# *Magnetic Resonance Imaging*

http://www.gehealthcare.com/euen/oncology/images/breast-health/onc_breast_pulmo_500.jpg

http://www.gehealthcare.com/euen/oncology/images/breast-health/onc_breast_wholemri.jpg

http://www.gehealthcare.com/euen/oncology/images/breast-health/onc_breast_bonemetasta_500.jpg

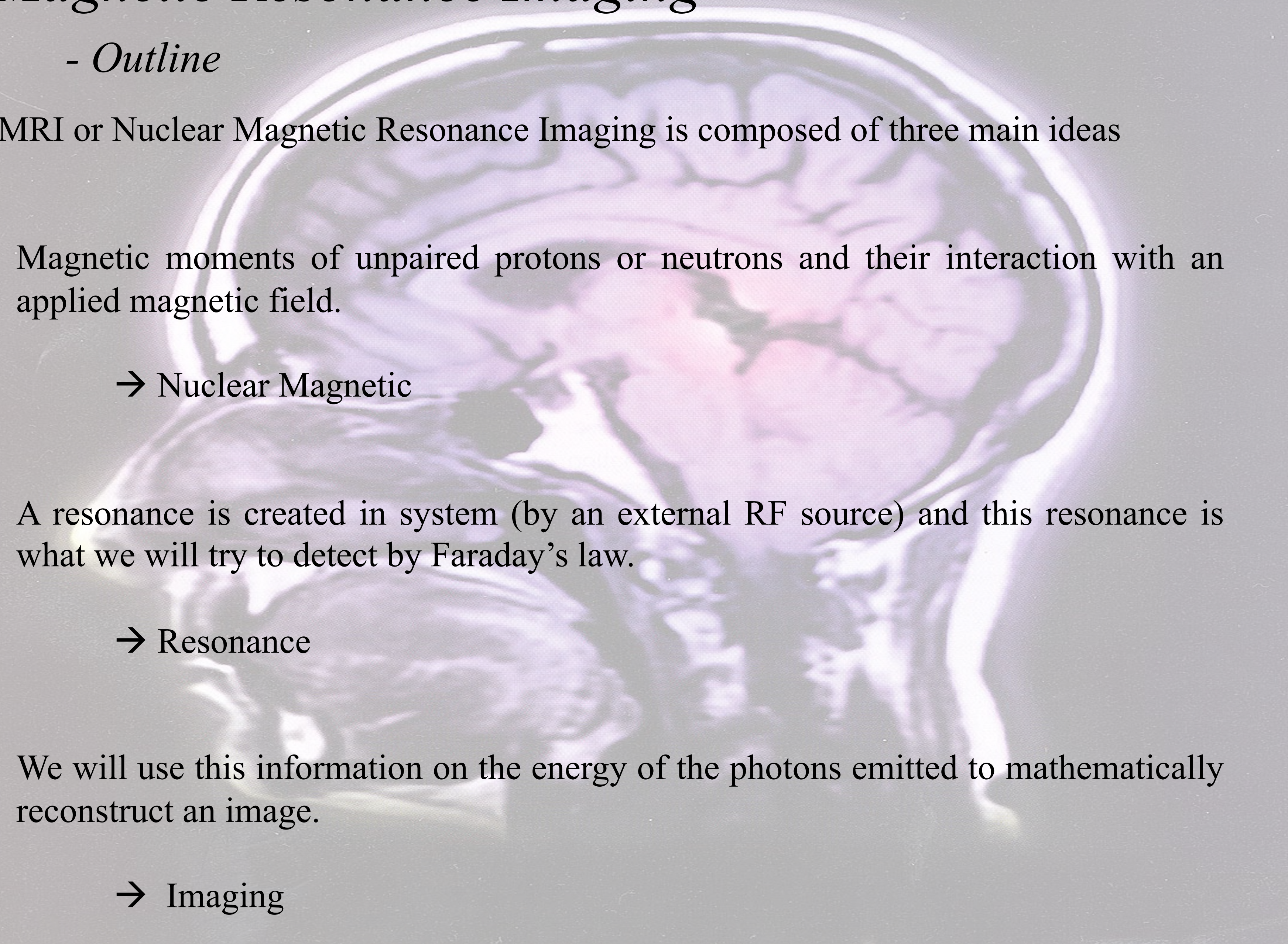
Background Image: http://www.ruf.rice.edu/~pomeran/images/brainmri.jpg

# *Magnetic Resonance Imaging*

## *- Outline*

- Magnetism, the nucleus, and precession of magnetic moments in a magnetic field
- Nuclear magnetic fields and nuclear magnetic moments
- Spin orientation, spin state transitions & Larmor frequency
- RF pulses & NMR signal
- T1 & T2 relaxation and MRI images
- Spin echo formation
- Contrast mechanisms
- MRI scanners and imaging

http://dynamicheartphantom.com/cardiac.jpg



# *Magnetic Resonance Imaging*

## *- Outline*

NMRI or Nuclear Magnetic Resonance Imaging is composed of three main ideas

- Magnetic moments of unpaired protons or neutrons and their interaction with an applied magnetic field.

  → Nuclear Magnetic

- A resonance is created in system (by an external RF source) and this resonance is what we will try to detect by Faraday's law.

