2 \times 10^{-4} \frac{\text{N}}{\text{C}}}{3 \times 10^8 \frac{\text{m}}{\text{s}}} = 6.67 \times 10^{-13} \text{ T}$ in the z-direction.

6. Unpolarized light of intensity I_o passes through a Polaroid with its transmission axis vertically oriented. What intensity emerges? If the transmitted light passes through a second Polaroid sheet with its transmission axis 60° to the vertical what fraction of the original incident light intensity I_o emerges?

Through the 1st polarizer, $\frac{1}{2}$ of the original intensity emerges. This vertically polarized light is then incident on a 2nd polarizer oriented at 60° to the vertical. The intensity that emerges

$$\text{is } S = \frac{S_o}{2} \cos^2 60 = 0.125 S_o = 12.5\% S_o.$$

7. Three polarizers arranged in series are each oriented at 30° from the previous one. If an unpolarized light beam travels through the three polarizers and emerges with an intensity of 0.2 W/m^2 , what was the intensity of the beam incident on the first polarizer?

Through the 1st polarizer, $\frac{1}{2}$ of the original intensity emerges, or $S_1 = \frac{S_o}{2}$. Through the

second polarizer, the intensity that emerges is $S_2 = S_1 \cos^2 30 = \frac{S_o}{2} \cos^2 30 = 0.375 S_o$.

This light is incident on a 3rd polarizer and the intensity that emerges is $S_3 = S_2 \cos^2 30 = 0.375 S_o \cos^2 30 = 0.281 S_o$. It is found that the actual intensity that emerges is 0.2 W/m^2 . Thus the initial intensity is $0.2 \frac{\text{W}}{\text{m}^2} = 0.281 S_o \rightarrow S_o = 0.71 \frac{\text{W}}{\text{m}^2}$.

8. Suppose a 50-kW radio station emits EM waves uniformly in all directions.
- How much energy per second crosses a 1.0-m^2 area 100 m from the transmitting antenna?
 - What is the rms magnitude of the \vec{E} field at this point, assuming the station is operating at full power?
 - What is the voltage induced in a 1.0-m-long vertical car antenna at this distance?

- (a) The radio waves have the same intensity in all directions, so the energy per unit area per unit time over a sphere centered at the source with a radius of 100 m is

$$S = \frac{P_o}{A} = \frac{P_o}{4\pi r^2} = \frac{(500 \times 10^3 \text{ W})}{4\pi (100 \text{ m})^2} = 0.398 \text{ W/m}^2.$$

Thus the power through the area is

$$P = SA = (0.398 \text{ W/m}^2)(1.0 \text{ m}^2) = \boxed{0.40 \text{ W.}}$$

- (b) We find the rms value of the electric field from

$$S = c\epsilon_0 E_{\text{rms}}^2;$$

$$0.398 \text{ W/m}^2 = (3.00 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) E_{\text{rms}}^2, \text{ which gives } E_{\text{rms}} = \boxed{12 \text{ V/m.}}$$

- (c) The voltage over the length of the antenna is

$$E_{\text{rms}} = V_{\text{rms}} = E_{\text{rms}} d = (12 \text{ V/m})(1.0 \text{ m}) = \boxed{12 \text{ V.}}$$

9. Repeat problem #8 for a distance of 100 km from the station.

- (a) The radio waves have the same intensity in all directions, so the energy per unit area per unit time over a sphere centered at the source with a radius of 100 km is

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \frac{(50 \times 10^3 \text{ W})}{4\pi (100 \times 10^3 \text{ m})^2} = 3.98 \times 10^{-7} \text{ W/m}^2.$$

Thus the power through the area is

$$P = SA = (3.98 \times 10^{-7} \text{ W/m}^2)(1.0 \text{ m}^2) = 3.98 \times 10^{-7} \text{ W} = \boxed{0.40 \mu\text{W}}.$$

- (b) We find the rms value of the electric field from

$$S = c\epsilon_0 E_{\text{rms}}^2;$$

$$3.98 \times 10^{-7} \text{ W/m}^2 = (3.00 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)E_{\text{rms}}^2,$$

$$\text{which gives } E_{\text{rms}} = \boxed{0.012 \text{ V/m}}.$$

- (c) The voltage over the length of the antenna is

$$E_{\text{rms}} = V_{\text{rms}} = E_{\text{rms}}d = (0.012 \text{ V/m})(1.0 \text{ m}) = \boxed{0.012 \text{ V}}.$$

10. What is the maximum power level of the radio station of Problem #8 so as to avoid electrical breakdown of air at a distance of 1.0 m from the antenna? Assume the antenna is a point source. Air breaks down in an electric field of about $3 \times 10^6 \text{ V/m}$.

The energy per unit area per unit time is

$$S = \frac{1}{2} c\epsilon_0 E_0^2;$$

$$= \frac{1}{2} (3.00 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3 \times 10^6 \text{ V/m})^2 = 1.20 \times 10^{10} \text{ W/m}^2.$$

The power output is

$$P = S4\pi r^2 = (1.20 \times 10^{10} \text{ W/m}^2)4\pi (1.0 \text{ m})^2 = \boxed{1.5 \times 10^{11} \text{ W}}.$$

11. An EM wave has frequency $9.66 \times 10^{14} \text{ Hz}$. What is its wavelength, and how would we classify it?

The wavelength is given from

$$c = f\lambda \rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8 \frac{\text{m}}{\text{s}}}{9.66 \times 10^{14} \text{ s}^{-1}} = 3.13 \times 10^{-7} \text{ m} = 313 \text{ nm}$$

which is in the ultraviolet portion of the EM spectrum.

12. Our nearest star (other than the Sun) is 4.2 light-years away. That is, it takes 4.2 years for the light it emits to reach Earth. How far away is it in meters?

A light year is the distance that light travels in one year. Since the nearest star (Proxima Centari) is 4.2 light-years away, the light has to travel a distance equal to

$$c = \frac{d}{t} \rightarrow d = ct = 3 \times 10^8 \frac{m}{s} \times \left(4.2 \text{ yrs} \times \frac{3.16 \times 10^7 s}{1 \text{ yr}} \right) = 3.98 \times 10^{16} m.$$