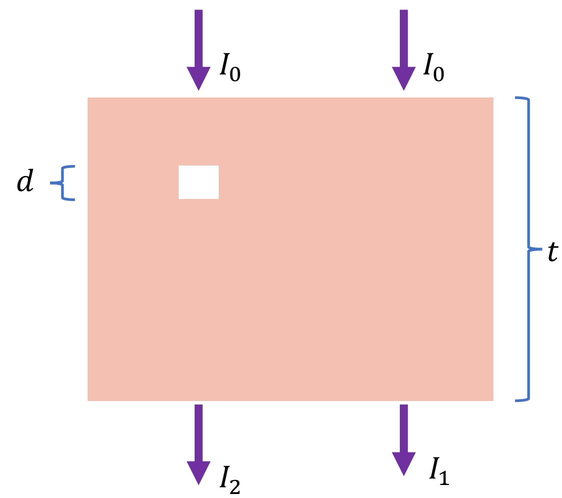
Chapter 5 X-rays and CT

1 Breast micro-calcification and contrast

a. Suppose that a beam of x-rays was incident on a piece of material (tissue) of thickness $t = 1mm$. Imbedded in the tissue is a spherical bead of calcium of diameter (thickness) $d = 200\mu m$ as shown below. This scenario could represent breast tissue in which there is a micro-calcification, and this could be indicative of breast cancer. If the x-ray beam does not scatter, what is the contrast C between the calcium bead and the tissue for x-ray energies $20keV$, $50keV$, and $100keV$? Use the table below for the attenuation coefficients.

Energy (keV)	$\mu_{Tissue}\ (cm^{-1})$	$\mu_{calcium}\ (cm^{-1})$
20	0.793	20.150
50	0.227	1.547
100	0.170	0.397



For $20keV$:

$$C = \left(1 - e^{-(\mu_c - \mu_t)x_c}\right) \times 100\% = \left(1 - e^{-(20.15cm^{-1} - 0.973cm^{-1})0.02cm}\right) \times 10\% = 32\%$$

For $50keV$:

$$C = \left(1 - e^{-(\mu_c - \mu_t)x_c}\right) \times 100\% = \left(1 - e^{-(1.547cm^{-1} - 0.227cm^{-1})0.02cm}\right) \times 10\% = 3\%$$

For $100keV$:

$$C = \left(1 - e^{-(\mu_c - \mu_t)x_c}\right) \times 100\% = \left(1 - e^{-(0.397cm^{-1} - 0.170cm^{-1})0.02cm}\right) \times 10\% = 0.5\%$$

b. What conclusion can you draw about the contrast and the photon energy? Which energy range gives the highest contrast? Which energy ranges gives the lowest? Which energy would you use to visualize something as small as this micro-calcification?

From the results of the previous part, as the photon energy increase the contrast between the structures decrease. This is called beam hardening. The highest contrast is with the lowest photon energy and the lowest contrast is with the highest energy. To visualize something this small, I'd look for something with the greatest contrast and use the lowest energy photons possible.

2. X-ray imaging of small objects

Accidental ingestion of foreign bodies is a common problem in children. One such recent hazard is the ingestion of small, rare-earth magnets from toys. When ingested, the magnets can have potentially lethal consequences if not immediately treated. The magnets may lodge in the esophagus, the stomach, or in any segment of the bowel. In the stomach or bowel, the magnets can bind together to form obstructions, which can lead to severe complications if left untreated. According to the American Academy of Pediatrics 100's of cases of magnet ingestion are reported annually by emergency rooms across the US. These toys sold could contain hundreds of small magnets and it's hard to tell if a few have gone missing by say a parent. An image of such a toy is shown below in Figure 1.

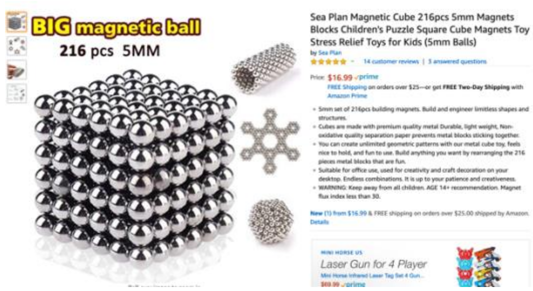


Figure 1: An ad from Amazon.com showing a set of 216 5-mm spherical magnets that you can by as a toy for

Consider the film x-ray image shown below (Figure 2) taken of a 3-year-old boy in the ER showing seven magnets lodged in his lower esophagus and upper stomach. The upper two magnets are in the esophagus while the remaining lower five are in the upper stomach.