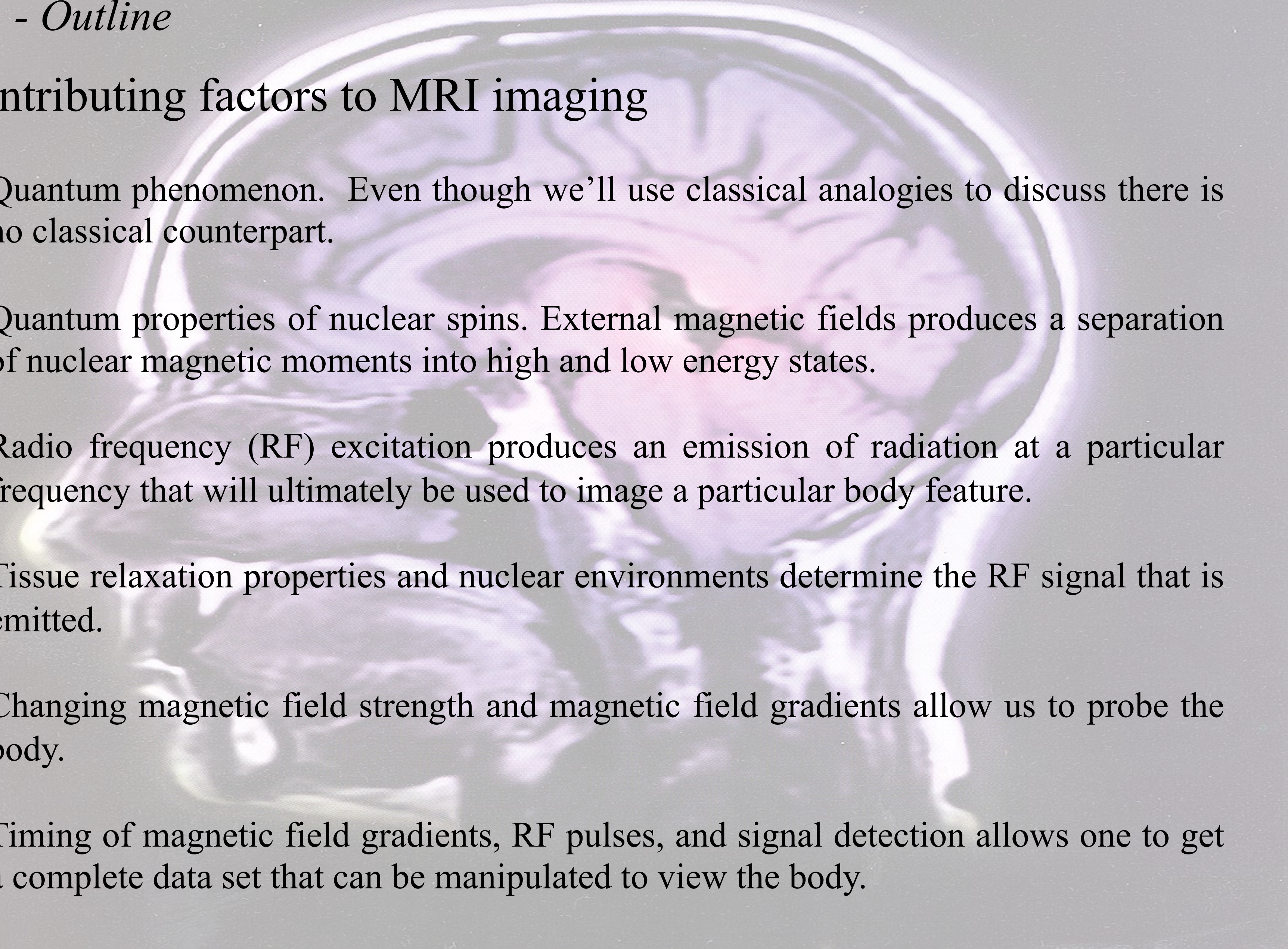
  → Resonance

- We will use this information on the energy of the photons emitted to mathematically reconstruct an image.

  → Imaging

# *Magnetic Resonance Imaging*

## *- Outline*



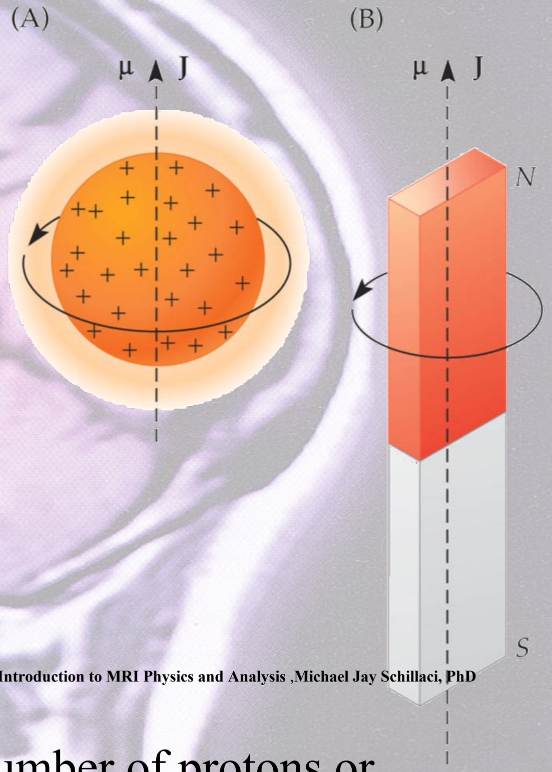
## Contributing factors to MRI imaging

- Quantum phenomenon. Even though we'll use classical analogies to discuss there is no classical counterpart.
- Quantum properties of nuclear spins. External magnetic fields produces a separation of nuclear magnetic moments into high and low energy states.
- Radio frequency (RF) excitation produces an emission of radiation at a particular frequency that will ultimately be used to image a particular body feature.
- Tissue relaxation properties and nuclear environments determine the RF signal that is emitted.
- Changing magnetic field strength and magnetic field gradients allow us to probe the body.
- Timing of magnetic field gradients, RF pulses, and signal detection allows one to get a complete data set that can be manipulated to view the body.

