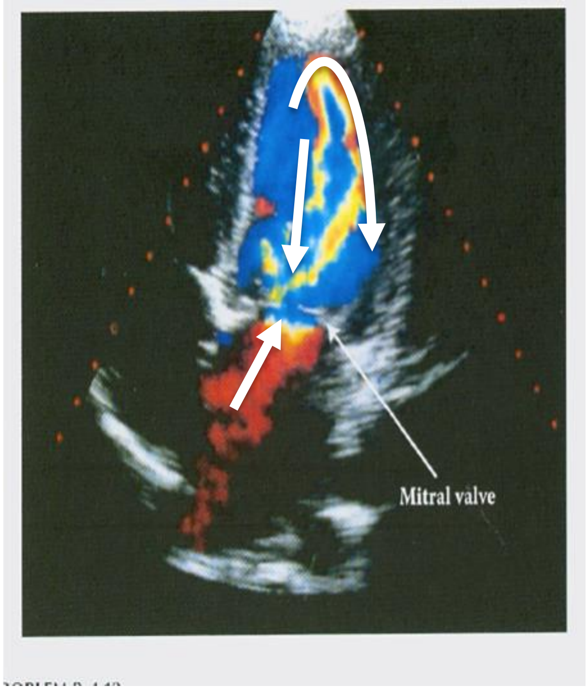
Chapter 4 – Seeing with Sound

Questions

Q4.6 No it does not mean there is no blood flow in the region. The color mappings are indications of changes in frequency of the US pulse and the changes in frequency are angle dependent. If the US transducer is perpendicular to the blood flow, then not frequency shift is detected and this would look like there is no flow, when in fact there could be.

Problems



P4.12 The picture below shows the directions and magnitudes of blood flow on several points on the color image using arrows whose lengths are related to the blood-flow speed. The fan-shaped sector scan indicates that the transducer is located as indicated at the top of the image, leading to the conclusion that the beam takes the radiating paths shown. By BART, the blue flow is away from the transducer and the red flow is toward the transducer. The blood is flowing from the left atria (red flow) towards the left ventricle through the mitral valve. The blood then rolls off the left ventricle wall and flows out to the aorta (blue flow). This means blood flow is measured only along this radial direction, and the flow perpendicular to the beams is not determined. A narrowing (**stenosis**) of the mitral valve as well as the blood rubbing against the ventricular wall causes turbulence in the blood, which is shown as a yellow/green region near the valve's opening.

P4.14 For this problem, the operating frequency is $f_0 = 3.5MHz$, and the smallest frequency shift measurable is $\Delta f = 0.1kHz$. From equation 4.20, the flow speed is $v = \frac{v_S \Delta f}{2 f_0 \cos\theta} = \frac{1540\frac{m}{s}\times 0.1\times 10^3 Hz}{2\times 3.5\times 10^6 Hz \cos 0} = 0.022\frac{m}{s} = 2.2\frac{cm}{s}$where we have chosen $\cos\theta = \cos 0 = 1$ at its maximum parallel to the flow. From this, flow velocities as slow as a few centimeters per second could be detected.

Chapter 5 X-rays and CT

Questions

Q5.1 There are many ways in which x-rays can interact with matter. The two main ones that we've discussed are the photoelectric absorption and Compton scattering. High Z materials are good for shielding

Q5.3 Air was used as a contrast medium compared to brain matter due to the attenuation coefficients of air and say fatty material. Air attenuates very little while the brain attenuates more. Using air in the veins/arteries in the brain will highlight the regions of maximum/minimum attenuation.

Q5.7 One measurement would be insufficient to measure bone density since all the absorption coefficients would be unable to be determined. Using DEXA we take measurements that both involve and do not involve bone. This way we can compensate for the soft tissue.

Problems

P5.1

a. *What is the operating voltage of the tube? What is the kVp?* These two quantities are based on the same quantity--the kVp is determined by the tube's operating voltage, because it represents the maximum amount of energy an electron has after crossing the voltage difference between the tube's cathode and anode. From the plot, we see that the maximum energy of the x-rays produced is approximately $87keV$. This is also the maximum energy of the electrons in the tube, hence the kVp and operating voltages are $87kV$.

