Physics 210
Medical Physics
Midterm Exam
Winter 2025
February 14, 2025

Name___________________________

Problem 1	/24
Problem 2	/24
Problem 3	/24
Total	/72

For the exam, you may use your in-class notes, any Power Point slides you'd like, your textbook (Kane), and the readings from Wolbarst. You may not use old exams, homework solutions or use worked out solutions to problems.

I affirm that I have carried out my academic endeavors with full academic honesty.

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Signature

1. *Ureteropyeloscopy by Ultrasound*

Renal calculi or kidney stones are hard calcified masses that form in the kidneys. If small enough, these stones may leave the kidney and pass through the ureter into the bladder where they may be passed out with the urine stream through the urethra. However, if the stones are large enough, they may become lodged in any number of places, such as the kidney, the ureter, or the urethra. To remove a lodged stone, lithotripsy maybe used to create a high-energy shock wave to fragment and the lodged stone. This technique may be done externally using focused ultrasound (called *ESWL* or *extracorporeal shock wave lithotripsy*) or internally using a combination of optical fibers scopes and lasers (called *laser lithotripsy* or *Ureteropyeloscopy*).

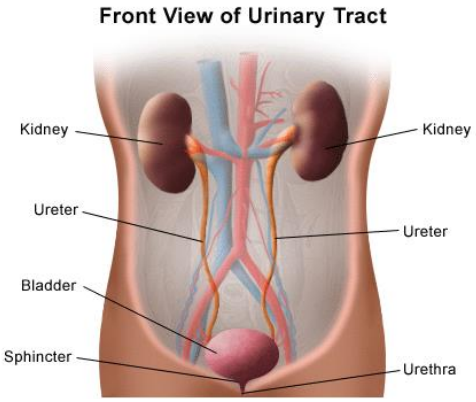


Figure 1A: Schematic of the human urinary system.
https://www.hopkinsmedicine.org/health/wellness-and-prevention/anatomy-of-the-urinary-system

a. Suppose that you wanted to image the stone in the ureter using ultrasound. What minimum frequency of ultrasound would be best to allow you to image the stone if the stone has an approximate diameter, the width of the ureter, $3mm$? Why would this be a minimum frequency? Explain ideally why you'd like a higher frequency and what are some limitations of these higher frequencies.

$$v_s = f\lambda \rightarrow f = \frac{v_s}{\lambda} = \frac{1540\frac{m}{s}}{3 \times 10^{-3}m} = 5.1 \times 10^5 Hz = 513kHz = 0.513MHz$$

Ideally, you'd want a higher frequency so you can see features in the body smaller than the $3mm$ quoted in the problem. To see smaller sizes (proportional to the wavelength of sound) you'd need to go to higher frequencies. Some limitations of going to very high frequencies are:

b. Figure 1B below shows an ultrasound scan of a ureter stone. The ultrasound beam passes through several layers of tissues labeled in Figure 1C. How long (in microseconds) would it take for an ultrasound to leave the transducer, placed on the top exterior portion of the abdominal wall, and return to the transducer if the sound reflects off structures along the line in Figure 1C? In addition, what could you say about the response time of the piezoelectric transducer if you wanted to be able to resolve these individual structures? Note you will be calculating 4 times in the problem, the time to reflect from the abdominal wall/bladder interface (AW/B), the bladder/upper ureter wall interface (B/UW), the ureter wall/stone interface (UW/S), and the stone/ureter wall interface (S/UW) on the lower ureter wall.