# *Magnetic Resonance Imaging*

## *- MRI Basics*

## What kinds of nuclei can be used for NMR?



An Introduction to MRI Physics and Analysis ,Michael Jay Schillaci, PhD

- Nucleus needs to have 2 properties:
  - Spin
  - Charge

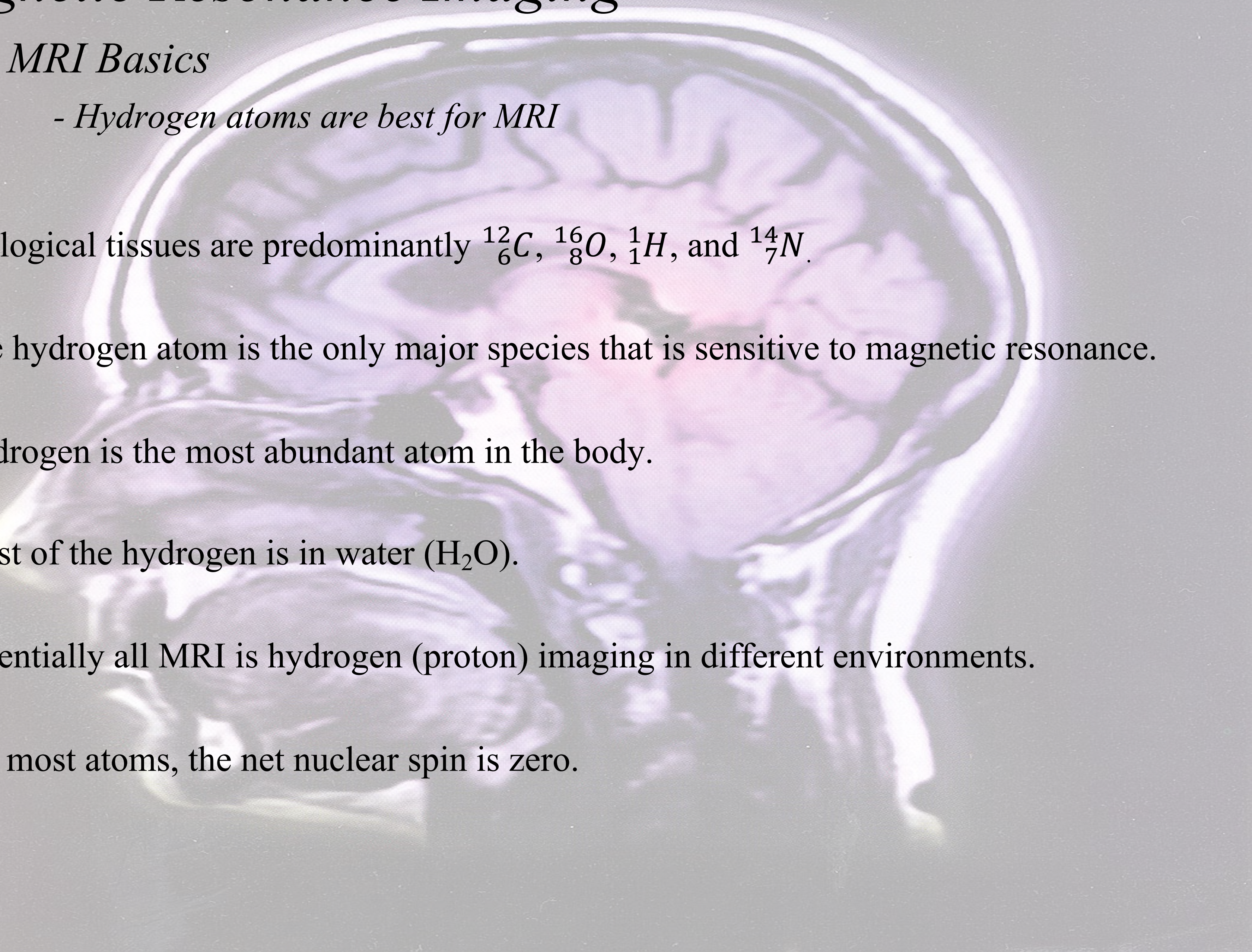
- Nuclei are made of protons and neutrons
  - Both are called spin ½ particles
  - Protons have charge

- Pairs of spins tend to cancel, so only atoms with an odd number of protons or neutrons have a net nuclear spin
  - Good nuclear choices for magnetic resonance are $^{1}H_{1}$, $^{13}C_{6}$, $^{19}F_{9}$, $^{23}Na_{11}$, $^{31}P_{15}$

# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- Hydrogen atoms are best for MRI*



- Biological tissues are predominantly ${}^{12}_{6}C$, ${}^{16}_{8}O$, ${}^{1}_{1}H$, and ${}^{14}_{7}N$.
- The hydrogen atom is the only major species that is sensitive to magnetic resonance.
- Hydrogen is the most abundant atom in the body.
- Most of the hydrogen is in water ($H_2O$).
- Essentially all MRI is hydrogen (proton) imaging in different environments.
- For most atoms, the net nuclear spin is zero.

# *Magnetic Resonance Imaging*

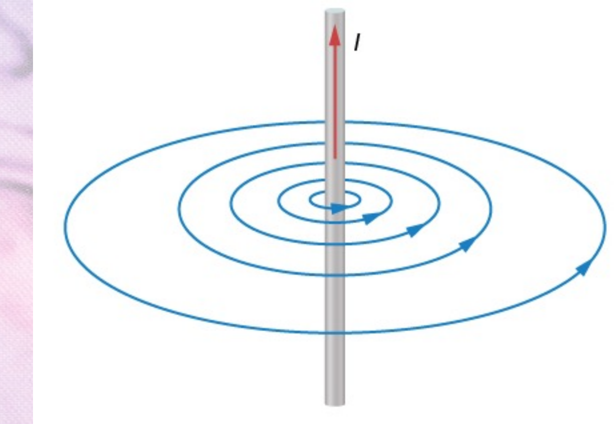
## *- MRI Basics*

### *- Origins of Magnetism*

- Macroscopic View
  - Moving charges (Current) in wire
  - Field wraps "around" wire

    Depends on current

    Depends on distance

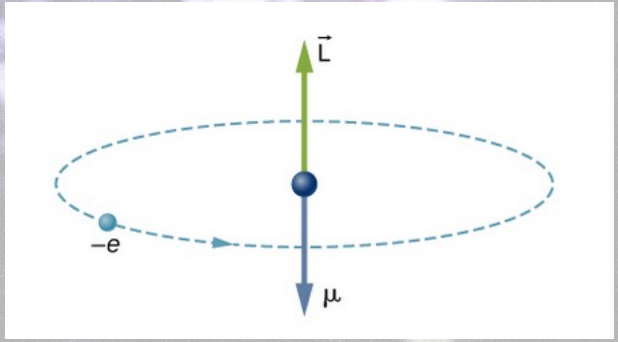


$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{r}$$

https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Book%3A_University_Physics_II_-_Thermodynamics_Electricity_and_Magnetism_(OpenStax)/11%3A_Magnetic_Forces_and_Fields/11.05%3A_Magnetic_Force_on_a_Current-Carrying_Conductor