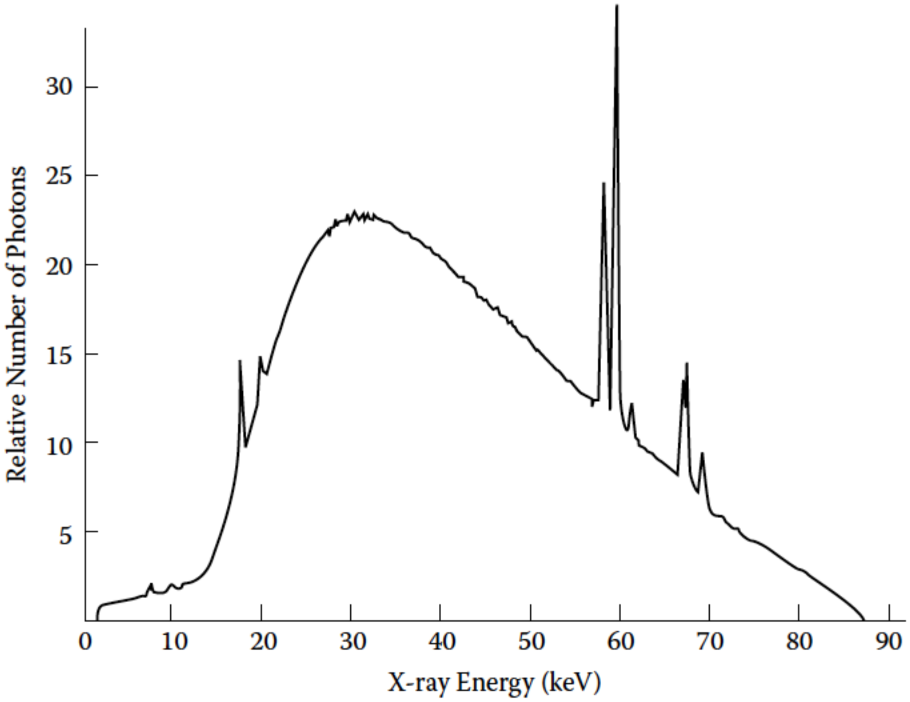
b. *What is the anode material?* We calculate the atomic number from the energy formula. From this plot, it's hard to tell the exact energy of the primary transition, but the energy is approximately $60keV$, which corresponds to $Z{\sim}78$ which is platinum.

$$\Delta E = 60000eV = -13.6eV(Z-1)^2\left(\frac{1}{n_{upper}^2} - \frac{1}{n_{lower}^2}\right)$$

$60000eV = -13.6eV(Z-1)^2\left(\frac{1}{2^2} - \frac{1}{1^2}\right) \rightarrow Z = 78$ which could be platinum.

c. *Explain which features of the curve correspond to bremsstrahlung and which to characteristic x-rays.* The broad, continuous spectrum of emitted x-rays correspond to bremsstrahlung--this process is not selective in the energies of x-rays produced; the sharp emission peaks correspond to characteristic x-ray emission--these peak locations are sensitively determined by the exact atomic composition of the anode material.

d. *How would the curve be changed qualitatively if the operating voltage of the tube were halved?* In that case, the kVp would move to half of its present value. *If the operating current were doubled?* The number of x-rays produced would double also if the number of



P5.5

a. The mass absorption coefficient and density of lead for $140keV$ x-rays are $\mu_m = 2\frac{cm^2}{g}$ and $\rho_{Pb} = 11.3\frac{g}{cm^3}$. The attenuation coefficient for $140keV$ x-rays from is $\mu = \mu_m \rho_{Pb} = 2\frac{cm^2}{g} \times 11.3\frac{g}{cm^3} = 22.6cm^{-1}$. X-ray attenuation follows an exponential decay, and for a distance of $x = 0.5mm = 0.05cm$ we have $I = I_0 e^{-\mu x} = I_0 e^{-22.6cm^{-1} \times 0.05cm} = 0.32I_0$ or about 32% of the x-rays are transmitted through the apron.