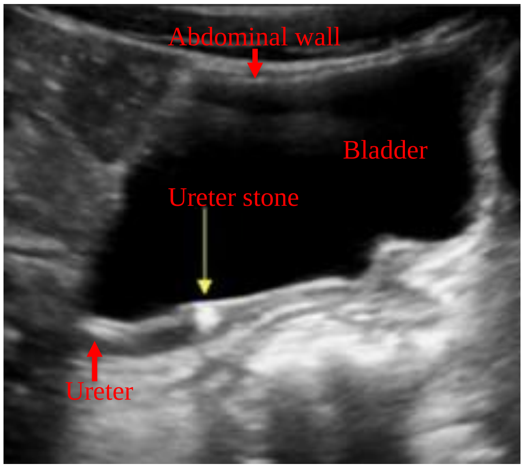


Figure 1B: US image of a ureter stone.
https://www.pocus.org/renal-point-of-care-ultrasound-pocus-for-nephrolithiasis/

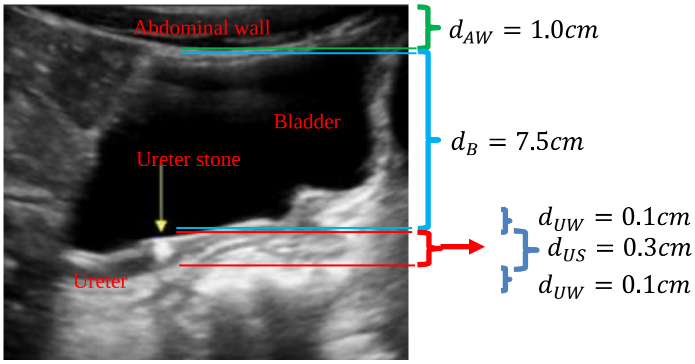


Figure 1C: US image of a ureter stone with some labeled distances from the transducer.
https://www.pocus.org/renal-point-of-care-ultrasound-pocus-for-nephrolithiasis/

The velocity for the sound in is given by $v_s = \frac{d_{in}}{t_{in}}$ and the time for the return of the US pulse (the echo) is $t = 2t_{in} = \frac{2d_{in}}{v_s}$.

$$t_{AW/B} = \frac{2d_{AW/B}}{v_s} = \frac{2\times 0.01m}{1540\frac{m}{s}} = 1.29\times 10^{-5}s = 13\mu s$$

$$t_{B/UW} = \frac{2d_{B/UW}}{v_s} = \frac{2\times 0.085m}{1540\frac{m}{s}} = 1.104\times 10^{-4}s = 110.4\mu s$$

$$t_{UW/S} = \frac{2d_{UW/S}}{v_s} = \frac{2\times 0.086m}{1540\frac{m}{s}} = 1.117\times 10^{-4}s = 111.7\mu s$$

$$t_{UW/S} = \frac{2d_{UW/S}}{v_s} = \frac{2\times 0.089m}{1540\frac{m}{s}} = 1.156\times 10^{-4}s = 115.6\mu s$$

You'd want the response time of the piezoelectric transducer to be smaller than a microsecond if you want to resolve the ureter walls and the stone as being separate structures.

c. How long would it take to perform a complete ultrasound scan the patient’s urinary system? Assume that the patient’s dimensions for the scan are shown in Figure 1D below with the depth of the patient taken to be $D = 35cm$. Assume that the ultrasound beam has a height and width given by $h = 1mm$ and $w = 0.5mm$ respectively.

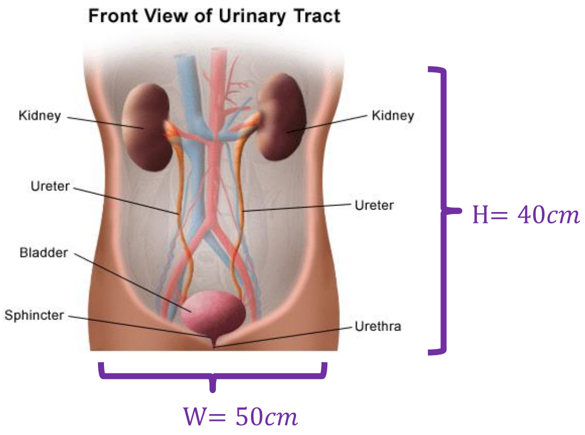


Figure 1D: Patient dimensions for an ultrasound scan of the urinary system.
https://www.hopkinsmedicine.org/health/wellness-and-prevention/anatomy-of-the-urinary-system