- Microscopic View
  - Moment of an atom
  - Field is "about" nucleus

    Depends on material



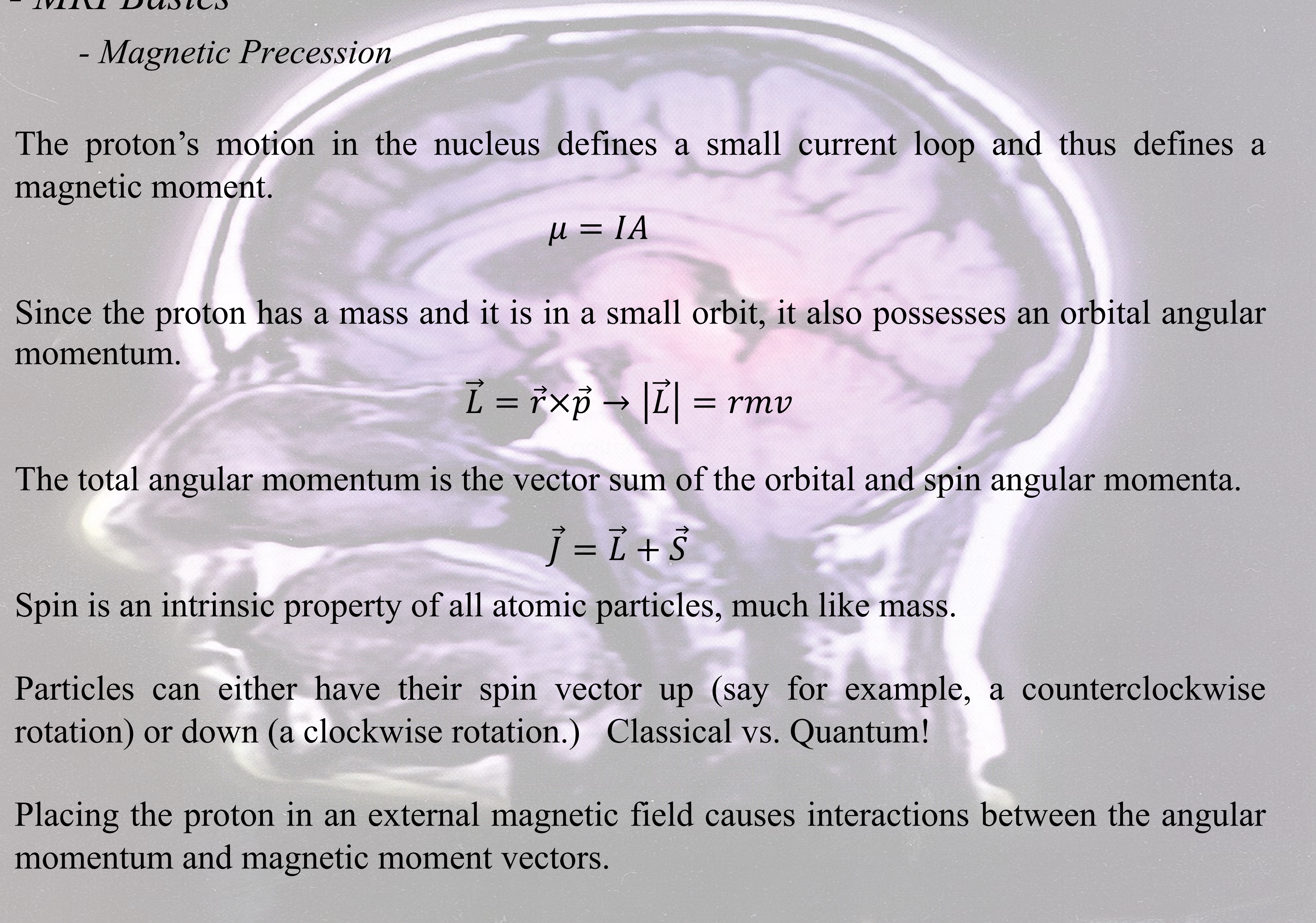
$$\mu = IA$$

https://openstax.org/books/university-physics-volume-3/pages/8-2-orbital-magnetic-dipole-moment-of-the-electron

# Magnetic Resonance Imaging

## - MRI Basics

### - Magnetic Precession



- The proton's motion in the nucleus defines a small current loop and thus defines a magnetic moment.

$$\mu = IA$$

- Since the proton has a mass and it is in a small orbit, it also possesses an orbital angular momentum.

$$\vec{L} = \vec{r} \times \vec{p} \rightarrow \left|\vec{L}\right| = rmv$$

- The total angular momentum is the vector sum of the orbital and spin angular momenta.

$$\vec{J} = \vec{L} + \vec{S}$$

- Spin is an intrinsic property of all atomic particles, much like mass.

- Particles can either have their spin vector up (say for example, a counterclockwise rotation) or down (a clockwise rotation.) Classical vs. Quantum!

- Placing the proton in an external magnetic field causes interactions between the angular momentum and magnetic moment vectors.

# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- Magnetic Precession*

- The electron in orbit defines a current $I$ and the time $T$ it takes the electron (of charge $-e$) to orbit one time about the proton is the orbital period.

- The orbital period $T$ can be related to the speed $v$ of the proton in orbit about the proton.

$$I = \frac{\Delta Q}{\Delta t} = -\frac{e}{T}$$

$$v = \frac{2\pi r}{T}$$

$$\longrightarrow \quad I = -\frac{e}{T} = -\frac{ev}{2\pi r} = -\frac{emv}{2\pi mr}$$

- From this we can calculate the magnitude of the magnetic moment $\mu$ of the electron about the proton.

$$\mu = IA = -\frac{emv}{2\pi mr} \times \pi r^2 = -\left(\frac{e}{2m}\right) rmv = -\left(\frac{e}{2m}\right) L$$

- The magnetic moment $\vec{\mu}$ is directly related to the electron's angular momentum $\vec{L}$ about the proton multiplied by a constant we call the gyromagnetic ratio $\gamma$.

$$\vec{\mu} = -\left(\frac{e}{2m}\right) \vec{L} = -\gamma \vec{L}$$

# Magnetic Resonance Imaging

## - MRI Basics

### - Magnetic Precession

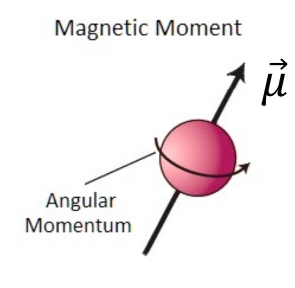
- This is analogous for the proton, except the proton has a positive charge.

$$\vec{\mu} = \gamma \vec{L}$$

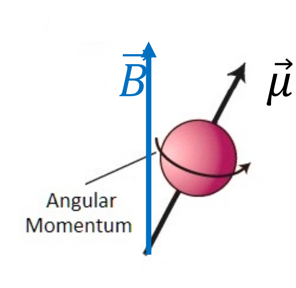
- The interaction of the magnetic moment of the proton and the external magnetic field exerts a torque on the system.