b. To reduce the transmitted intensity of $8keV$ x-rays (these are probably copper) to 1% of its original value we use the exponential decay for attenuation of x-rays with $\mu_m = 232\frac{cm^2}{g}$. We have $I = 0.01I_0 = I_0 e^{-\mu x} = I_0 e^{-\left(232\frac{cm^2}{g} \times 11.3\frac{g}{cm^3}\right)x} \rightarrow x = 1.8 \times 10^{-3}m = 1.8mm$. This is a very, very thin piece of lead indeed. Here we have that shielding low energy x-rays seems relatively easy. It requires only a very thin lead foil. The apron from part a would give excellent shielding of these x-rays. To shield higher energy x-rays, those with energies corresponding to the high end of those used in diagnostic imaging, would requires much thicker lead shielding. A lead apron with the thickness given in part a would be adequate to shield lower energy x-rays commonly used in imaging, for example, but a thicker layer of lead would be necessary to shield higher energy x-rays.

P5.6

a. For $20keV$ and a $1cm$ thick piece of rib bone, ($\mu_{bone} = 4.8cm^{-1}$), embedded in $20cm$ of soft tissue ($\mu_{st} = 0.76cm^{-1}$), the transmission is $I = I_0 e^{-(\mu_{st}x_{st}+\mu_{bone}x_{bone})} = I_0 e^{-(0.76cm^{-1}\times 20cm+4.8cm^{-1}\times 1cm)} = 2.1 \times 10^{-9} I_0$, or $2.1 \times 10^{-7}\%$. For $60keV$ and a $1cm$ thick piece of rib bone ($\mu_{bone} = 0.55cm^{-1}$) embedded in $20cm$ of soft tissue ($\mu_{st} = 0.2cm^{-1}$), the transmission is $I = I_0 e^{-(\mu_{st}x_{st}+\mu_{bone}x_{bone})} = I_0 e^{-(0.2cm^{-1}\times 20cm+0.55cm^{-1}\times 1cm)} = 0.011 I_0$ or 0.11%. Clearly x-rays of higher energy have a much greater transmission. However, this does not mean that we can increase the energy indefinitely.

b. For $20keV$ x-rays through the $4cm$ region of breast tissue, we have $\mu_{breast} = 0.76cm^{-1}$and the percent of x-ryas transmitted is $I = I_0 e^{-\mu x} = I_0 e^{-0.76cm^{-1}\times 4cm} = 0.048 I_0$ or 0.48% transmitted. For $60keV$ x-rays through the $4cm$ region of breast tissue, we have $\mu_{breast} = 0.20cm^{-1}$and the percent of x-ryas transmitted is $I = I_0 e^{-\mu x} = I_0 e^{-0.20cm^{-1}\times 4cm} = 0.45 I_0$ or 45% transmitted. For x-rays of higher energy transmission of those x-rays through breast tissue to develop the image receptor increases. However, raising the energy degrades the contrast.

P5.8

a. For the case of the microcalcification, we have $x = 0.1mm = 0.01cm$ and for $20keV$ x-rays, $\mu_{breast} = 0.5cm^{-1}$ and $\mu_{bone} = 4.8cm^{-1}$. The contrast is
$C = 1 - e^{-(\mu_{bone}-\mu_{breast})x_{bone}} = 1 - e^{-(4.8cm^{-1}-0.5cm^{-1})\times 0.01cm} = 0.042$, or 4.2%.

b. For the case of the microcalcification, we have $x = 0.1mm = 0.01cm$ and for $60keV$ x-rays, $\mu_{breast} = 0.17$ and $\mu_{bone} = 0.55cm^{-1}$. The contrast is
$C = 1 - e^{-(\mu_{bone}-\mu_{breast})x_{bone}} = 1 - e^{-(0.55cm^{-1}-0.17cm^{-1})\times 0.01cm} = 0.0038$, or 0.38%
where I've chosen the greatest difference in attenuation coefficients to give the best possible contrast. The x-ray contrast can be positive or negative. The positive sign in both cases means that the microcalcification is less transmitting/more absorbing than fat, so that the transmission through fat alone is greater than the transmission through fat plus the microcalcification. For $20keV$ x-rays, a much greater difference in transmission occurs for the two tissues than is the case for $60keV$ x-rays. In fact, only an image made with $20keV$ x-rays would be able to distinguish the microcalcification given an x-ray film/phosphor combination only sensitive to contrasts greater than about 2%.