The time for a single US beam to pass through the patient and return to the transducer over the depth of the person is given by:

$$v_s = \frac{D}{t_D} \rightarrow T = 2t_D$$

$$T = 2\frac{0.35m}{1540\frac{m}{s}} = 4.56 \times 10^{-4}s$$

$$T = 456\mu s$$

The number of scans it takes over the width of the person is:

$$N_{horizontal\ scans} = \frac{W}{w} = \frac{50mm}{0.5mm} = 1000$$

And thus, the time it takes to scan the complete with of the person is

$$T_W = 1000 scans \times 456\frac{\mu s}{scan} = 4.6 \times 10^5 \mu s = 0.46s$$

The number of scans it takes over the height of the person is:

$$N_{vertical\ scans} = \frac{H}{h} = \frac{400mm}{1mm} = 400$$

And thus, the time it takes to scan the complete with of the person is

$$T_H = 400 scans \times 0.46\frac{s}{scan} = 184s = 3\ minutes$$

The total time for the patient’s scan is $184s \sim 3\ minutes$ which seems reasonable for a diagnostic scan.

d. Suppose that you decide that you want to focus the US beam onto the stone to try to shatter it using the energy contained in the US beam. What would be some advantages and disadvantages in using the US beam to shatter the ureter stone?

Advantages: Non-invasive
Higer energies potentially incident on the stone to shatter it.
Less likely to have any post-operative treatments.
Less likely to have an infection or complications from the surgery.

Disadvantages: Focused beam can heat up the tissues involved, and this could lead to tissue damage.
Could cause cavitation or the formation of bubbles in the fluid.
May not shatter into small pieces.

2. *Laser ablation in Uteropyloscopy*

After consultation with some physicians and ultrasonograhers you decide that it may not be a good idea to use the US beam to break up the stone. Instead, you decide that you'd like to use a laser to shatter the stone in the ureter. To do this you use a fiber optic scope coupled with a laser. The procedure called laser lithotripsy utilizes an extremely high-powered pulsed holmium-YAG laser to disrupt kidney stones and/or other painful mineralized deposits in the urinary system. The procedure is usually performed under general anesthesia by introducing a small telescope (called a ureteroscope) through the urethra, into the bladder, and up the ureter as shown in Figure 2A below. With direct visualization of the stone within the ureter, a laser fiber can be used to fragment the stone into smaller pieces. When laser light exits the optical fiber, the intense laser beam instantly vaporizes the water at the very tip of the optical fiber, creating a shock wave, which destroys the stone as shown in Figure 2B. (No focusing of the laser is needed for the high instantaneous intensities used.)

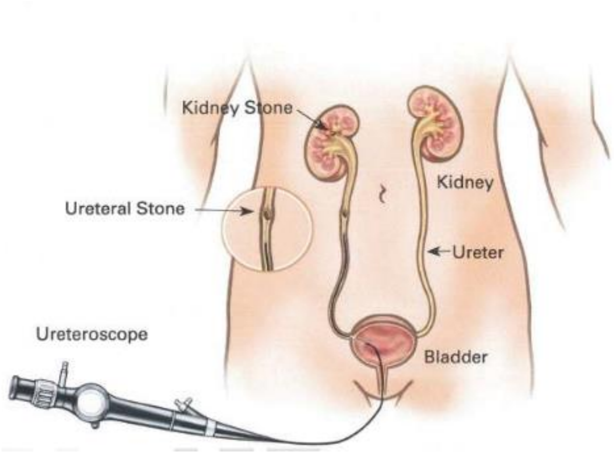


Figure 2A: Schematic of the urinary system.
https://darwinurology.com.au/wp-content/uploads/2019/04
Ureteric-Kidney-Stones-Ureteroscopy-Pyeloscopy-1.3.201

