Physics in Modern Medicine **Winter 2023**

Homework #3

Chapter 3 – Lasers in Medicine

Questions

Q3.5 Since the chief pigment in the endometrial tissue is blood, a good choice of laser would be one that is in the blue portion of the spectrum and one that would be inappropriate would be a laser whose output is in the red portion of the spectrum. An appropriate choice for the laser would be something in the $400nm - 600nm$ range, say an Argon laser or maybe a copper vapor. An inappropriate choice for the laser would be lasers whose wavelength is in the $600nm$ and larger range, say He-Ne, Krypton, ruby and any IR laser.

Q3.7 a. To use a laser as a surgical tool, you need to know the absorption properties of the tissues that will be operated on, the penetration depth for the laser, the power density, and the mode of operation of the laser.

b. The tissue's absorption spectrum should be known in order to select the appropriate laser. The concentration of absorbing molecules should also be known since this will determine the penetration depth.

Q3.9 Excimer lasers, which are ultraviolet lasers, are used in photorefractive surgery because the cornea is transparent to any visible light, such as that in the argon laser. The cornea is a good absorber of UV radiation. The retina on the other hand has many blood vessels and to target these in a surgery one would want the argon laser since it operates in the blue-green portion of the EM spectrum and would be preferentially absorbed by the red blood cells here.

Problems

3P.5

a. To achieve a power density of $80\frac{mW}{cm^2}$ with a 1W CW laser, you would need a spot size given through (Eq. 3.7) $I = \frac{P}{\pi r^2}$, where I is the power density (or intensity) and P is the power. Solving for the spot radius, r, we get:

$$r = \sqrt{\frac{P}{\pi I}} = \sqrt{\frac{1W}{\pi \times 80 \times 10^{-3}\frac{W}{cm^2}}} = 2cm.$$

b. The intensity is the power per unit area and the power is the energy per unit time. Thus, the exposure time would be given by the energy per unit area divided by the intensity. We define the energy per unit area the fluence F.The exposure time would be given by:

$$I = \frac{P}{A} = \frac{E}{tA} \rightarrow t = \frac{E/A}{I} = \frac{F}{I} = \left[\frac{100\frac{J}{cm^2}}{80\times10^{-3}\frac{W}{cm^2}}\right] \times \frac{1min}{60s} = 20.8min \sim 21min$$

This is within a reasonable time limit for a therapeutic procedure.

c. The goals would be to maximize absorption for the photosensitizer, in this case

phthalocyanine, and at the same time to minimize hemoglobin absorption, which is not specific to tumors. Since phthalocyanine has an absorption peak and hemoglobin absorbs relatively weakly at the region between $600nm$ to $720\ nm$, the red gold vapor laser ($630nm$), Krypton laser ($676.4nm$), and dye laser operating in this wavelength range would be reasonable choices. The helium-neon laser has a good match, but in fact the powers achievable tend to be too low for this use. The ruby laser is a similarly good wavelength match, but the pulsed operation is not desirable in this application.

d. The reason lasers are a good choice involves both their spatial coherence, necessary because of the need for relatively high power densities over a small region of tissue, and their temporal coherence, which is important to guarantee that the power density delivered is concentrated at the absorption peak of the photosensitizer, while avoiding high levels of absorption by other body tissues, such as blood. The very similar absorption properties of hemoglobin and many photosensitizers especially make the latter criterion highly important.

Chapter 4 – Seeing with Sound

Questions

Q4.1 An US transducer is placed on the patient and coupled to the patient using a gel. This minimizes reflections at the air/skin interface. The US that propagates into the body reflects off the internal structures in the body. The strength of the echoes depends on the differences in acoustic impedance between different structures. The US transducer not only emits sound, but also is used to detect the echoes from inside of the body. The time it takes the US to enter the body and then for the echo to return can be used (knowing the velocity of sound in the body) to determine distances to the various structures in the body. In addition, the intensity of the reflected sound waves can be coded into a shade of grey at each of these distances

Q4.2 One reason that bats have evolved US is to conceal themselves while hunting. If bats produced audible sound as their prey, then the prey would hear them hunting. Another reason is that since the velocity of sound is more or less constant in air (it depends on temperature) audible sound will have a longer wavelength than US. The longer wavelength sound produces less spatial resolution than does US. The bat can therefore detect objects as small as the wavelength of sound it emits. The US will "see" the insects and other prey of the bat, where audible sound would not.

Q4.3 Low frequency foghorns have long wavelengths. These long wavelengths, when they disperse through space, can envelope the ship so that anyone on the ship can hear the sound. If high frequency sound were used, with a shorter wavelength, then you would only target perhaps, a small area of the ship and the ship may not be aware of the danger if no one hears the sound.

Q4.4 Increasing the frequency may improve your resolution, but the penetration depth of the sound into the sample would go to zero. The half intensity depth is inversely related to the frequency. As the frequency approaches infinity, the penetration depth approaches zero.