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{\mu} \times \vec{B}$$

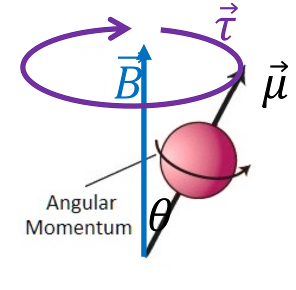
- This torque tries to align the magnetic moment with the external field and results in the magnetic moment precessing about the external magnetic field.



https://my-ms.org/mri_physics.htm



https://my-ms.org/mri_physics.htm



https://my-ms.org/mri_physics.htm

$$\vec{\tau} = \langle \mu_x, 0, \mu_z \rangle \times \langle 0,0,B_z \rangle = \langle 0, -\mu_x B_z, 0 \rangle \rightarrow \tau = \mu B \sin\theta$$

# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- RF Photon Energy, Absorption, Emission and Spin*

When the magnetic moment interacts with the external magnetic field the system experiences a torque.

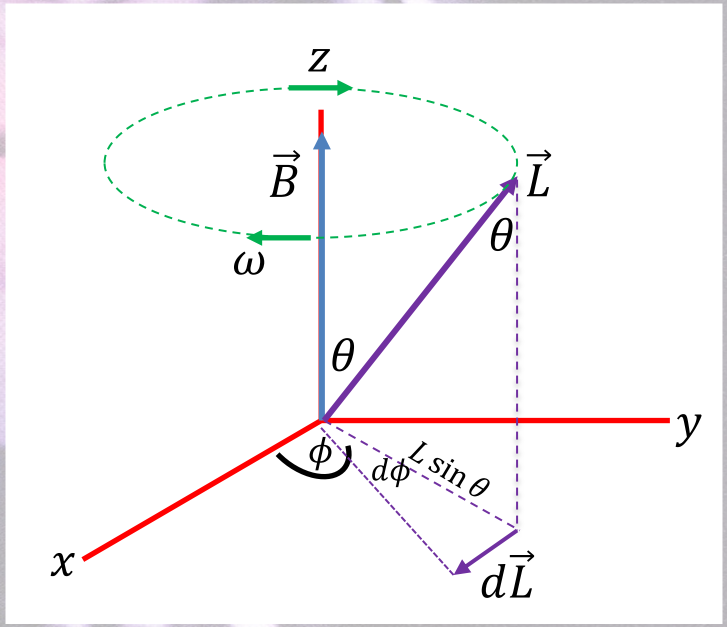
$$\vec{\tau} = \vec{\mu}\times\vec{B} = \frac{d\vec{L}}{dt} = \gamma\vec{L}\times\vec{B}$$

$$\sin d\phi \sim \tan d\phi \sim d\phi = \frac{dL}{L\sin\theta} = \frac{\tau dt}{L\sin\theta}$$

$$d\phi = \frac{\tau dt}{L\sin\theta}$$

$$d\phi = \frac{\mu\sin\theta\, B dt}{\frac{\mu}{\gamma}\sin\theta} \longrightarrow \omega = \frac{d\phi}{dt} = \gamma B$$

This is the the Larmor or precession frequency.



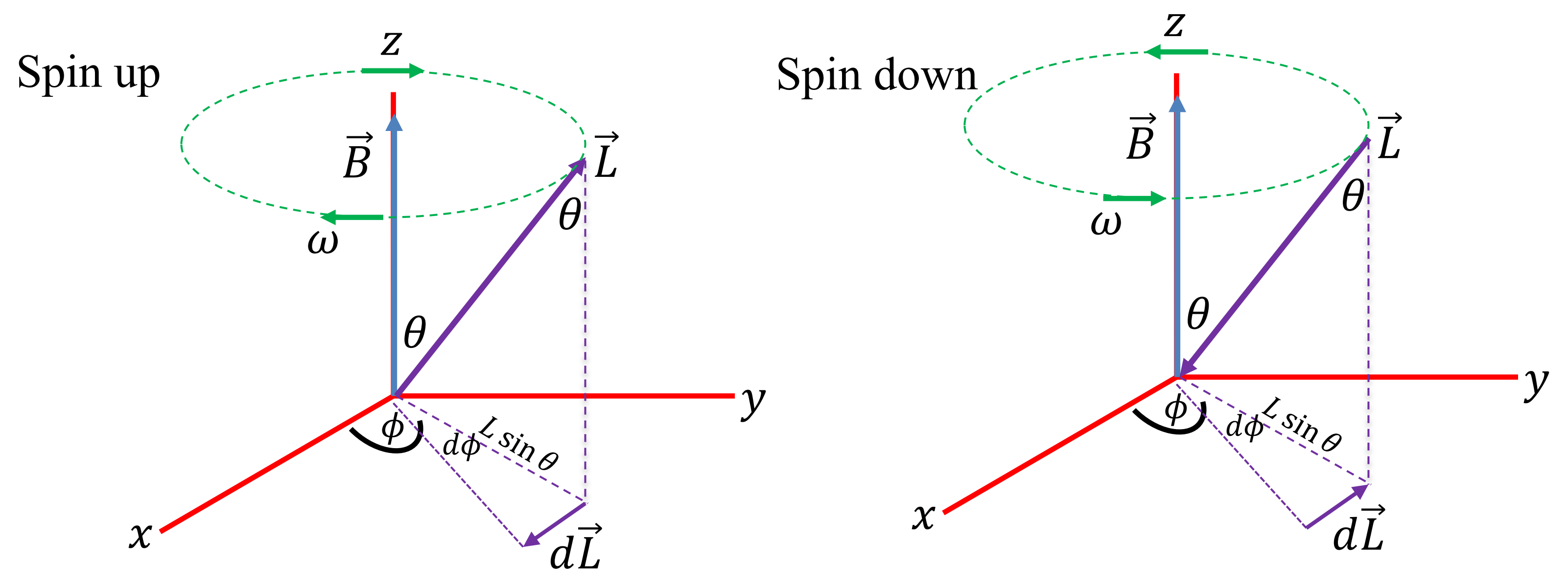
Experimentally it’s seen that: $\gamma = \frac{5.59e}{2m}$

This considers the effects of spin on the system which doesn’t have a classical analogue and the fact that the proton not a point particle

# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- Static magnetic field*



- In the presence of a static field the magnetic moment tries to align (or anti-align) with the applied field.
- If the magnetic moment is aligned *exactly* with the applied field, then the proton does not precess.
- If the magnetic moment is *not aligned exactly* with the field, then the proton precesses about the static field.
- There are high and low energy states that are separated out by the static field.

