Physics 210
Medical Physics
Midterm Exam
Winter 2023
February 10, ,2023

Name__________________________

Problem 1	/35
Problem 2	/35
Total	/70

For the exam, you may use your in-class notes, any Power Point slides you'd like, your textbook (Kane), and the readings from Wolbarst. You may not use old exams, homework solutions or use worked out solutions to problems.

I affirm that I have carried out my academic endeavors with full academic honesty.

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Signature

1. *Hepatic Cysts*

Hepatic cysts are fluid-filled cavities in and on the liver. There are many types of hepatic cysts with simple being the most common. These are generally not malignant (and thus are not cancerous), contain a clear fluid, and range in size from a few millimeters to tens of centimeters in diameter. These simple cysts are made of a thin outer layer that contains the fluid.

Consider the following US image of a simple hepatic cyst, where a $5MHz$ US beam was incident on the patient normal to the patient's surface. The beam travels through the skin (which is acoustically coupled to the transducer by a gel), through the of the abdominal wall (assumed $2cm$ thick and made of fat) and finally to the cyst on the liver. Some US properties for tissues in the body are given in the Table 1 attached.

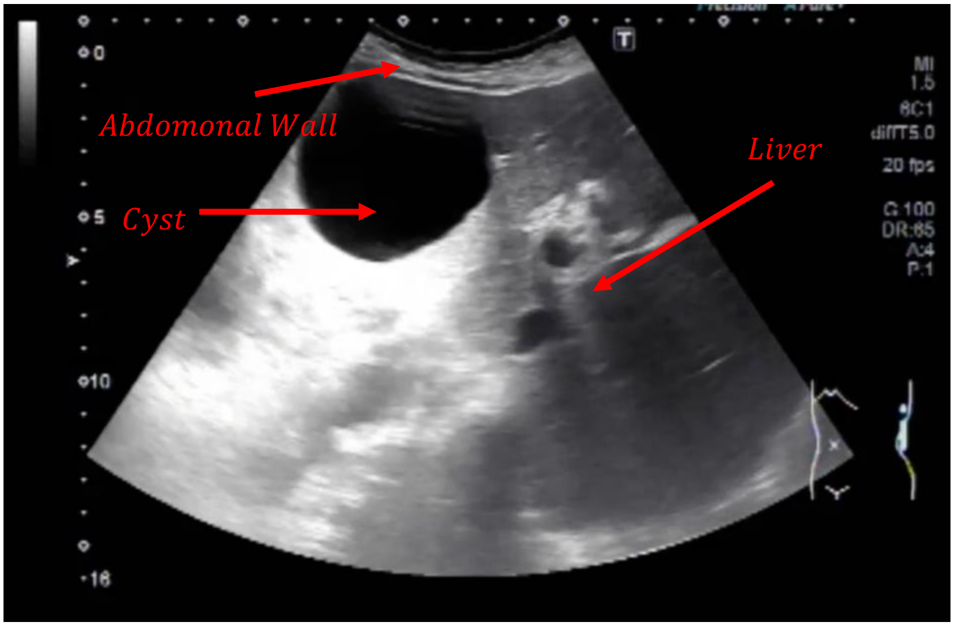


Figure 1: Ultrasound scan of a hepatic cyst.
https://www.youtube.com/watch?v=YNXHPxqkJ1U

a. Suppose that the speed of sound in tissue is $v_{st} = 1540\frac{m}{s}$ and that the cyst is found to be $7cm$ in diameter. If the US sound signal is incident on the patient at a time $t_i = 0s$, at what times will the signals from the front and back surfaces of the cyst reach the transducer?