c. For the case of the lump, we have $x = 0.1cm$ and for $20keV$ x-rays, $\mu_{breast} = 0.5cm^{-1}$ and $\mu_{lump} = 0.76cm^{-1}$. The contrast is
$C = 1 - e^{-(\mu_{lump}-\mu_{breast})x_{lump}} = 1 - e^{-(0.76cm^{-1}-0.5cm^{-1})\times 0.1cm} = 0.026$, or 2.6%.

d. For the case of the lump, we have $x = 0.1cm$ and for $60keV$ x-rays, $\mu_{breast} = 0.17$ and $\mu_{lump} = 0.20cm^{-1}$. The contrast is
$C = 1 - e^{-(\mu_{lump}-\mu_{breast})x_{lump}} = 1 - e^{-(0.20cm^{-1}-0.17cm^{-1})\times 0.1cm} = 0.003$, or 0.3%

The contrast is again significantly higher for $20keV$ case compared to the $60keV$ case, and here, only the $20keV$ case would be detectable on the film/phosphor combination. Thus, we see that for both cases, only the $20keV$ case would correspond to a detectable image, even neglecting the effects of scattering, noise, etc. Although the $60keV$ x-rays provide a higher x-ray dose than $20keV$ x-rays would, they are essential for imaging the possible signs of a tumor. In both cases, we see that these numbers come out close to the limits of detectability anyway, showing that microcalcifications smaller than $0.1mm$ are undetectable with most current mammography setups, and that small solid tumors are also difficult to distinguish. Earlier mammography systems used in the early 1970's was unable to perform at this level, and consequently did not provide adequate mammograms for detecting early breast cancer. It is hoped that ongoing improvements in x-ray imaging will lead to even better detection rates, *improving* the rates of breast cancer detection and cures relative to those observed in the population studies to date. (In fact, these studies could not really assess improvements available since the mid-1980's, since not enough time has elapsed since then to evaluate their effectiveness.)

X-Ray Problem Set I & II

5.1 X-ray generation

Consider the x-ray intensity (number of photons) versus energy spectrum shown below.

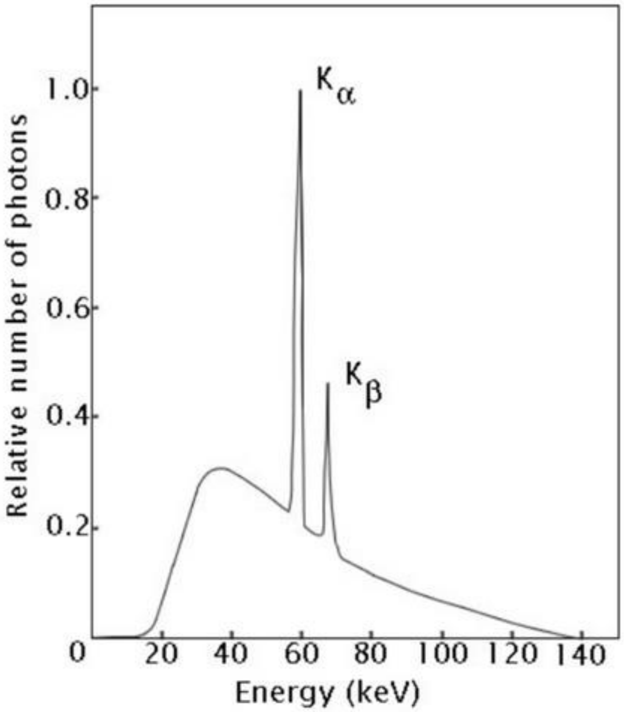
a. What is the operating voltage of the tube?
The operating tube potential is approximately $140kV$.

b. What is the anode material?
To determine the anode material, we use the $K_\alpha$peak energy, which is approximately $60keV$. The identity of the material is determined from $\Delta E = 60keV = -13.6eV(Z-1)^2\left(\frac{1}{2^2}-\frac{1}{1^2}\right)$, where the upper state the electron transitions from is $n_{upper} = 2$ to the lower state $n_{lower} = 1$. Therefore $Z = 78$ and looking this up in a periodic table, we have the anode made of Platinum.