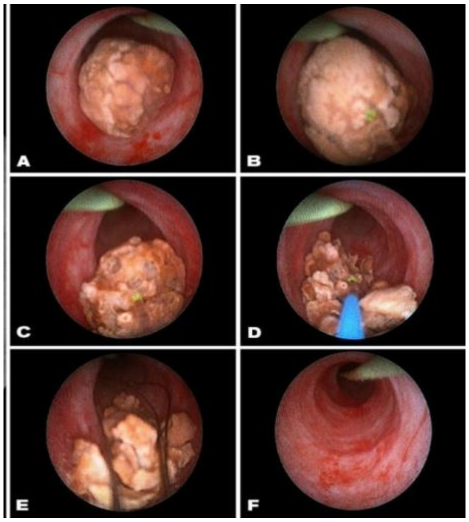


Figure 2B: Calcified stone in the ureter. The figures show the stone (panels A, B, & C), the fracture of the stone (panels D & E) and the clear ureter (panel F).
https://www.dcurology.net/procedures/ureteroscopy-with-laser-lithotripsy.php

a. Suppose that the laser light is incident on the front surface of an endoscope at an angle θ, where the light on the front surface of the scope is incident from the air and is shown in Figure 2C. The endoscope core has an index of refraction $n_{core} = 1.7$ and is coated with a material (the cladding) with an index of refraction $n_{cladding} = 1.5$. If the light needs to be internally reflected in the scope, at what angle θ should the light be incident on the front surface of the endoscope from the air?

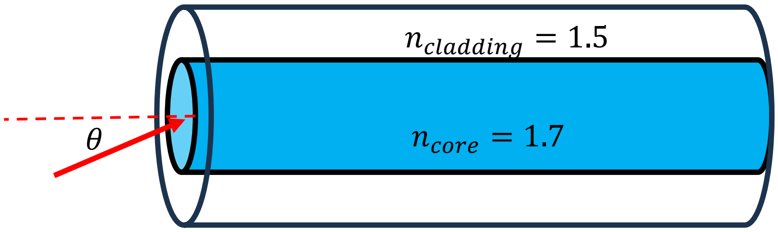


Figure 2C: Light is incident on the front surface of an endoscope. The light needs to be totally internally reflected in the endoscope.

For total internal reflection on the upper surface of the guide at the core/cladding interface, we have:

$$n_{core} \sin\theta_{core} = n_{cladding} \sin\theta_{cladding}$$

$$n_{core} \sin\theta_{crit,core} = n_{cladding} \sin 90 = n_{cladding} \rightarrow \sin\theta_{crit,core} = \frac{n_{cladding}}{n_{core}} = \frac{1.5}{1.7}$$

$$\theta_{crit,core} = \sin^{-1}(0.8824) = 61.9^0$$

Using the geometry in the problem, the light must enter the core (from the air) at $\phi = 90^0 - 61.9^0 = 28.1^0$

Then at the air/core interface we can determine θ.

$$n_{air} \sin\theta = n_{core} \sin\phi \rightarrow \sin\theta = \frac{n_{core}}{n_{air}} \sin\phi = \frac{1.7}{1.0} \sin 28.1 = 0.8007$$

$$\theta = \sin^{-1}(0.8007) = 53.2^0$$

b. Since the ureter stone is mostly water and minerals, the laser light should be chosen to target the water. If the intensity of the laser light can be made high enough, then we could vaporize the water in the stone and shatter the stone into smaller pieces. Which laser or lasers from the list below would be an ideal choice to use to break up the ureter stone? Be sure to explain your selections and why you chose the one(s) you did.

Laser	$\lambda(nm)$	$E(keV)$	Type
Ho-YAG	2100	0.59	IR
Carbon Dioxide	10600	0.12	IR
ND-YAG	532, 1064	2.4, 1.2	Visible, IR
He-Ne	630	2.0	Visible
Alexandrite	755	1.7	Visible
Ruby	694	1.8	Visible
Eximer	157, 391	7.9, 3.2	UV
Argon Ion	334, 351	3.7, 3.5	UV

Water absorbs strongly in the infrared and in the ultraviolet. Thus, the choices of laser would be one that lases in the IR or UV. However, in terms of energies available to raise the temperature of water in the stone to vaporize it and to minimize potential treatment times, I'd opt for a higher energy laser an choose one of the two UV lasers.

c. Unfortunately, you accidentally nicked the inner wall of the bladder with a tool on the endoscope, and the inner wall of the bladder begins to bleed. This is not a major problem, and you decide to cauterize the small bleed. Will your current laser do the job? If yes, explain why it will and what makes it effective to stop the bleed. If no, explain why it will not and what type of laser you would need to do the cauterization and why you would need this laser.