$$v = \frac{d}{t} \rightarrow t = \frac{d}{v}$$

$$t_{AW} = \frac{d_{AW}}{v_s} = \frac{0.02m}{1540\frac{m}{s}} = 1.3 \times 10^{-5}s \rightarrow t_{trans} = 2t_{AW} = 2.6 \times 10^{-5}s$$

$$t_{cyst,back} = \frac{d_{cyst,back}}{v_s} = \frac{0.09m}{1540\frac{m}{s}} = 5.8 \times 10^{-5}s \rightarrow t_{trans} = 2t_{cyst,back} = 1.16 \times 10^{-4}s$$

b. What are the reflection coefficients for US waves from the abdominal wall/cyst interface and from the cyst/muscle interface? On the axes of the graph below, draw the approximate A-mode scan for the sound waves as they travel through the abdominal wall, the cyst, and reflect off the muscle on the back side of the cyst. The initial US pulse incident I_0 is drawn for scale.

$$R_{AW/Cyst} = \left(\frac{z_{AW} - z_{cyst}}{z_{AW} + z_{cyst}}\right)^2 = \left(\frac{1.4 \times 10^6 rayl - 1.48 \times 10^6 rayl}{1.4 \times 10^6 rayl + 1.48 \times 10^6 rayl}\right)^2 = 0.000772$$

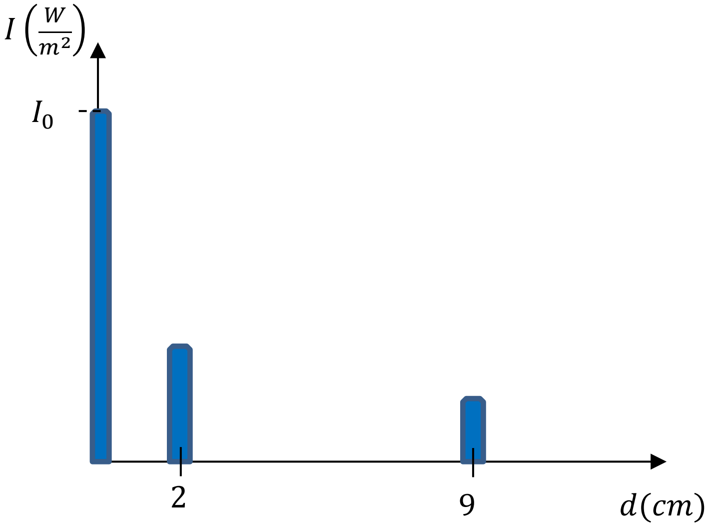
$$R_{AW/cyst} = \frac{I_{reflected\ off\ AW/cyst}}{I_0} \rightarrow I_{reflected\ off\ AW/cyst} = R_{AW/cyst} I_0 = 0.000772 I_0$$

$$T_{into\ cyst} = 1 - R_{\frac{AW}{cyst}} = 0.99923 \rightarrow I_{trans\ into\ cyst} = 0.99923 I_0$$

$$R_{cyst/muscle} = \left(\frac{z_{cyst} - z_{muscle}}{z_{cyst} + z_{muscle}}\right)^2 = \left(\frac{1.48 \times 10^6 rayl - 1.7 \times 10^6 rayl}{1.48 \times 10^6 rayl + 1.7 \times 10^6 rayl}\right)^2 = 0.0049$$

$$R_{cyst/muscle} = \frac{I_{reflected\ off\ AW/cyst}}{0.99923 I_0} \rightarrow I_{reflected\ off\ cyst/mucle} = R_{cyst/muscle}(0.99923 I_0)$$

$$I_{reflected\ off\ cyst/mucle} = 0.0048 I_0$$



c. Suppose that the intensity of the US beam entering the patient were $I_{incident} = 10\frac{mW}{cm^2}$. What is the intensity of the beam that returns to the transducer from the back of the cyst after the US has passed through the abdominal wall and the cyst and reflects and how many decibels of energy loss does this correspond? Assume that whatever US pulse gets transmitted back into the abdominal wall is the US pulse that the transducer will hear.

