Physics in Modern Medicine **Winter 2023**

Homework #1

Chapter 2 – Telescopes for Inner Space

Questions

Q2.1 Small optical fibers provide for imaging of very small feature. The more optical fibers packed into a bundle the greater the image quality.

Q2.2 Incoherent bundles of optical fibers do not retain the orientation of the individual fibers from end to end. Therefore, they are only useful for transmitting light for purposes of illumination, and they cannot transmit images. Coherent fiber optic bundles are organized with each optical fiber in the same orientation with respect to its neighbors at each end, so they *can* be used to transmit images.

Problems

2P.1 Reflection and Refraction

a. By the law of reflection, the angle of reflection is 20^0. The angle of refraction is determined by Snell's law and is

$$n_{inc} \sin\theta_{inc} = n_{ref} \sin\theta_{ref} \rightarrow \theta_{ref} = \sin^{-1}\left(\frac{n_{inc}\sin\theta_{inc}}{n_{ref}}\right)$$

$$= \sin^{-1}\left(\frac{1.0 \sin 20}{1.33}\right) = 14.9^0$$

b. By the law of reflection, the angle of reflection is 5^0. The angle of refraction is determined by Snell's law and is

$$n_{inc} \sin\theta_{inc} = n_{ref} \sin\theta_{ref} \rightarrow \theta_{ref} = \sin^{-1}\left(\frac{n_{inc}\sin\theta_{inc}}{n_{ref}}\right)$$

$$= \sin^{-1}\left(\frac{1.33 \sin 5}{1.00}\right) = 6.7^0$$

2P.4 Twisted fiber optics

a. Even though they use two different types of glass for the optical fiber and the filling between the optical fibers, Bob and Lynn will still be able to have total internal reflection within the fibers as long as they use the higher index material for the optical fibers and the lower index material for the spaces in between. We will use the equation for the critical angle: $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.5}{1.65}\right) = 65.4^0$where $n_2 = 1.50$for the filling material between the fibers and $n_1 = 1.65$for the fibers.

b. Bob and Lynn may have twisted senses of humor, but no amount of twisting or knotting will distort the image through their fiber optic scopes, if the fibers are in the same positions with respect to each other at the entrance end as at the exit end (Cases A and B.) This is because the light from each part of the image is confined to lie within the fiber by total internal reflection, even when the fibers are bent as shown. Scrambling the ends of the scope will distort the image, since each fiber carries part of the image, and their orientations must be carefully preserved to reconstruct the image properly at the viewing end (Case C.) The light source fibers need not be kept in any configuration since they do not carry an image.

2P.6 Poor optical fiber design

In order too effectively guide the light the cladding must have an index of refraction that is less than the core. For this situation, the core has an index of refraction that is greater than the cladding and thus there will be no angles of incidence that will exceed the critical angle. Moreover, the critical angle is given as

$n_1 \sin\theta_c = n_2 \sin\theta_2 \rightarrow \theta_c = \sin^{-1}\left(\frac{n_2 \sin\theta_2}{n_1}\right) = \sin^{-1}\left(\frac{1.60 \sin 90}{1.55}\right)$ which is undefined.

2P.8 A rod cell

a. The critical angle is $n_1 \sin\theta_c = n_2 \sin\theta_2 \rightarrow \theta_c = \sin^{-1}\left(\frac{n_2 \sin\theta_2}{n_1}\right) = \sin^{-1}\left(\frac{1.38 \sin 90}{1.48}\right) = 68.8^0$.

b. The angle of refraction is determined by $n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.38 \sin 13}{1.48}\right) = 12.1^0$

c. Since the critical angle is 68.8^0, the only ray that has an angle of incidence greater than the critical angle, is ray D.

d. Rod cells, or rods, are photoreceptor cells in the retina of the eye that can function in less intense light than can the other type of photoreceptor, the cone cells. Since rod cells are more sensitive to light, rods are responsible for night vision. Named for their cylindrical shape, rods are concentrated at the outer edges of the retina and are used in peripheral vision. In low light conditions you would want a cell to act as a light pipe and guide the incoming light to the retina to form an image.

P2.9 The intensity of light through a fiber optic

a. The reflected intensity is given by

$S_{R,front} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 S_{incident} = \left(\frac{1.5-1.0}{1.5+1.0}\right)^2 S_{incident} = 0.04 S_{incident} \rightarrow 4\% S_{incident}$.

This means that of the incident light, 4% gets reflected so 96% gets transmitted.

b. The 96% of the incident light that propagates down the fiber optic, when it reaches the end, reflects from the surface, and transmits into the patient.

$S_{R,back} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 S'_{incident} = \left(\frac{1.5 - 1.0}{1.5 + 1.0}\right)^2 0.96 S_{incident} = 0.038 S_{incident} \rightarrow 3.8\% S_{incident}$.

This means that 3.8% of the total incident light on the front surface that made it to the back surface gets reflected. Then, of the light incident on the back surface, the amount that gets transmitted into the patient is $0.96 S_{incident} - 0.038 S_{incident} = 0.922 S_{incident} \rightarrow 92.2\% S_{incident}$. Of the original light incident, $S_{incident}$, only 92.2% gets transmitted into the patient.