Physics in Modern Medicine **Winter 2023**

Homework #4

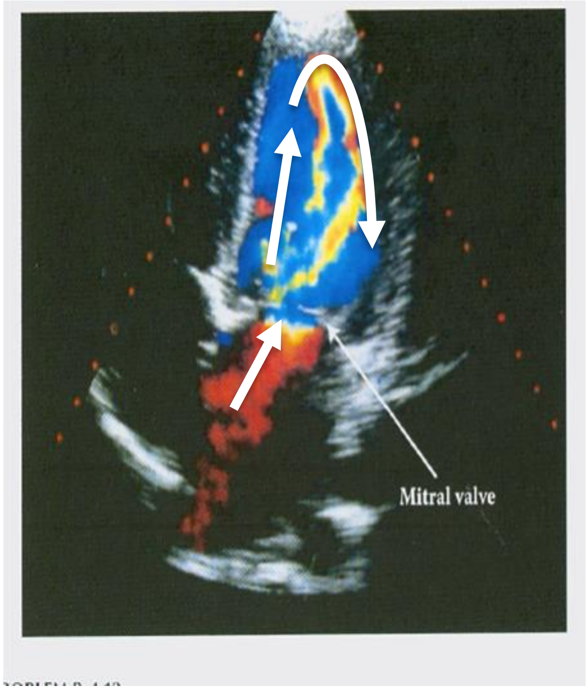
Chapter 4 – Seeing with Sound

Questions

Q4.6 No it does not mean there is no blood flow in the region. The color mappings are indications of changes in frequency of the US pulse and the changes in frequency are angle dependent. If the US transducer is perpendicular to the blood flow, then not frequency shift is detected and this would look like there is no flow, when in fact there could be.

Problems

P4.12 The picture below shows the directions and magnitudes of blood flow on several points on the color image using arrows whose lengths are related to the blood-flow speed. The fan-shaped sector scan indicates that the transducer is located as indicated at the top of the image, leading to the conclusion that the beam takes the radiating paths shown. This means blood flow is measured only along this radial direction, and the flow perpendicular to the beams is not determined. A narrowing (**stenosis**) of the mitral valve as well as the blood rubbing against the ventricular wall causes turbulence in the blood, which is shown as a yellow/green region near the valve's opening.



P4.14 For this problem, the operating frequency is $f_0 = 3.5MHz$, and the smallest frequency shift measurable is $\Delta f = 0.1kHz$. From equation 4.20, the flow speed is $v = \frac{v_s \Delta f}{2 f_0 \cos\theta} = \frac{1540\frac{m}{s} \times 0.1 \times 10^3 Hz}{2 \times 3.5 \times 10^6 Hz \cos 0} = 0.022\frac{m}{s} = 2.2\frac{cm}{s}$ where we have chosen $\cos\theta = \cos 0 = 1$ at its maximum parallel to the flow. From this, flow velocities as slow as a few centimeters per second could be detected.

Chapter 5 – X-ray Vision

Questions

Q5.1 There are many ways in which x-rays can interact with matter. The two main ones that we've discussed are the photoelectric absorption and Compton scattering. High Z materials are good for shielding

Q5.3 Air was used as a contrast medium compared to brain matter due to the attenuation coefficients of air and say fatty material. Air attenuates very little while the brain attenuates more. Using air in the veins/arteries in the brain will highlight the regions of maximum/minimum attenuation.

Q5.7 One measurement would be insufficient to measure bone density since all the absorption coefficients would be unable to be determined. Using DEXA we take measurements that both involve and do not involve bone. This way we can compensate for the soft tissue.