From part b,
$I_{reflected\ off\ cyst/mucle} = 0.0048I_0$

And off the cyst/abdominal wall the US reflects again

$$R_{AW/Cyst} = \left(\frac{z_{AW} - z_{cyst}}{z_{AW} + z_{cyst}}\right)^2 = \left(\frac{1.4 \times 10^6 rayl - 1.48 \times 10^6 rayl}{1.4 \times 10^6 rayl + 1.48 \times 10^6 rayl}\right)^2 = 0.00077$$

What, then, gets to the transducer is $T_{tranducer} = 1 - R_{\frac{AW}{Cyst}} = 0.99923$

So, the intensity that gets to the transducer
$I_{tranducer} = 0.99923 \times (0.0048I_0) = 0.00478I_0$

The number of decibels of energy loss

$$dB = -10\log\frac{I_{transducer}}{I_0} = -10\log\frac{0.00478I_0}{I_0} = 23dB$$

d. Hepatic cysts have been known to rupture into the abdominal cavity. To see into the abdominal cavity a laparoscopic procedure was performed, and the following image (Figure 2 below) was obtained. Explain in as much detail as possible, how this image was produced.

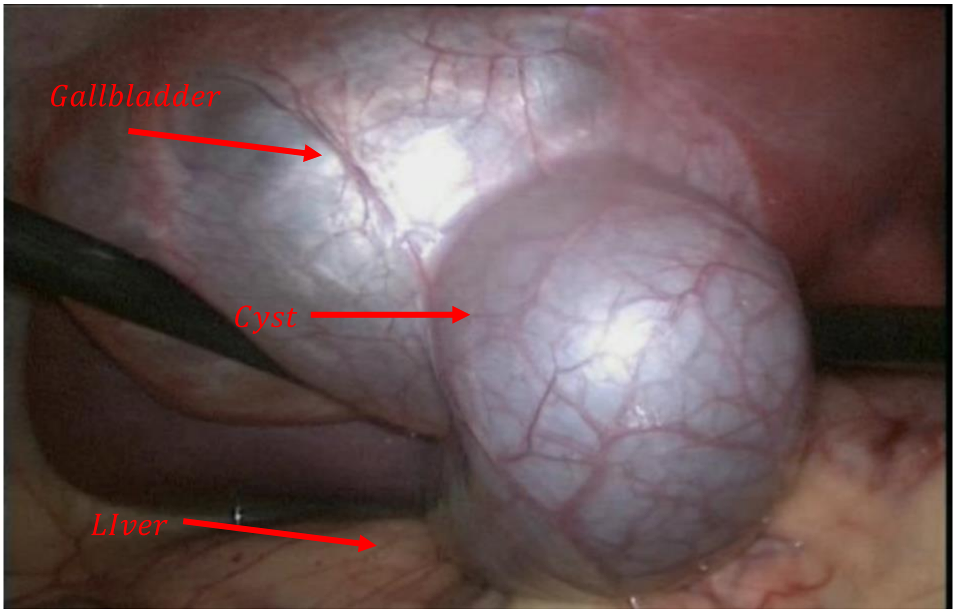


Figure 2: Laparoscopic view of the cyst showing the major anatomy involved.
https://www.sciencedirect.com/science/article/pii/S2213576619301617

Several small incisions were probably made in the abdominal wall through which an endoscope was inserted. The endoscope most likely has an incoherent bundle of fibers to illuminate the object and a coherent bundle of fibers used to transmit the light by total internal reflection back to the exterior of the patient. The resulting image is sent to a computer and to a TV screen so the surgeon can see inside of the patient.

e. Suppose that the surgeon wants to remove the cyst before it has the option to rupture. A surgical resection is performed, and the cyst is removed from the gallbladder and the liver as shown in Figure 3 below. To cauterize the resulting wound at the edge of the gallbladder, the surgeon opts for a laser. What type(s) of lasers could the surgeon choose? Be as specific as possible in your answer and be sure to explain why you chose the laser(s) you did.