c. Explain the features of the curve. What are the parts of the curve generated by?
The background is bremsstrahlung radiation from the electrons decelerating in the anode material. The electrons, as they decelerate, produce a continuous distribution of x-ray energies. The larger peaks on top of the background are x-rays characteristic of the anode material.

d. Qualitatively, what would happen to the spectrum produced if the operating voltage of the tube say were halved? What about if the tube current were doubled?

If the operating voltage of the tube were halved, there would be less energy for the electrons and when they decelerate in the anode material, they would still produce the bremsstrahlung background, but it would be smaller since they have less energy. Halving the tube voltage, you'd still be above the energy needed for characteristic x-ray production, so you would produce both the K_α and K_β (most likely – it's hard to tell from the graph) x-rays from platinum. If you doubled the tube current, then there would be more electrons incident on the anode and more electrons means more x-rays would be produced.

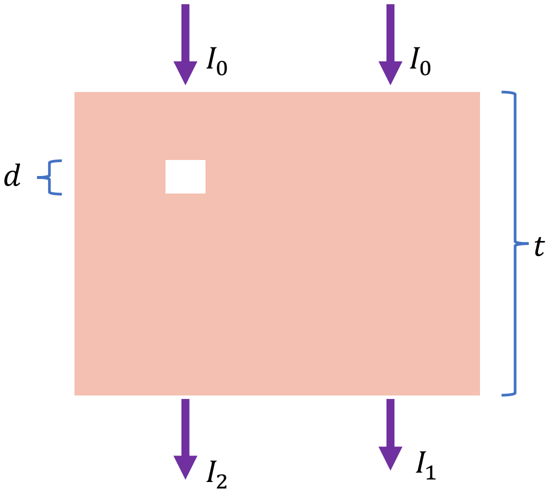


X-Ray Problems II

1 Breast micro-calcification and contrast

a. Suppose that a beam of x-rays was incident on a piece of material (tissue) of thickness $t = 1mm$. Imbedded in the tissue is a spherical bead of calcium of diameter (thickness) $d = 200\mu m$ as shown below. This scenario could represent breast tissue in which there is a micro-calcification, and this could be indicative of breast cancer. If the x-ray beam does not scatter, what is the contrast C between the calcium bead and the tissue for x-ray energies $20keV$, $50keV$, and $100keV$? Use the table below for the attenuation coefficients.

Energy (keV)	$\mu_{Tissue}\ (cm^{-1})$	$\mu_{calcium}\ (cm^{-1})$
20	0.793	20.150
50	0.227	1.547
100	0.170	0.397



For $20keV$:
$$C=\left(1-e^{-(\mu_c-\mu_t)x_c}\right)\times 100\%=\left(1-e^{-\left(20.15cm^{-1}-0.973cm^{-1}\right)0.02cm}\right)\times 10\%=32\%$$

For $50keV$:
$$C=\left(1-e^{-(\mu_c-\mu_t)x_c}\right)\times 100\%=\left(1-e^{-\left(1.547cm^{-1}-0.227cm^{-1}\right)0.02cm}\right)\times 10\%=3\%$$

For $100keV$:
$$C=\left(1-e^{-(\mu_c-\mu_t)x_c}\right)\times 100\%=\left(1-e^{-\left(0.397cm^{-1}-0.170cm^{-1}\right)0.02cm}\right)\times 10\%=0.5\%$$

b. What conclusion can you draw about the contrast and the photon energy? Which energy range gives the highest contrast? Which energy ranges gives the lowest? Which energy would you use to visualize something as small as this micro-calcification?

From the results of the previous part, as the photon energy increase the contrast between the structures decrease. This is called beam hardening. The highest contrast is with the lowest photon energy and the lowest contrast is with the highest energy. To visualize something this small, I'd look for something with the greatest contrast and use the lowest energy photons possible.