Problems

P5.1

a. *What is the operating voltage of the tube? What is the kVp?* These two quantities are based on the same quantity--the kVp is determined by the tube's operating voltage, because it represents the maximum amount of energy an electron has after crossing the voltage difference between the tube's cathode and anode. From the plot, we see that the maximum energy of the x-rays produced is approximately $87keV$. This is also the maximum energy of the electrons in the tube, hence the kVp and operating voltages are $87kV$.

b. *What is the anode material?* We calculate the atomic number from the energy formula. From this plot, it's hard to tell the exact energy of the primary transition, but the energy is approximately $60keV$, which corresponds to $Z{\sim}78$ which is platinum.

$$\Delta E = 60000eV = -13.6eV(Z-1)^2\left(\frac{1}{n_{upper}^2} - \frac{1}{n_{lower}^2}\right)$$

$60000eV = -13.6eV(Z-1)^2\left(\frac{1}{2^2} - \frac{1}{1^2}\right) \rightarrow Z = 78$ which could be platinum.

c. *Explain which features of the curve correspond to bremsstrahlung and which to characteristic x-rays.* The broad, continuous spectrum of emitted x-rays correspond to bremsstrahlung--this process is not selective in the energies of x-rays produced; the sharp emission peaks correspond to characteristic x-ray emission--these peak locations are sensitively determined by the exact atomic composition of the anode material.

d. *How would the curve be changed qualitatively if the operating voltage of the tube were halved?* In that case, the kVp would move to half of its present value. *If the operating current were doubled?* The number of x-rays produced would double als

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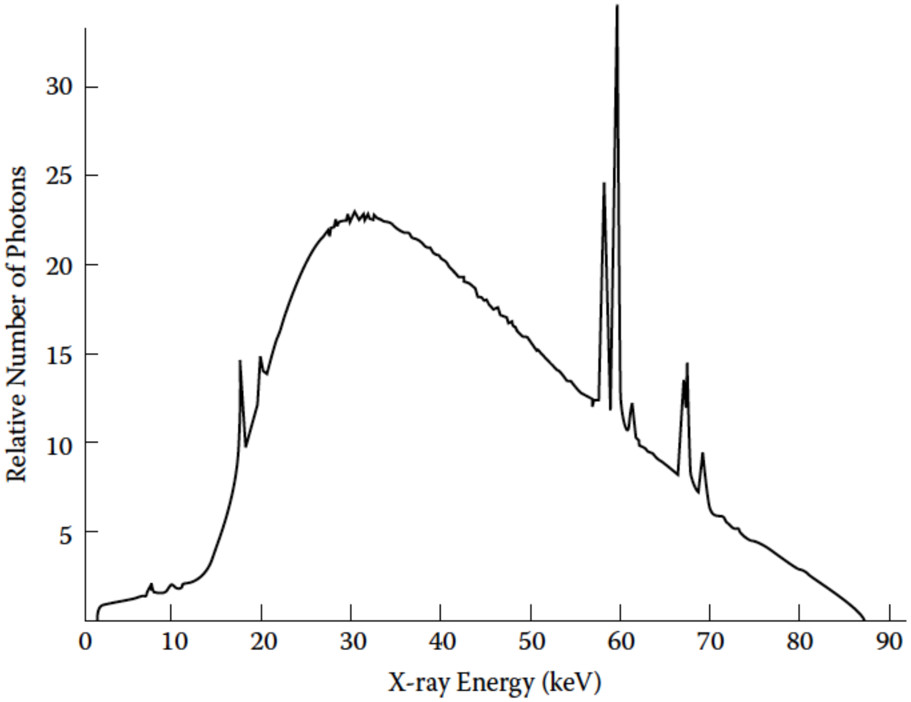


P5.5

a. The mass absorption coefficient and density of lead for $140keV$ x-rays are $\mu_m = 2\frac{cm^2}{g}$ and $\rho_{Pb} = 11.3\frac{g}{cm^3}$. The attenuation coefficient for $140keV$ x-rays from is $\mu = \mu_m \rho_{Pb} = 2\frac{cm^2}{g} \times 11.3\frac{g}{cm^3} = 22.6cm^{-1}$. X-ray attenuation follows an exponential decay, and for a distance of $x = 0.5mm = 0.05cm$ we have $I = I_0 e^{-\mu x} = I_0 e^{-22.6cm^{-1}\times 0.05cm} = 0.32I_0$ or about 32% of the x-rays are transmitted through the apron.