The IR or UV lasers from part b would not cauterize the blood. The intensity is most likely to high to achieve photocoagulation of the small tear in the bladder wall. In addition, the blood contains heme which appears red, and which strongly absorbs in the visible portion of the EM spectrum, and at short wavelengths. Thus, I'd like something in the blue/violet portion of the spectrum. None of these are in the violet portion of the EM spectrum. Therefore, of the choices I have, I'd most likely choose the ND-YAG laser to perform a photocoagulation procedure.

d. To ultimately shatter the stone by vaporizing the water in the stone you need to raise the temperature of the minerals and the water in the stone to approximately 120^0C. This is an approximately $\Delta T = 83^0C$ increase above the body's normal temperature 37^0C. Suppose that the laser beam has an output intensity at the stone's surface of $I_{laser} = 120\frac{W}{cm^2}$. How long would it take to shatter the stone and is this time reasonable or not? Be sure to explain your answer for reasonable or not. Some hints: The stone has a density $\rho = 1.36\frac{g}{cm^3}$ and it is spherical with a radius $0.62mm$. The specific heat of water and stone are $4.18\frac{J}{g^0C}$ and $0.9\frac{J}{g^0C}$ respectively. and that you need to heat ***both*** the minerals and the water in the ureter stone. Assume that the minerals in the ureter stone account for 40% of the volume of the ureter stone and that the water accounts for remaining 60% of the ureter stone's volume.

The intensity is given by: $I = \frac{Energy}{time \times area} = \frac{heat}{time \times area} = \frac{Q}{tA}$.

The treatment time therefore would be given by $t = \frac{Q}{IA}$

The area of the stone presented to the laser from Figure 2B is a from a circle of radius $r = 0.62mm$. Thus, the area presented is $A = \pi r_{stone}^2 = \pi\left(0.62mm \times \frac{1cm}{10mm}\right)^2 = 0.0121cm^2$

The energy delivered from the beam shows up as heat in the minerals of the stone and in the water to vaporize the water. The heat is:

$Q = Q_{minerals} + Q_{water} = m_{minerals}c_{minerals}\Delta T + m_{water}c_{water}\Delta T$

$Q = (0.4\rho_{minerals}V)c_{minerals}\Delta T + (0.6\rho_{water}V)c_{water}\Delta T$

$Q = \left(1.36\frac{g}{cm^3} \times 0.4 \times 0.9\frac{J}{g^0C} + 1\frac{g}{cm^3} \times 0.6 \times 4.18\frac{J}{g^0C}\right) \times \left[\frac{4}{3}\pi(0.062cm)^3\right] \times 83^0C$

$Q = 0.249J$

The treatment time:

$$t = \frac{0.249J}{120\frac{W}{cm^2} \times 0.0121cm^2} = 0.17s = 170ms$$

This seems very reasonable. The intensity is very large, so I wouldn't expect it would take long to vaporize the ureter stone.

3. *X-rays in Mammography*

Breast cancer is a disease in which malignant (cancer) cells form in the tissues of the breast. The damaged cells can invade surrounding tissue, but with early detection and treatment, most people continue a normal life. One in eight women will be diagnosed with breast cancer in her lifetime. Breast cancer is the most diagnosed cancer in women and is the second leading cause of cancer death among women. Each year it is estimated that over 275,000 women in the United States will be diagnosed with breast cancer and more than 40,000 will die. Although breast cancer in men is rare, an estimated 2,600 men will be diagnosed with breast cancer and approximately 500 will die each year. Over 3.5 million breast cancer survivors are alive in the United States today. (Facts from http://www.nationalbreastcancer.org/breast-cancer-facts). To that end, women with a family history of breast cancer or women over the age of 45 are encouraged to get a mammogram yearly. A mammogram is an x-ray image of the breast.

a. Figure 3A below shows an x-ray energy spectrum for molybdenum ($Z = 42$). Explain in as much detail as possible, how x-ray spectrum (given by the non-shaded line in the figure) was generated, how x-rays interact with the subject being "photographed" and then with a piece of photographic film to produce an image.