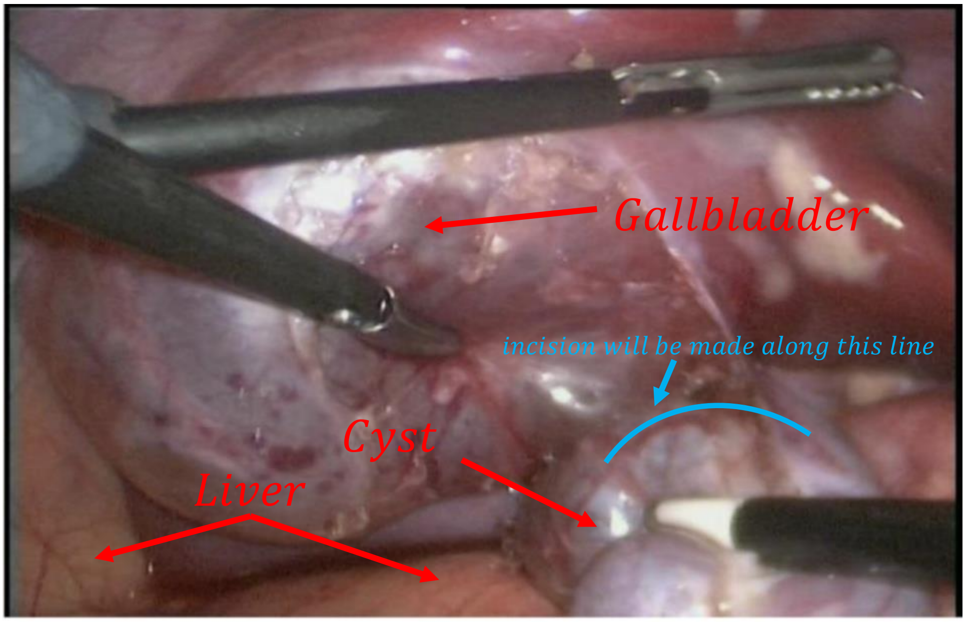
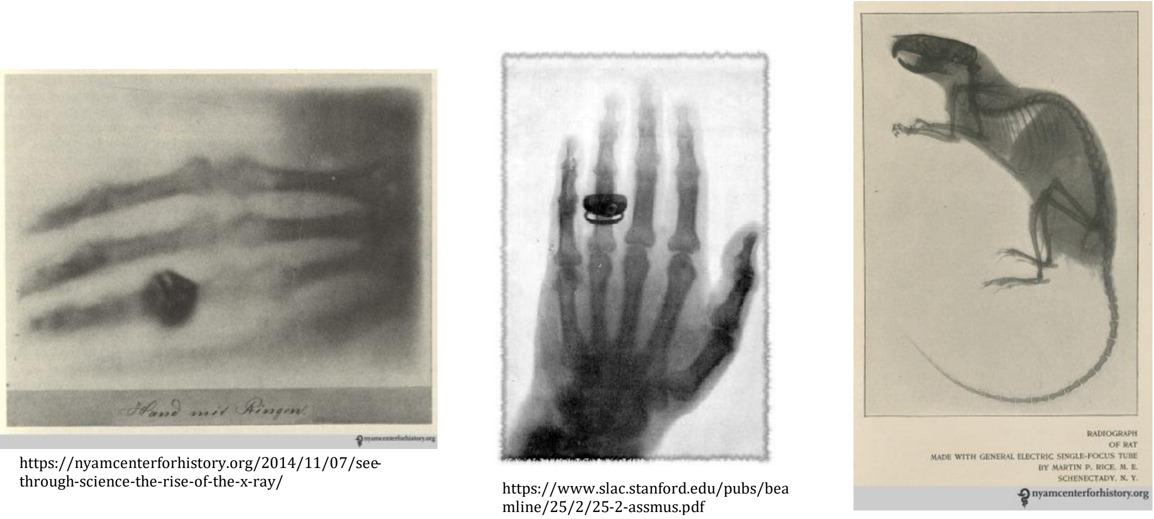


Figure 3: Photograph of the cyst being removed from the liver (on the bottom in the photograph) and the gallbladder (on the top in the photograph).
https://www.sciencedirect.com/science/article/pii/S2213576619301617

Since the incision will most likely involve blood and since the gallbladder appears red, an appropriate choice of laser would be something in the blue/green/UV portion of the electromagnetic spectrum. Some choices could be Excimer (UV), Nd:Yag (green), Argon (green), or a copper vapor (green) In addition, we may want to use the laser to not only to make the incision and to cauterize the wound. Thus, we need the intensity of the laser to be high ($> 100\frac{W}{cm^2}$). Of the lasers listed, only the Nd:Yag would have an intensity high enough to perform the surgery.