b. To reduce the transmitted intensity of $8keV$ x-rays (these are probably copper) to 1% of its original value we use the exponential decay for attenuation of x-rays with $\mu_m = 232\frac{cm^2}{g}$. We have $I = 0.01I_0 = I_0 e^{-\mu x} = I_0 e^{-\left(232\frac{cm^2}{g}\times 11.3\frac{g}{cm^3}\right)x} \rightarrow x = 1.8 \times 10^{-3}m = 1.8mm$. This is a very, very thin piece of lead indeed. Here we have that shielding low energy x-rays seems relatively easy. It requires only a very thin lead foil. The apron from part a would give excellent shielding of these x-rays. To shield higher energy x-rays, those with energies corresponding to the high end of those used in diagnostic imaging, would requires much thicker lead shielding. A lead apron with the thickness given in part a would be adequate to shield lower energy x-rays commonly used in imaging, for example, but a thicker layer of lead would be necessary to shield higher energy x-rays.

P5.6

a. For $20keV$ and a $1cm$ thick piece of rib bone, ($\mu_{bone} = 4.8cm^{-1}$), embedded in $20cm$ of soft tissue ($\mu_{st} = 0.76cm^{-1}$), the transmission is $I = I_0 e^{-(\mu_{st}x_{st} + \mu_{bone}x_{bone})} = I_0 e^{-\left(0.76cm^{-1}\times 20cm + 4.8cm^{-1}\times 1cm\right)} = 2.1 \times 10^{-9} I_0$, or $2.1 \times 10^{-7}\%$. For $60keV$ and a $1cm$ thick piece of rib bone ($\mu_{bone} = 0.55cm^{-1}$) embedded in $20cm$ of soft tissue ($\mu_{st} = 0.2cm^{-1}$) , the transmission is $I = I_0 e^{-(\mu_{st}x_{st} + \mu_{bone}x_{bone})} = I_0 e^{-\left(0.2cm^{-1}\times 20cm + 0.55cm^{-1}\times 1cm\right)} = 0.011I_0$ or 0.11%. Clearly x-rays of higher energy have a much greater transmission. However, this does not mean that we can increase the energy indefinitely.

b. For $20keV$ x-rays through the $4cm$ region of breast tissue, we have $\mu_{breast} = 0.76cm^{-1}$and the percent of x-ryas transmitted is $I = I_0 e^{-\mu x} = I_0 e^{-0.76cm^{-1}\times 4cm} = 0.048I_0$ or 0.48% transmitted. For $60keV$ x-rays through the $4cm$ region of breast tissue, we have $\mu_{breast} = 0.20cm^{-1}$ and the percent of x-ryas transmitted is $I = I_0 e^{-\mu x} = I_0 e^{-0.20cm^{-1}\times 4cm} = 0.45I_0$ or 45% transmitted. For x-rays of higher energy transmission of those x-rays through breast tissue to develop the image receptor increases. However, raising the energy degrades the contrast.

P5.8

a. For the case of the microcalcification, we have $x = 0.1mm = 0.01cm$ and for $20keV$ x-rays, $\mu_{breast} = 0.5cm^{-1}$ and $\mu_{bone} = 4.8cm^{-1}$. The contrast is $C = 1 - e^{-(\mu_{bone}-\mu_{breast})x_{bone}} = 1 - e^{-(4.8cm^{-1}-0.5cm^{-1})\times 0.01cm} = 0.042$, or 4.2%.
b. For the case of the microcalcification, we have $x = 0.1mm = 0.01cm$ and for $60keV$ x-rays, $\mu_{breast} = 0.17$ and $\mu_{bone} = 0.55cm^{-1}$. The contrast is $C = 1 - e^{-(\mu_{bone}-\mu_{breast})x_{bone}} = 1 - e^{-(0.55cm^{-1}-0.17cm^{-1})\times 0.01cm} = 0.0038$, or 0.38% where I've chosen the greatest difference in attenuation coefficients to give the best possible contrast. The x-ray contrast can be positive or negative. The positive sign in both cases means that the microcalcification is less transmitting/more absorbing than fat, so that the transmission through fat alone is greater than the transmission through fat plus the microcalcification. For $20keV$ x-rays, a much greater difference in transmission occurs for the two tissues than is the case for $60keV$ x-rays. In fact, only an image made with $20keV$ x-rays would be able to distinguish the microcalcification given an x-ray film/phosphor combination only sensitive to contrasts greater than about 2%.
c. For the case of the lump, we have $x = 0.1cm$ and for $20keV$ x-rays, $\mu_{breast} = 0.5cm^{-1}$ and $\mu_{lump} = 0.76cm^{-1}$. The contrast is $C = 1 - e^{-(\mu_{lump}-\mu_{breast})x_{lump}} = 1 - e^{-(0.76cm^{-1}-0.5cm^{-1})\times 0.1cm} = 0.026$, or 2.6%.