# Magnetic Resonance Imaging

## - MRI Basics

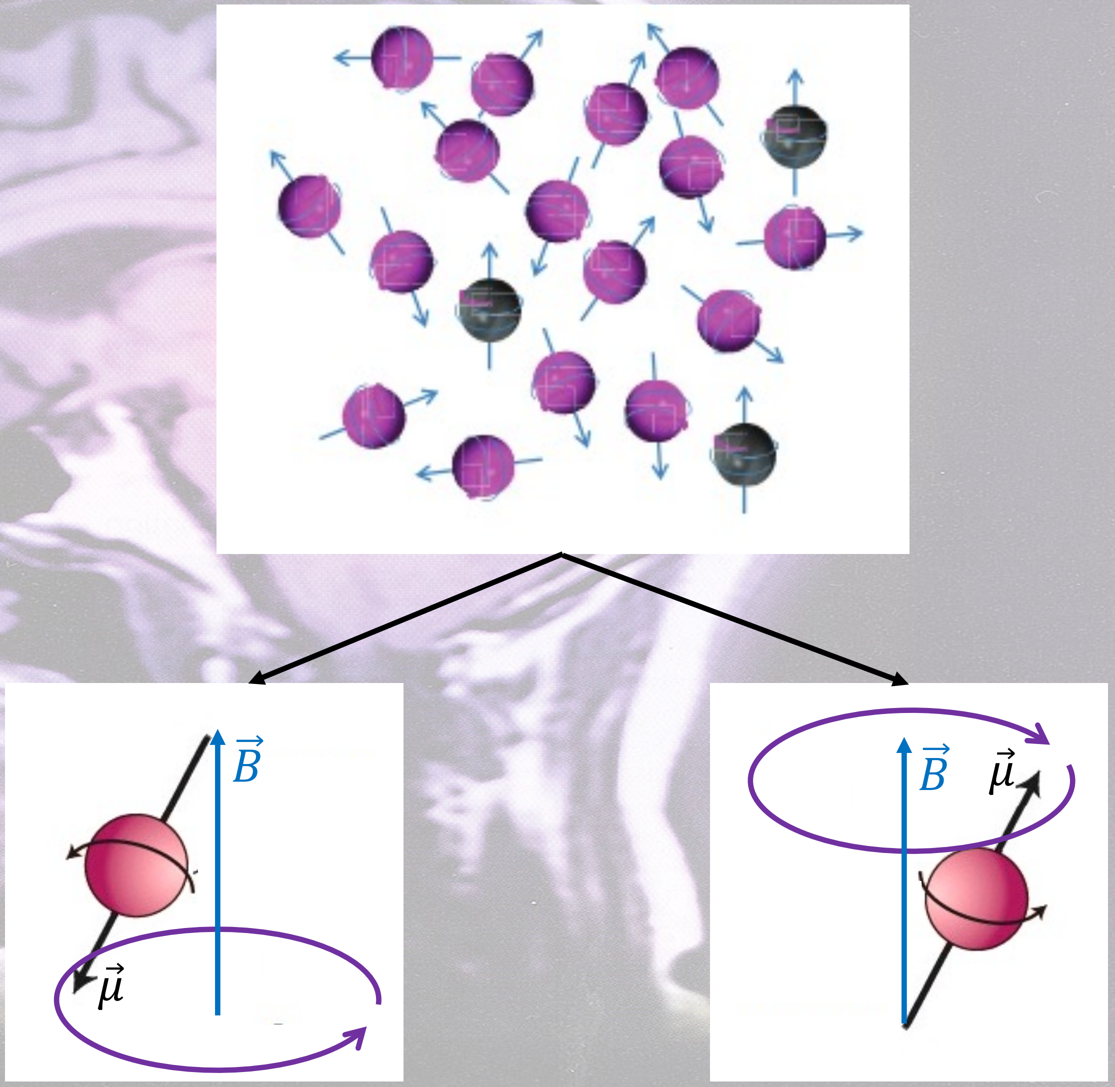
### - Magnetic Precession

- A collection of nuclei has their magnetic moments pointing in all random directions.