2. X-ray Imaging

X-rays were discovered on November 8, 1895, by Wilhelm Röntgen (1845–1923) Working in his laboratory, he found that an active vacuum tube, shielded with a cardboard sleeve so that no visible or ultraviolet light could come out, caused fluorescence on a light- and UV-sensitive screen some distance away. When he inadvertently passed between the tube and the fluorescing screen, he saw a projection of his own skeleton. Röntgen took the first x-ray image of a human by taking a picture of his wife's hand just before Christmas in 1895. His "photographs" as you can imagine, rather quickly ushered in the field of medical imaging. Everything was placed in front of the Röntgen rays as some people called to see what was inside. Several examples of the first x-ray images made by Röntgen are shown in the first two photographs on the left in Figure 4 and one of a rat made here at GE in Schenectady shown on the right of Figure 4. Röntgen won the first Nobel prize in Physics in 1901. To make these images, all that was necessary at the time was a vacuum tube (containing a filament and an anode) and a photographic plate. A photographic plate is essentially a piece of film.



https://nyamcenterforhistory.org/2014/11/07/see-through-science-the-rise-of-the-x-ray/

https://www.slac.stanford.edu/pubs/beamline/25/2/25-2-assmus.pdf

https://nyamcenterforhistory.org/2014/11/07/see-through-science-the-rise-of-the-x-ray/

Figure 4: Some original x-ray images.

a. For any of the images above, x-rays were produced and passed through an open room. The subject was then put in front of the rays and what emerged exposed a piece of photographic film. Explain in as much detail as possible, how the x-rays are generated, how they interact with the subject being “photographed” and then with the photographic film.

A filament cathode (usually made of tungsten) is heated, and electrons are emitted by thermal emission. A potential difference applied across the cathode to anode and the electrons are accelerated to a high energy (usually several to hundreds of keV’s) where they interact with atoms in the anode. The anode is made from atoms of a specific type (say tungsten, nickel, copper, gold, etc.). Most of the electrons decelerate when they interact with orbital electrons or nuclei in the anode. This is called bremsstrahlung or breaking radiation and there is an emission of x-rays with all sorts of energies. This creates a background in the energy spectrum. However, there is a small probability that the incident electron may eject an inner shell orbital electron from an atom in the anode. This creates a vacancy in an inner shell that will be filled by an electron from a higher orbital with an emission of an x-ray characteristic of the atoms in the anode. These characteristic x-rays from the atoms in the anode are what we’ll use to image the patient.

The x-rays generated are incident on the patient and the interactions of the x-rays and the tissues of the patient fall into three categories.

Photoelectric effect: x-rays are absorbed by high Z materials in the body, ejecting an orbital electron from the material. This takes photons out of the primary propagating beam.

Compton effect: an inelastic scattering event occurring mostly in low Z materials in the body. The photon scatters off an outer shell electron and gives some of its energy to the electron. This scatters the photon and photons are removed (changes direction from) of the primary propagating beam.