2. *X-ray imaging of small objects*

Accidental ingestion of foreign bodies is a common problem in children. One such recent hazard is the ingestion of small, rare-earth magnets from toys. When ingested, the magnets can have potentially lethal consequences if not immediately treated. The magnets may lodge in the esophagus, the stomach, or in any segment of the bowel. In the stomach or bowel, the magnets can bind together to form obstructions, which can lead to severe complications if left untreated. According to the American Academy of Pediatrics 100's of cases of magnet ingestion are reported annually by emergency rooms across the US. These toys sold could contain hundreds of small magnets and it's hard to tell if a few have gone missing by say a parent. An image of such a toy is shown below in Figure 1.

Sea Plan Magnetic Cube 216pcs 5mm Magnets Blocks Children's Puzzle Square Cube Magnets Toy Stress Relief Toys for Kids (5mm Balls)

by Sea Plan

14 customer reviews | 3 answered questions

Price: $16.99 prime

FREE Shipping on orders over $25—or get FREE Two-Day Shipping with Amazon Prime

- 5mm set of 216pcs building magnets. Build and engineer limitless shapes and structures.
- Cubes are made with premium quality metal Durable, light weight, Non-oxidative quality separation paper prevents metal blocks sticking together.
- You can create unlimited geometric patterns with our metal cube toy, feels nice to hold, and fun to use. Build anything you want by rearranging the 216 pieces metal blocks that are fun.
- Suitable for office use, used for creativity and craft decoration on your desktop. Endless combinations. It is up to your patience and creativeness.
- WARNING: Keep away from all children. AGE 14+ recommendation. Magnet flux index less than 30.

New (1) from $16.99 & FREE shipping on orders over $25.00 shipped by Amazon. Details

MINI HORSE US

Laser Gun for 4 Player

Mini Horse Infrared Laser Tag Set 4 Gun...

Figure 1: An ad from Amazon.com showing a set of 216 5-mm spherical magnets that you can by as a toy for

Consider the film x-ray image shown below (Figure 2) taken of a 3-year-old boy in the ER showing seven magnets lodged in his lower esophagus and upper stomach. The upper two magnets are in the esophagus while the remaining lower five are in the upper stomach.

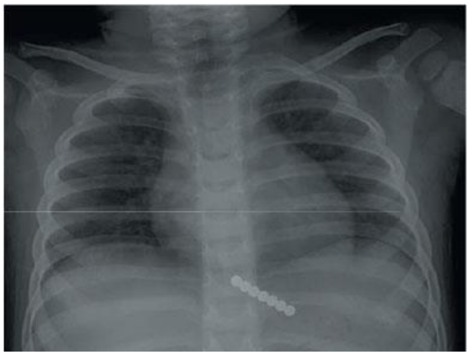


Figure 2: X-ray image of a 3-year-old child with a set of magnets lodged in his gastro-intestinal tract.
http://www.scielo.org.za/scielo.php?script=sci_arttext&pid=S0256-95742014000400017#f1

a. To form the x-ray image, suppose that the beam of x-rays was directed through the child from front-to-back as in the image shown in Figure 2. Let the cartoon diagram, shown in Figure 3 represent the structures in the body that the x-ray beam passed through. Further, let the intensity of the emerging x-ray beam on the detector be 0.04% of the incident beam intensity, or $I_{detector} = 0.0004 I_0$. From the information in Table 1 and using Figure 3, how thick was the magnet the child ingested? The magnet is colored blue in Figure 3. Assume that the x-ray beam goes through $0.5cm$ of fat on the belly, $9cm$ of liver tissue, $0.4cm$ of stomach wall muscle, $5.2cm$ of air in the stomach, another $0.4cm$ of stomach wall muscle, $4cm$ of the vertabra of the spine, $0.5cm$ of fat on the back, and of course the magnet in the stomach. Absorption coefficients and densities of the various structures in the body are given in Table 1.

Structure	$m_m\left(\frac{cm^2}{g}\right)$	$r\left(\frac{g}{cm^3}\right)$
Magnet	10.3	6.9
Fat/Liver	0.1974	0.95
Stomach/Muscle	0.2048	1.05
Bone	0.3148	1.92
Air	0.1875	0.0012

Table 1: X-ray mass attenuation coefficients and densities of various materials.