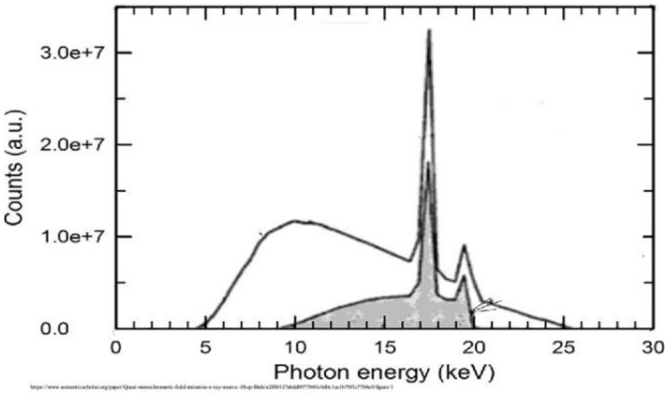


Figure 3A: X-ray energy spectrum for a molybdenum anode showing the intensity (number of x-rays) as a function of their energy.

The non-shaded spectrum is made by accelerating electrons from a filament toward an anode made from molybdenum. When the electrons interact with the anode they decelerate and radiate. This produces the background called bremsstrahlung. The characteristic x-ray peaks are due to ejection of electrons from the inner shells of the anode material. When electrons in the anode material transition from higher energy states to fill the vacancy, they release a photon of light. This photon of light is in the x-ray portion of the electromagnetic spectrum and the x-rays that are produced are characteristic of the material out of which the anode is made. The molybdenum x-rays have energies approximately 17.5keV and 19.5keV.

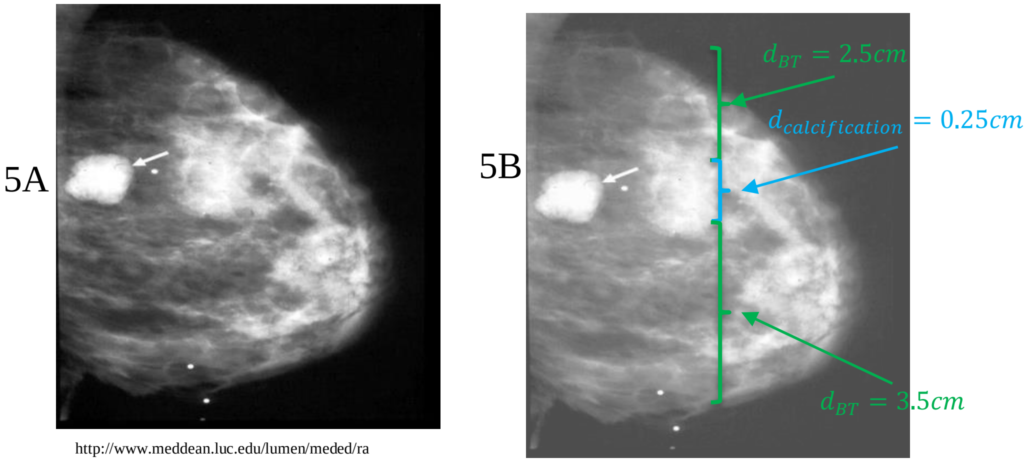
The x-rays interact with the patient by the photoelectric and Compton effects primarily. The x-rays get absorbed by the tissues/bones of the patient and the probability of absorption increases with increasing Z of the material and density of the material. Lower energy photos are more likely to be photoelectrically absorbed than higher energy photons, which are more likely to be Compton scattered. These two processes take photons out of the beam and this attenuates the beam. The photons that make it out are used to form the image on the detector.