Q4.5 Miscalibration of the US scanner will produce distortions in the US image.
To correct for miscalibrations the sonographer or technician would use test samples, or phantoms for calibration. Other effects that will cause distortions are refraction of the sound waves as the waves pass from one material into another and reverberations. Refraction produces echoes that are displaced from the expected path. Refraction can produce double images and there's not much to fix this problem. Reverberations are multiple high-intensity echoes from two strongly reflecting interfaces. Acoustic shadowing is another problem. US scanners compensate for the fact that the US pulse intensity decreases with increasing depth into the sample by artificially increasing the brightness. Acoustic shadows are created if a strong absorbing or very reflective surface is encountered by the US wave and the scanner software over compensates.

Problems

P4.2

a. We wish to have an ultrasound wavelength of $0.50mm$ in soft tissue. Since the speed of sound in soft tissue is $v_s = 1540\frac{m}{s}$, the frequency needed is
$$f = \frac{v_s}{\lambda} = \frac{1540\frac{m}{s}}{0.5 \times 10^{-3}m} = 3 \times 10^6 s^{-1} = 3MHz$$
This is indeed in the range of frequencies used for abdominal and obstetrical ultrasound imaging.

b. For a $2.0MHz$ US beam and a feature size of $1.75mm$ (which we'll take as the wavelength), the speed of sound is
$$v_s = f\lambda = 2 \times 10^6 s^{-1} \times 1.75 \times 10^{-3}m = 3500\frac{m}{s}$$
This, by the way is the speed of sound in bone.

P4.3

a. Since the velocity of sound in the body is roughly constant, the time it takes to send the US pulse into the body and for it to reflect off a structure can be translated into a distance. For a $20ms$ pulse traveling with a velocity $v_s = 1540\frac{m}{s}$ the distance traveled by the US pulse is
$$v_s = \frac{distance}{time} = \frac{2d}{t} \rightarrow d = \frac{v_s t}{2} = \frac{1540\frac{m}{s} \times 20 \times 10^{-3}s}{2} = 15.4m$$

b. If the interfaces are $\Delta D = 5cm$ apart, we can estimate how much time would elapse between the echoes from the front and back surfaces. We first need to note that the extra distance traveled by the echo from the second interface (i.e., the one farthest from the transducer) will be $10cm$ since the transmitted pulse must travel to the interface ($5cm$) and travel an extra $5cm$ on its return trip. We can then compute the extra time for the second echo pulse to reach the transducer from:

$$v_s = \frac{distance}{time} = \frac{2\Delta D}{t} \rightarrow t = \frac{2\Delta D}{v_s} = \frac{2 \times 5 \times 10^{-2} m}{1540 \frac{m}{s}} = 6.5 \times 10^{-5} s$$

4P.5

a. To determine how long it takes an US scanner to make a complete scan, we need to make some rough estimates of the size of the abdomen. Taking some measurements we have, as a rough estimate, the depth of the abdomen $20cm$ front-to-back, and a width side-to-side of $30cm$. We assume that the scanner is linear and that it is oriented to take one line of data along one ultrasound beam at a time (into and out of the body from front-to-back) and then we'll fan out the beam across the width of the body (side-to-side). The transit time for ultrasound pulses can be computed, assuming we orient the transducer, so they travel along the $20cm$ direction:
$$v_s = \frac{distance}{time} = \frac{2d}{t} \rightarrow t_{scan} = \frac{2d}{v_s} = \frac{2 \times 0.2m}{1540 \frac{m}{s}} = 2.6 \times 10^{-4} s = 0.26ms$$
b. If we assume that the beam width has a value of $2mm$, a typical value for abdominal ultrasound scans as given in the text, then the number of scans is
$n = \frac{0.3m}{2 \times 10^{-3} m} = 150.$
The total time for a scan is the number of scans times the time per scan.
$t = n \times t_{scan} = 150 \times 0.26ms = 39ms$
c. Body motions as breathing, the beating of the heart, and motions associated with digestion could be easily resolved with this fast of a scan time.

P4.7

a. We can quickly rank the echo intensities by noting that the greatest echo intensities come from the greatest mismatch in acoustic impedance. Thus, echoes from fat-cartilage > muscle-blood > muscle-fat. The order in which each tissue is encountered does not affect the echo intensity. We want to plot the echo intensity, i_{echo} (as a percentage of the original pulse) vs. time (seconds). We need to compute the reflected intensity
$$\frac{I_{echo}}{I_{indident}} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

For interfaces after the first, we must keep track of the transmitted intensity, $I_{trans} = I_{incident} - I_{echo}$ since this represents the total intensity for the next interface. To compute the echo-return times, we use
$t = \frac{2d}{v_s}$ where the distances are from the transduce on the abdominal to each interface.

From the figure in the text, I need to determine some approximate distances to the structures. I made a model of the "me" and the figure in the text below. I'm assuming that I'm $25cm$ from front-to-back and that my abdominal wall is $1cm$ thick. From the abdominal wall to the aorta ($3cm$ thick) the distance is roughly $10cm$. From the aorta to my vertebrae in my spine ($3cm$ thick) I'll

assume the distance is about $8cm$. Adding up all these distances we get $25cm$. In the following table, the interfaces encountered are listed starting from the left side of the figure a moving in the direction of the US pulse into the body. Each value of echo intensity and transmitted intensity considers the amount by which the incident beam was diminished by previous interfaces.