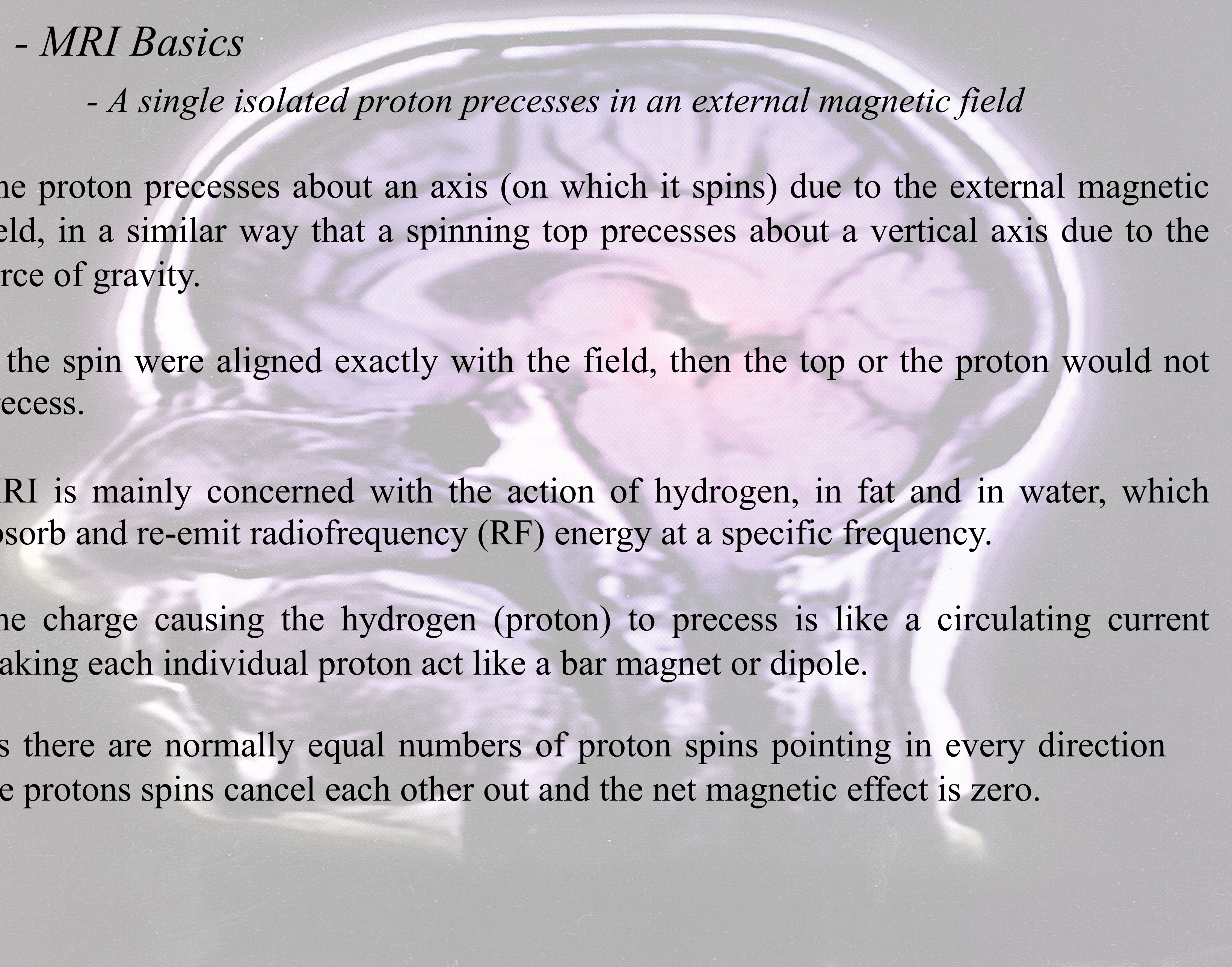
- When an external magnetic field is turned on, the magnetic moments experience a torque and try to either align or anti-align with the external magnetic field.

- This separates the set into two spin-states, spin up (say more aligned with the external magnetic field) and spin down (more anti-aligned).

- The net torque causes a precession of the magnetic moment about the external magnetic field.



https://my-ms.org/mri_physics.htm



# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- A single isolated proton precesses in an external magnetic field*

- The proton precesses about an axis (on which it spins) due to the external magnetic field, in a similar way that a spinning top precesses about a vertical axis due to the force of gravity.

- If the spin were aligned exactly with the field, then the top or the proton would not precess.

- MRI is mainly concerned with the action of hydrogen, in fat and in water, which absorb and re-emit radiofrequency (RF) energy at a specific frequency.

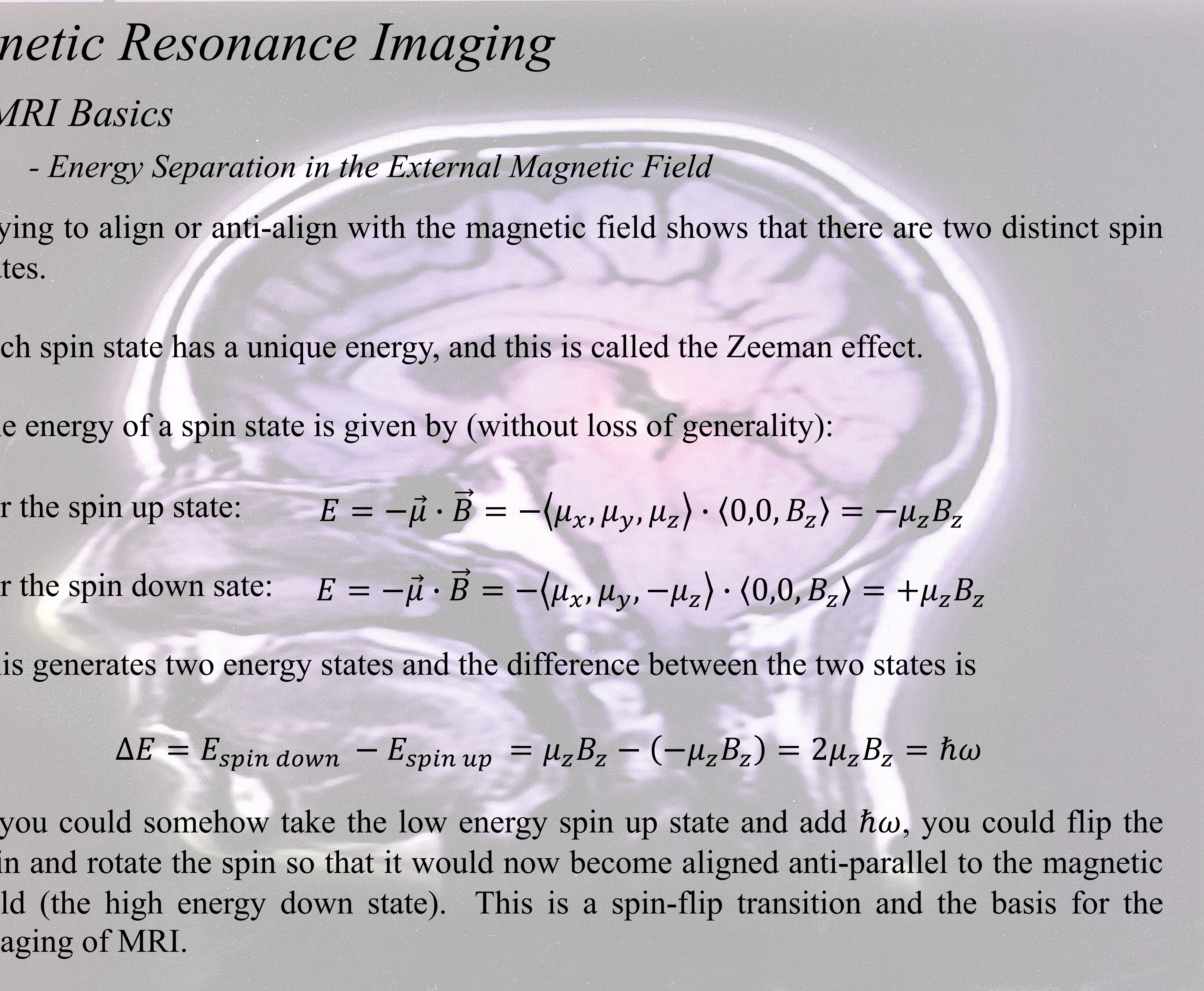
- The charge causing the hydrogen (proton) to precess is like a circulating current making each individual proton act like a bar magnet or dipole.