If a piece of film is used, the photons interact with crystals of silver-bromide on the film. The more photons that strike a location the higher the degree of sensitization (excitement of the silver bromide crystals.). During the development of the film the sensitized silver bromide crystals are washed away and what's left is a photographic negative of the attenuated photons and this is the image seen on the film.

b. What is the efficiency of x-ray production for the molybdenum x-ray tube?

$$\varepsilon = 0.9 \times 10^{-9}\frac{C}{J} \times ZV = 0.9 \times 10^{-9}\frac{C}{J} \times 42 \times 26 \times 10^{3}V = 0.00098 = 0.001 = 0.1\%$$

c. Suppose that we have the x-ray image of a female breast shown below in Figure 5A. X-rays with intensity I_o are incident on the upper surface of the breast and as they pass through the breast some fraction of them are absorbed by the tissues. The intensity of the emerging x-ray beam was measured to be 0.19% of the incident intensity. Using the information in Figure 5B below, what is the x-ray absorption coefficient $\mu_{calcification}$ for the macrocalcification in the breast tissue? For x-rays, the mass attenuation coefficient for breast tissue is $\mu_{m,BT} = 0.6889\frac{cm^2}{g}$ and we can model the breast tissue as muscle.



http://www.meddean.luc.edu/lumen/meded/radio/curriculum/surgery/mammography1.htm

http://www.meddean.luc.edu/lumen/meded/radio/curriculum/surgery/mammography1.htm

Figure 5: X-ray image of (Figure 5A) and labeled image of (Figure 5B) a female breast.

$$I = 0.0019I_0 = I_0e^{-(\mu_{BT}x_{BT}+\mu_{calcification}x_{calcification})}$$

$$\ln\left(\frac{I}{I_0}\right) = -(\mu_{BT}x_{BT} + \mu_{calcification}x_{calcification})$$

$$\ln 0.0019 = -6.266 = -(\mu_{m,BT}\rho_{BT}x_{BT} + \mu_{calcification}x_{calcification})$$

$$6.266 = \left(0.6889\frac{cm^2}{g} \times 1.06\frac{g}{cm^3} \times 6cm\right) + (0.25cm \times \mu_{calcification})$$

$$\rightarrow \mu_{calcification} = 7.54cm^{-1}$$

d. What is the contrast between the microcalcification and the layer of breast tissue surrounding the microcalcification? Comment on the result you get.

$$C = 1 - e^{-(\mu_{calcificatiion} - \mu_{BT})x_{calcification}}$$

$$C = 1 - e^{-\left(7.54cm^{-1} - 0.6889\frac{cm^2}{g} \times 1.06\frac{g}{cm^3}\right)0.25cm} = 0.818 = 81.8\%$$

Since the contrast between the microcalcification and the surrounding tissue is $\sim 82\%$, I'd clearly be able to distinguish the microcalcification from the breast tissue.

Table 2: Some useful data:

Material	Speed (m/s)	$Z\ (\times 10^6{}_{Rayl})$	$\rho\ \left(\frac{g}{cm^3}\right)$	US intensity loss (dB/cm)	$\mu_m\ \left(\frac{cm^2}{g}\right)$
Air	343	0.0004	0.0013	12	0.7790
Blood	1570	1.6	1.057	0.15	0.8428
Bone	3500	7.8	1.900	14.2	?
Stones/Calcium Deposit	4080	4.7	1.550	20	0.5831
Fat/Soft Tissue	1460	1.4	1.000	0.6	0.8230
Water/Urine	1480	1.48	1.000	0.0022	0.8096
Abdominal Wall/Ureter Wall/Muscle	1580	1.7	1.060	1.4	0.8205

Table 2: Speeds of sound, acoustic impedance, density of common materials, US intensity loss in various materials, and x-ray mass absorption coefficients. Values are taken from http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html, http://www.nist.gov/pml/data/xraycoef/, and Physics of Radiology, 2nd Ed., Anthony Wolbarst, p120.