Interface	Z_1 $(\times 10^6)Rayl$	Z_2 $(\times 10^6)Rayl$	I_{refl}	I_{trans}	I_{echo}	$t = \frac{2d}{v_s}$ (s)
Muscle-fat	1.30	1.65	0.0146	0.9854	0.0146	1.30×10^{-6}
Fat-aorta muscle left side	1.30	1.65	0.0146	0.9854 $\times$ 0.9854 $=$ 0.97101	0.0146 $\times$ 0.9854 $=$ 0.01439	1.43×10^{-4}
Aorta muscle left side-blood	1.65	1.66	9.13×10^{-6}	0.9711 $\times$ 0.999991 $=$ 0.9710	0.97101 $\times$ 9.13 $\times 10^{-6}$ $= 8.887 \times 10^{-6}$	1.49×10^{-4}
Blood-aorta muscle right side	1.66	1.65	9.13×10^{-6}	0.9710 $\times$ 0.999991 $=$ 0.97099	0.9710 $\times$ 9.13 $\times 10^{-6}$ $= 8.865 \times 10^{-6}$	1.75×10^{-4}
Aorta muscle right side-fat	1.65	1.30	0.0146	0.97099 $\times$ 0.9854 $=$ 0.9568	0.97099 $\times$ 0.0146 $=$ 0.01418	1.82×10^{-4}
Fat-cartilage	1.30	2.20	0.0661	0.9339 $\times$ 0.9568 $=$ 0.8936	0.0661 $\times$ 0.9568 $=$ 0.06320	2.86×10^{-4}

Using the diagram below, the times to each structure are calculated.

$$t_{muscle-fat\ through\ abdominal\ wall} = \frac{2(0.01m)}{1540\frac{m}{s}} = 1.30 \times 10^{-6}s$$

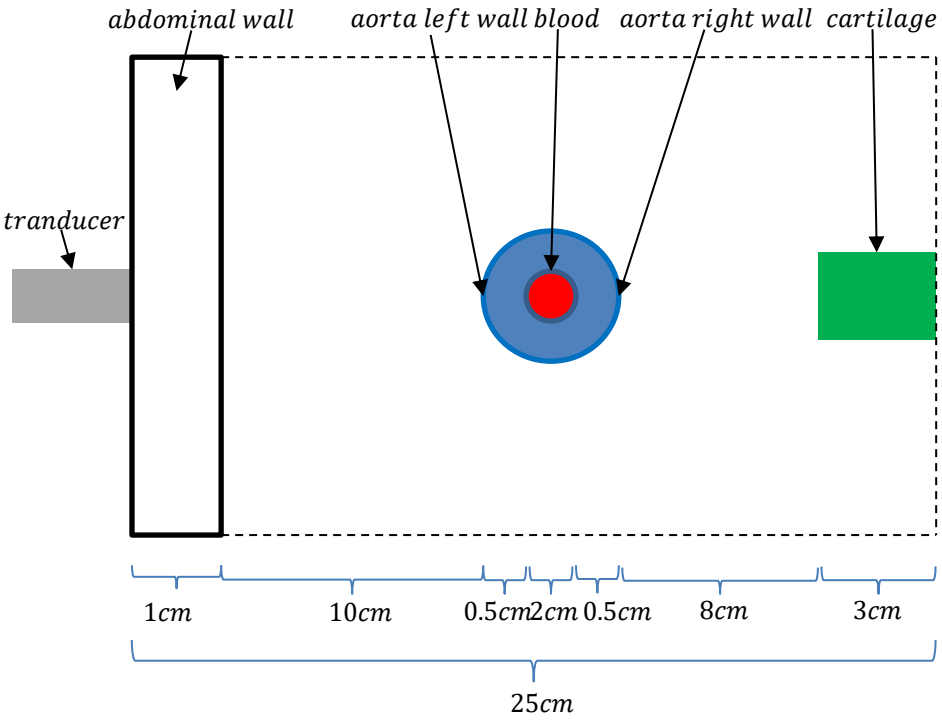
$$t_{fat-aorta\ wall\ left} = \frac{2(0.11m)}{1540\frac{m}{s}} = 1.43 \times 10^{-4}s$$

$$t_{aorta\ wall\ left-blood} = \frac{2(0.115m)}{1540\frac{m}{s}} = 1.49 \times 10^{-4}s$$

$$t_{blood-aorta\ wall\ right} = \frac{2(0.135m)}{1540\frac{m}{s}} = 1.75 \times 10^{-4}s$$

$$t_{aorta\ wall\ right-fat} = \frac{2(0.14m)}{1540\frac{m}{s}} = 1.82 \times 10^{-4}s$$

$$t_{fat-cartilage} = \frac{2(0.22m)}{1540\frac{m}{s}} = 2.86 \times 10^{-4}s$$



b. The intensity versus time graph – not drawn to scale.