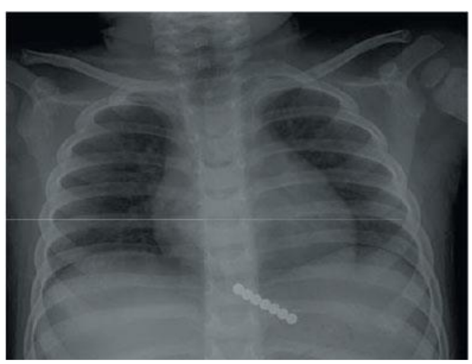


Figure 2: X-ray image of a 3-year-old child with a set of magnets lodged in his gastro-intestinal tract.
http://www.scielo.org.za/scielo.php?script=sci_arttext&pid=S0256-95742014000400017#f1

a. To form the x-ray image, suppose that the beam of x-rays was directed through the child from front-to-back as in the image shown in Figure 2. Let the cartoon diagram, shown in Figure 3 represent the structures in the body that the x-ray beam passed through. Further, let the intensity of the emerging x-ray beam on the detector be 0.04% of the incident beam intensity, or $I_{detector} = 0.0004I_0$. From the information in Table 1 and using Figure 3, how thick was the magnet the child ingested? The magnet is colored blue in Figure 3. Assume that the x-ray beam goes through $0.5cm$ of fat on the belly, $9cm$ of liver tissue, $0.4cm$ of stomach wall muscle, $5.2cm$ of air in the stomach, another $0.4cm$ of stomach wall muscle, $4cm$ of the vertabra of the spine, $0.5cm$ of fat on the back, and of course the magnet in the stomach. Absorption coefficients and densities of the various structures in the body are given in Table 1.

Structure	$\mu_m\left(\frac{cm^2}{g}\right)$	$\rho\left(\frac{g}{cm^3}\right)$
Magnet	10.3	6.9
Fat/Liver	0.1974	0.95
Stomach/Muscle	0.2048	1.05
Bone	0.3148	1.92
Air	0.1875	0.0012

Table 1: X-ray mass attenuation coefficients and densities of various materials.

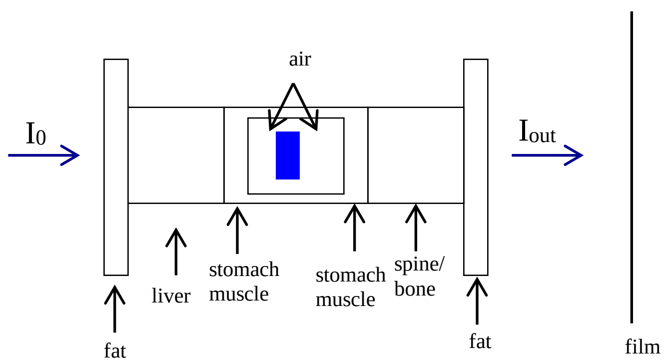


Figure 3: Cartoon version of the path that the x-ray beam takes through the child.

The attenuation coefficients used in this problem are calculated using $\mu = \mu_m \rho$ and are shown under Table 2.

$$I = I_0 e^{-\mu_{effective} x_{effective}} \rightarrow 0.0004 I_o = I_0 e^{-\mu_{effective} x_{effective}} \rightarrow \mu_{effective} x_{effective} = -\ln\left(\frac{0.0004 I_o}{I_0}\right) = 7.82$$

$$\mu_{effective} x_{effective} = 2\mu_{fat} x_{fat} + \mu_{liver} x_{liver} + 2\mu_{stomach} x_{stomach} + \mu_{air}\left(x_{air} - x_{magnet}\right) + \mu_{bone} x_{bone} + \mu_{magnet} x_{magnet}$$

$$7.82 = \left(2 \times 0.5 \times 0.1875\right) + \left(9cm \times 0.1875\right) + \left(2 \times 0.4 \times 0.2150\right) + 0.00025\left(5.2 - x_{magnet}\right) + \left(4 \times 0.6044\right) + \left(x_{magnet} \times 71.1\right)$$

$$7.82 = 0.1875 + 1.6875 + 0.172 + 0.0012 - 0.00023x + 2.4176 + 71.1x$$

$$\therefore \; x = 0.047cm \sim 0.5mm$$

b. Instead of magnet, suppose that a patient presents in the ER complaining of chronic (happening for a long time) stomach pain. It is believed by the ER physician that the patient may be suffering from a stomach ulcer. Stomach ulcers are painful sores that develop in the stomach lining. The attending ER physician and a new resident physician have different ideas on how to best see the ulcer. The resident physician would like to image the ulcer using an x-ray scan while the ER physician would like to try something else. Suppose that the ulcer can be modeled by blood $\left(\mu_{m,blood} = 0.2057\frac{cm^2}{g};\ \rho_{blood} = 1.06\frac{g}{cm^3}\right)$ and the resident physician wanted to see a $0.25cm$ thick ulcer in the lining of the stomach wall, what is the contrast between the ulcer and the stomach wall? Assume that the lining of the stomach can be modeled by muscle and use Table 1.

$$C = \left(1 - e^{-(\mu_m - \mu_{blood})x}\right) \times 100\% = \left(1 - e^{-\left(\left[0.2057 \times 1.06\right]cm^{-1} - 0.2150\,cm^{-1}\right) \times 0.25cm}\right) \times 100\%$$

$$C = 0.08\%$$

c. Comment on the result that you get using x-rays to image an ulcer. Do you think the resident physician is right? Can you image the ulcer on an x-ray scan? Explain why or why not.

Since the contrast between the ulcer in the stomach lining and the stomach lining is 0.08%, these would be almost impossible to distinguish on an x-ray. Thus, the ER physician wins this one. There are other methods that can be used to image the ulcer.

d. Assuming the resident physician is incorrect and that you cannot image the ulcer on an x-ray image, suggest at least one way the attending ER physician would probably use to see the ulcer.

One way to see the ulcer in the stomach lining would be through an endoscopic procedure where the physician will put an endoscope down the esophagus of the person and get it into the stomach. Here, the physician will be able to see if there is an ulcer or not in the lining of the stomach.