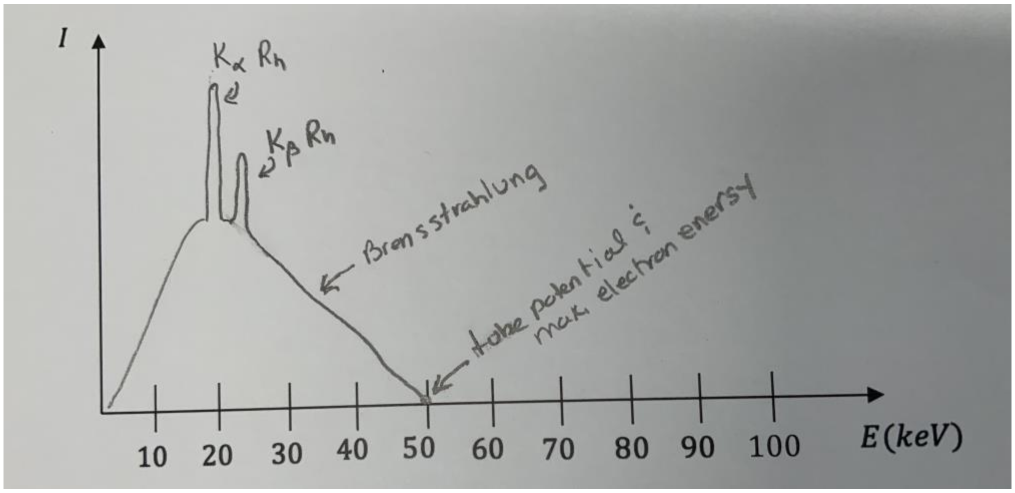
Coherent scattering: an elastic scattering event in which the x-ray scatters from an orbital electron essentially losing no energy to the collision. This changes the direction of the x-rays effectively removing them from the primary propagating beam.

The x-rays are attenuated as they pass through the body and those that emerge are used to form the image on a piece of film. Film has an emulsion of silver-bromide and when an x-ray strikes a silver-bromide grain it excites the grain. The more grain excitement in an area the more x-rays that struck that area and vice versa. When the film is developed, the areas that are not excited are washed away leaving a shade of grey that is interpreted by the physician.

b. Back in 1895 people didn't know about the dangers of exposure to x-rays as we do now, but the technology for the modern x-ray image has essentially remined unchanged over the last 120 years, but with some safety features added today. Suppose that you wanted to generate x-rays from a rhodium anode (${}^{A}_{Z}X = {}^{103}_{45}Rh$) from a tube that operated at $50kV$ producing $30mA$ of electron current on the anode. What is the energy of the K_α x-ray produced by rhodium and what would the spectrum of the intensity of x-rays produced as a function of energy look like? Use the axes below and note that the graph should be qualitatively correct, and you should explain the main features of what you drew.

$$\Delta E = E_{upper} - E_{lower} = -13.6eV(Z-1)^2\left(\frac{1}{n_{upper}^2} - \frac{1}{n_{lower}^2}\right)$$

$$\Delta E = -13.6eV(45-1)^2\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 19.7keV$$



c. What is the efficiency of x-ray production from the rhodium anode?

$$\varepsilon = \frac{P_r}{P_d} = \frac{3W}{1500W} = 0.002 \rightarrow 0.2\%$$

$$P_d = IV = 30 \times 10^{-3}A \times 50 \times 10^3 V = 1500W$$

$$P_r = \left(0.9 \times 10^{-9}\frac{C}{J}\right) ZV^2 I = \left(0.9 \times 10^{-9}\frac{C}{J}\right) \times 45 \times 30 \times 10^{-3}A \times (50 \times 10^3 V)^2 = 3W$$

d. The adult human liver varies in size as the person ages and with your sex. Suppose that the adult human liver is roughly circular with a diameter $D = 5in = 12cm$ as shown in Figure 5 below. In Figure 5 several metastases of the liver are also seen (the black arrows) on the CT scan. Suppose that there are two paths taken by an x-ray beam, shown in the diagram by the red and blue arrows. The x-ray beam not only has to pass through the liver but also $0.5cm$ of muscle on the anterior and posterior abdominal walls and of $5cm$ of air. Following the red line, x-rays that are incident on the anterior abdominal wall exit from the posterior abdominal wall, the back. Using the x-ray attenuation coefficients from the table attached, what fraction of the incident x-rays emerge from the back? Note: Just as note in passing, rhodium probably would not be used to image the liver in a CT scan.