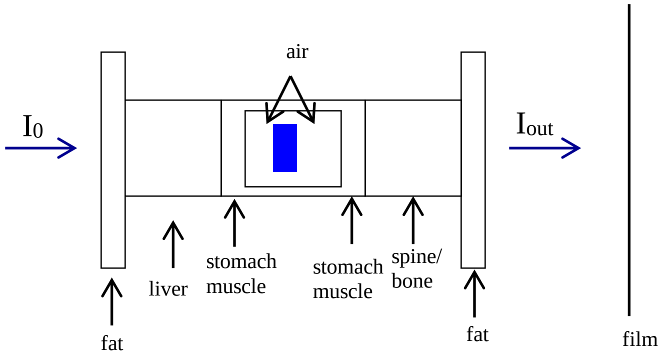


Figure 3: Cartoon version of the path that the x-ray beam takes through the child.

The attenuation coefficients used in this problem are calculated using $\mu = \mu_m \rho$ and are shown under Table 2.

$$I = I_0 e^{-\mu_{effective} x_{effective}} \rightarrow 0.0004 I_o = I_0 e^{-\mu_{effective} x_{effective}} \rightarrow \mu_{effective} x_{effective} = -\ln\left(\frac{0.0004 I_o}{I_0}\right) = 7.82$$

$$\mu_{effective} x_{effective} = 2\mu_{fat} x_{fat} + \mu_{liver} x_{liver} + 2\mu_{stomach} x_{stomach} + \mu_{air}\left(x_{air} - x_{magnet}\right) + \mu_{bone} x_{bone} + \mu_{magnet} x_{magnet}$$

$$7.82 = \left(2 \times 0.5 \times 0.1875\right) + \left(9cm \times 0.1875\right) + \left(2 \times 0.4 \times 0.2150\right) + 0.00025\left(5.2 - x_{magnet}\right) + \left(4 \times 0.6044\right) + \left(x_{magnet} \times 71.1\right)$$

$$7.82 = 0.1875 + 1.6875 + 0.172 + 0.0012 - 0.00023x + 2.4176 + 71.1x$$

$$\therefore \quad x = 0.047cm \sim 0.5mm$$

b. Instead of magnet, suppose that a patient presents in the ER complaining of chronic (happening for a long time) stomach pain. It is believed by the ER physician that the patient may be suffering from a stomach ulcer. Stomach ulcers are painful sores that develop in the stomach lining. The attending ER physician and a new resident physician have different ideas on how to best see the ulcer. The resident physician would like to image the ulcer using an x-ray scan while the ER physician would like to try something else. Suppose that the ulcer can be modeled by blood $\left(\mu_{m,blood} = 0.2057\frac{cm^2}{g};\ \rho_{blood} = 1.06\frac{g}{cm^3}\right)$ and the resident physician wanted to see a $0.25cm$ thick ulcer in the lining of the stomach wall, what is the contrast between the ulcer and the stomach wall? Assume that the lining of the stomach can be modeled by muscle and use Table 1.

$$C = \left(1 - e^{-\left(\mu_m - \mu_{blood}\right)x}\right) \times 100\% = \left(1 - e^{-\left(\left[0.2057 \times 1.06\right]cm^{-1} - 0.2150cm^{-1}\right) \times 0.25cm}\right) \times 100\%$$

$$C = 0.08\%$$

c. Comment on the result that you get using x-rays to image an ulcer. Do you think the resident physician is right? Can you image the ulcer on an x-ray scan? Explain why or why not.

Since the contrast between the ulcer in the stomach lining and the stomach lining is 0.08%, these would be almost impossible to distinguish on an x-ray. Thus, the ER physician wins this one. There are other methods that can be used to image the ulcer.

d. Assuming the resident physician is incorrect and that you cannot image the ulcer on an x-ray image, suggest at least one way the attending ER physician would probably use to see the ulcer.

One way to see the ulcer in the stomach lining would be through an endoscopic procedure where the physician will put an endoscope down the esophagus of the person and get it into the stomach. Here, the physician will be able to see if there is an ulcer or not in the lining of the stomach.