d. For the case of the lump, we have $x = 0.1cm$ and for $60keV$ x-rays, $\mu_{breast} = 0.17$ and $\mu_{lump} = 0.20cm^{-1}$. The contrast is $C = 1 - e^{-(\mu_{lump}-\mu_{breast})x_{lump}} = 1 - e^{-(0.20cm^{-1}-0.17cm^{-1})\times 0.1cm} = 0.003$, or 0.3%

The contrast is again significantly higher for $20keV$ case compared to the $60keV$ case, and here, only the $20keV$ case would be detectable on the film/phosphor combination. Thus, we see that for both cases, only the $20keV$ case would correspond to a detectable image, even neglecting the effects of scattering, noise, etc. Although the $60keV$ x-rays provide a higher x-ray dose than $20keV$ x-rays would, they are essential for imaging the possible signs of a tumor. In both cases, we see that these numbers come out close to the limits of detectability anyway, showing that microcalcifications smaller than $0.1mm$ are considered to be undetectable with most current mammography setups, and that small solid tumors are also difficult to distinguish. Earlier mammography systems used in the early 1970's was unable to perform at this level, and consequently did not provide adequate mammograms for detecting early breast cancer. It is hoped that ongoing improvements in x-ray imaging will lead to even better detection rates, *improving* the rates of breast cancer detection and cures relative to those observed in the population studies to date. (In fact, these studies could not really assess improvements available since the mid-1980's, since not enough time has elapsed since then to evaluate their effectiveness.)

X-Ray Problem Set

5.1 X-ray generation

Consider the x-ray intensity (number of photons) versus energy spectrum shown below.

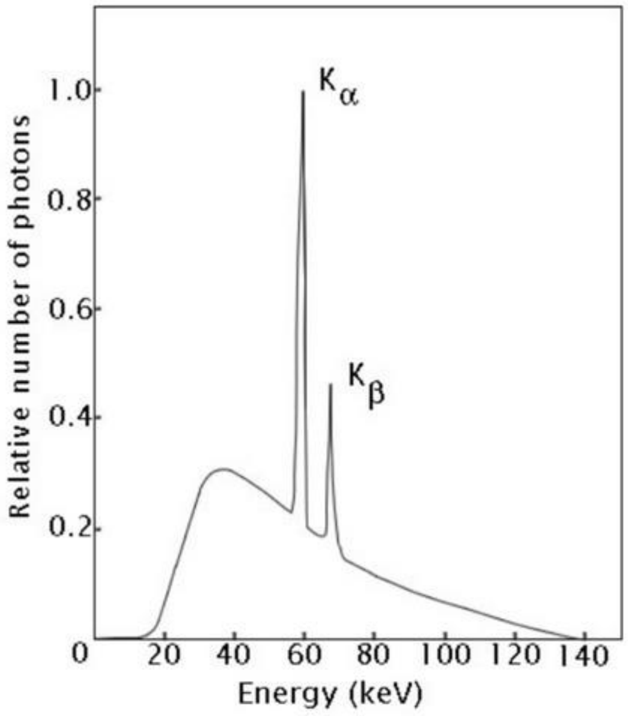
a. What is the operating voltage of the tube?
The operating tube potential is approximately $140kV$.