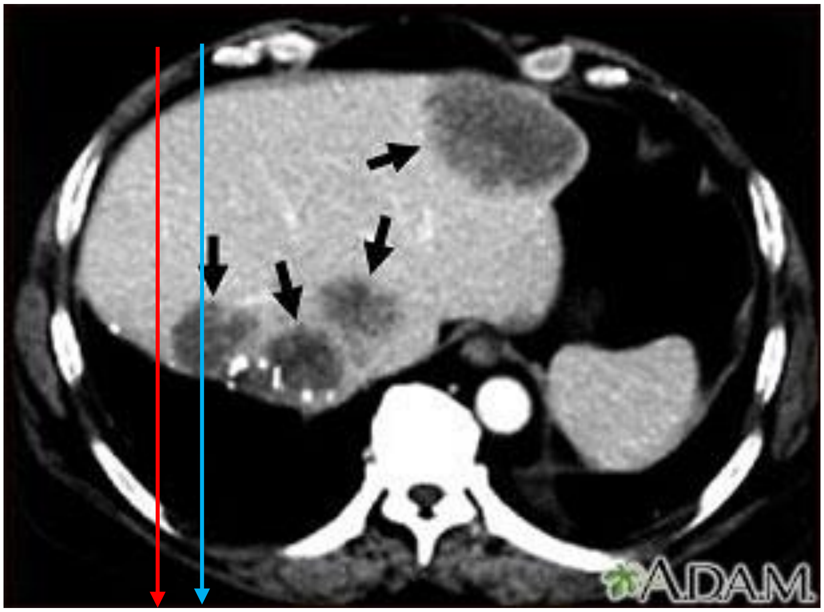


Figure 5: CT scan of the liver showing several large metastases.
https://medlineplus.gov/ency/imagepages/1180.htm

$$I = I_o e^{-\left(2\mu_{AW}x_{AW}+\mu_L x_L+\mu_{air}x_{air}\right)}$$

$$I = I_0 e^{-\left(2\times 0.8205\frac{cm^2}{g}\times 1.06\frac{g}{cm^3}\times 0.5cm+0.8230\frac{cm^2}{g}\times 1.0\frac{g}{cm^3}\times 12cm+0.779\frac{cm^2}{g}\times 0.0013\frac{g}{cm^3}\times 5cm\right)}$$

$$I = 2.1\times 10^{-5} I_0$$

e. Following the blue line, if the metastasis is in $3cm$ diameter and is imbedded in the liver, what is the contrast between the liver (modeled as soft tissue) and the metastasis? Comment on your result.

$$C = 1 - e^{-(\mu_2 - \mu_1)x_2} = 1 - e^{-\left(0.6889\frac{cm^2}{g}\times 0.901\frac{g}{cm^3} - 1.00\frac{cm^2}{g}\times 0.823\frac{g}{cm^3}\right)\times 3cm} = -0.84$$

Since the contrast is negative which means the liver metastasis is less absorbing to x-rays than the surrounding liver tissue. The contrast is also about 84% so you'd be able to see the metastases imbedded in the liver tissue, as the image shows.

Some useful data:

Material	Speed (m/s)	$Z\ (\times 10^6{}_{Rayl})$	$\rho\left(\frac{g}{cm^3}\right)$	US intensity loss (dB/cm)	$\mu_m\left(\frac{cm^2}{g}\right)$
Air	343	0.0004	0.0013	12	0.7790
Blood	1570	1.6	1.057	0.15	0.8428
Bone	3500	7.8	1.900	14.2	4.000
Calcium Deposit	4080	4.7	1.550	20	0.5831
Fat/Soft Tissue	1460	1.4	1.000	0.6	0.8230
Water	1480	1.48	1.000	0.0022	0.8096
Muscle	1580	1.7	1.060	1.4	0.8205
Liver Metastasis	1547	1.6	0.901	8	0.6889

Table 1: Speeds of sound, acoustic impedance, density of common materials, US intensity loss in various materials, and x-ray mass absorption coefficients. Values are taken from http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html, http://www.nist.gov/pml/data/xraycoef/, and Physics of Radiology, 2nd Ed., Anthony Wolbarst, p120.