The limits of m and n.

1. From class, we saw the limits of m have to be in integer from $e^{im\phi} = e^{im(\phi+2\pi)} \rightarrow 1 = e^{2im\pi} \rightarrow 1 = \cos(2m\pi) + i\sin(2m\pi)$ which can only be true if $m = 0, \pm 1, \pm 2, ...$

2. From class we have the index n defined by $n = l+1+j$, where j is from the power series solution to the reduced radial equation. The values of the index n can be $n = 1,2,3,4,...$ Setting this equal to $l+1+j$, we have $n = l+1+j \rightarrow n-1 = l+j$. This was where my mistake was in class, j cannot just be a *single value* $j = 0,1,2,3,...$, but rather there are allowed values of l *and* j. Here are some of the allowed values of l and j for a given n.

n	$n-1$	l	j
1	0	0	0
2	1	0	1
		1	0
3	2	0	2
		1	1
		2	0

If we continue this pattern, we see that for any n, the index l can assume values $l = 0,1,2,3,...,(n-1)$. This is the limits l of for a given n.

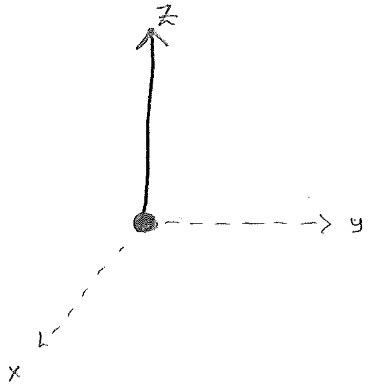
In addition, we can construct the values of $L_Z = m\hbar$ and $L = \sqrt{l(l+1)}\hbar$ for any given l and m.
We have:

l	m	L_z	L
0	0	0	0
1	-1	$-\hbar$	$\sqrt{2}\hbar$
	0	0	
	1	$\hbar$	
2	-2	$-2\hbar$	$\sqrt{6}\hbar$
	-1	$-\hbar$	
	0	0	
	1	$\hbar$	
	2	$2\hbar$	

The vector diagrams that we drew at the end of class illustrating $L = \sqrt{l(l+1)}\hbar$ and $L_Z = m\hbar$ are attached.

$l = 0$

$m = 0$

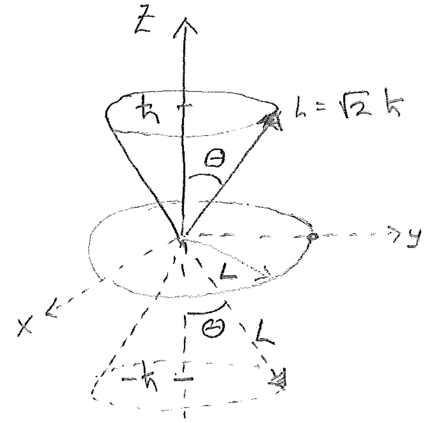


$L_z = 0$

$L = 0$

$l = 1$

$m = -1, 0, 1$



3 allowed States of angular momentum.

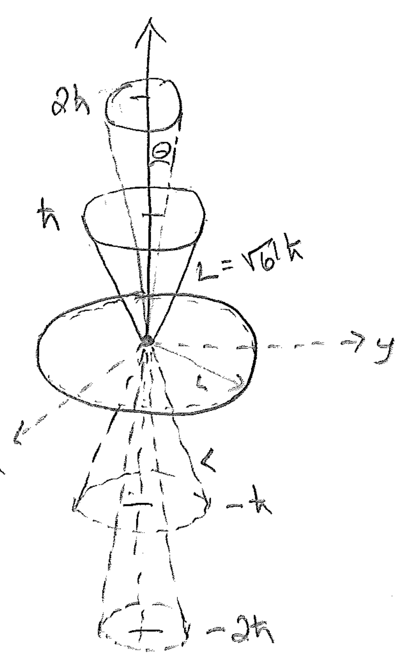
The Components L_x & L_y are unknown and are Smeared around a Cone of angle θ.

$$L_z = \begin{cases} -\hbar \\ 0 \\ \hbar \end{cases}$$

$L = \sqrt{2}\,\hbar$

$l = 2$

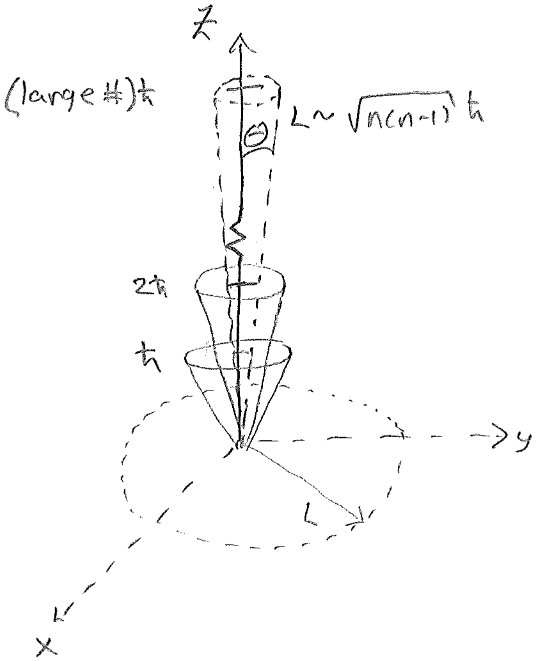
$m = -2, -1, 0, 1, 2$



$$L_z = \begin{cases} -2\hbar \\ -\hbar \\ 0 \\ \hbar \\ 2\hbar \end{cases}$$

$L = \sqrt{6}\,\hbar$

In the limit of large n, l is large and there are more allowed states of angular momentum.



1) $L = \sqrt{l(l+1)}\,\hbar \sim \sqrt{(n-1)(n)}\,\hbar$

2) $\cos\theta = \dfrac{L_z}{L} \doteq 1$ in the limit that θ is small (as $n \to \infty$).

$L \sim L_z$ and L becomes fixed in space. That means the mag. & dir. are fixed and this is the Classical Limit.

Since L points in a given direction w/ known magnitude

$\rightarrow$ no uncertainty.

all components are known.