b. What is the anode material?
To determine the anode material, we use the K_α peak energy, which is approximately $60keV$. The identity of the material is determined from
$\Delta E = 60keV = -13.6eV(Z-1)^2\left(\frac{1}{2^2}-\frac{1}{1^2}\right)$, where the upper state the electron transitions from is $n_{upper} = 2$ to the lower state $n_{lower} = 1$. Therefore $Z = 78$ and looking this up in a periodic table, we have the anode made of Platinum.

c. Explain the features of the curve. What are the parts of the curve generated by?
The background is bremsstrahlung radiation from the electrons decelerating in the anode material. The electrons, as they decelerate, produce a continuous distribution of x-ray energies. The larger peaks on top of the background are x-rays characteristic of the anode material.

d. Qualitatively, what would happen to the spectrum produced if the operating voltage of the tube say were halved? What about if the tube current were doubled?

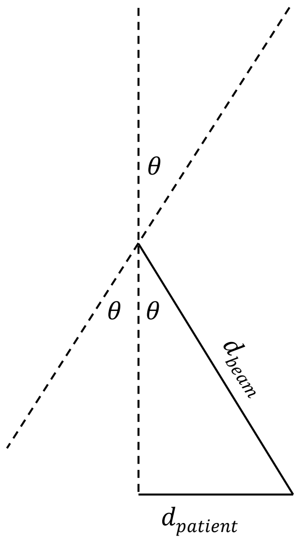
If the operating voltage of the tube were halved, there would be less energy for the electrons and when they decelerate in the anode material, they would still produce the bremsstrahlung background, but it would be smaller since they have less energy. Halving the tube voltage, you'd still be above the energy needed for characteristic x-ray production, so you would produce both the K_α and K_β (most likely – it's hard to tell from the graph) x-rays from platinum. If you doubled the tube current, then there would be more electrons incident on the anode and more electrons means more x-rays would be produced.

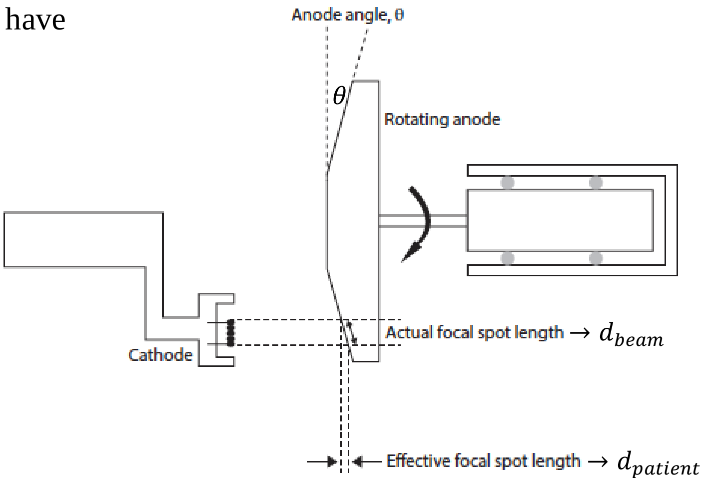


5.2 X-ray beam spot size

a. Consider the diagram below of a rotating anode x-ray generating system. The beam is incident on the anode and makes a spot of size d_{beam}. The beam that is directed to the patient has some effective size $d_{patient}$ and can vary from as small as $0.1mm$ (mammography) to $1.5mm$ (radiography/CT). Using the diagram below, derive an expression relating d_{beam} to $d_{patient}$ and the anode angle θ.

From the geometry in the system, we have





Thus, $\sin\theta = \frac{d_{patient}}{d_{beam}} \rightarrow d_{patient} = d_{beam}\sin\theta$

b. For an anode angle of $\theta = 12^0$, what beam size d_{beam} would be needed so that the effective focal spot size is $d_{patient} = 1.2mm$. Note: you can vary d_{beam} by collimating the beam of electrons from the cathode heading to the anode and by controlling d_{beam}, you can control $d_{patient}$.

$$d_{patient} = d_{beam} \sin\theta \rightarrow d_{beam} = \frac{d_{patient}}{\sin\theta} = \frac{1.2mm}{\sin 12} = 5.8mm$$