- As there are normally equal numbers of proton spins pointing in every direction the protons spins cancel each other out and the net magnetic effect is zero.

# Magnetic Resonance Imaging

## - MRI Basics

### - Energy Separation in the External Magnetic Field



- Trying to align or anti-align with the magnetic field shows that there are two distinct spin states.

- Each spin state has a unique energy, and this is called the Zeeman effect.

- The energy of a spin state is given by (without loss of generality):

- For the spin up state: $E = -\vec{\mu} \cdot \vec{B} = -\langle \mu_x, \mu_y, \mu_z \rangle \cdot \langle 0,0,B_z \rangle = -\mu_z B_z$

- For the spin down sate: $E = -\vec{\mu} \cdot \vec{B} = -\langle \mu_x, \mu_y, -\mu_z \rangle \cdot \langle 0,0,B_z \rangle = +\mu_z B_z$

- This generates two energy states and the difference between the two states is

$$\Delta E = E_{spin\ down} - E_{spin\ up} = \mu_z B_z - (-\mu_z B_z) = 2\mu_z B_z = \hbar\omega$$

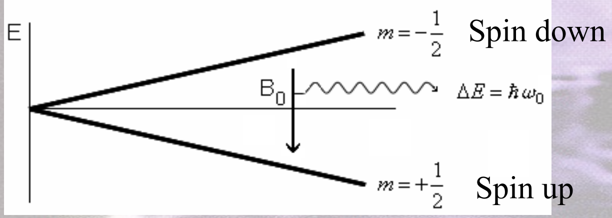
- If you could somehow take the low energy spin up state and add $\hbar\omega$, you could flip the spin and rotate the spin so that it would now become aligned anti-parallel to the magnetic field (the high energy down state). This is a spin-flip transition and the basis for the imaging of MRI.

# *Magnetic Resonance Imaging*

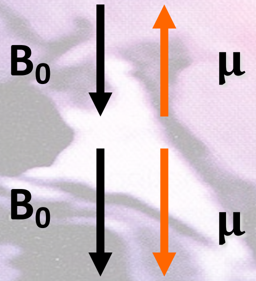
## *- MRI Basics*

### *- RF Photon Energy, Absorption, Emission and Spin*

- The static field "splits" the states in to high and low
  - This is called the Zeeman effect.

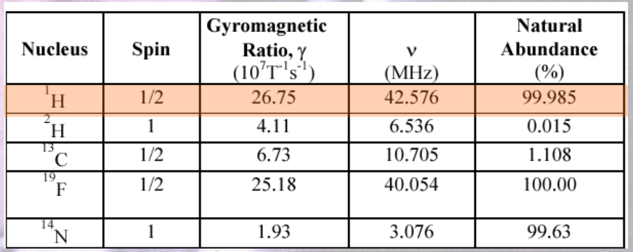


**An Introduction to MRI Physics and Analysis Michael Jay Schillaci, PhD**



- To influence the down energy state to change its spin orientation, another field, called the transverse RF Field (or oscillating field), is needed to perturb and "rotate" the magnetic moments of the protons away from aligned with to anti-aligned with the external magnetic field.

*$B_0 = 1.0T$*



| **Nucleus** | **Spin** | **Gyromagnetic Ratio, γ** ($10^7 T^{-1} s^{-1}$) | **ν** (MHz) | **Natural Abundance** (%) |
|---|---|---|---|---|
| $^{1}H$ | 1/2 | 26.75 | 42.576 | 99.985 |
| $^{2}H$ | 1 | 4.11 | 6.536 | 0.015 |
| $^{13}C$ | 1/2 | 6.73 | 10.705 | 1.108 |
| $^{19}F$ | 1/2 | 25.18 | 40.054 | 100.00 |
| $^{14}N$ | 1 | 1.93 | 3.076 | 99.63 |

- For comparison: In the Earth's magnetic field (0.00005 T ), hydrogen precesses at ~2100 Hz.

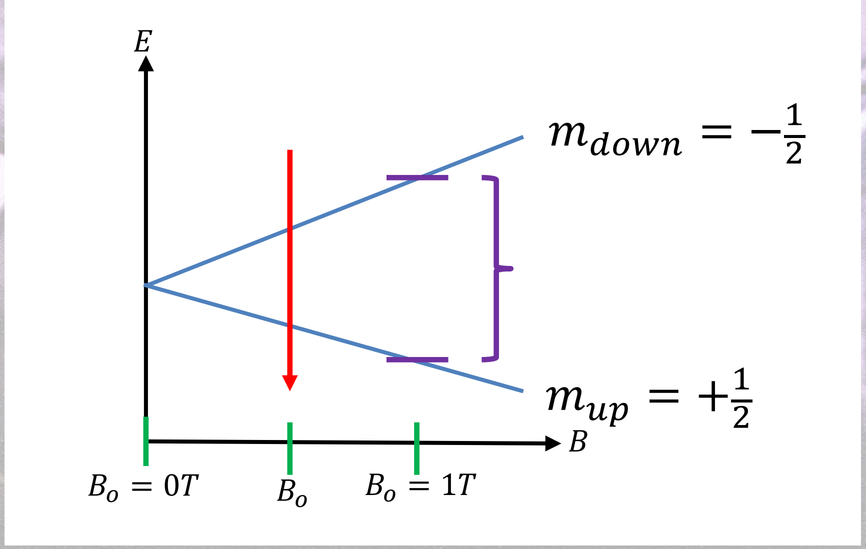
# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- Static magnetic field*

- MRI scanners are generational, meaning that the size of the static magnetic field depends on when the scanner was produced. Suppose you have a first-generation scanner that's operating with an axial static external magnetic field of $1T$. What is the Larmor frequency of protons?

- What is the Larmor frequency if the static field is $3T$ (a 3rd generation scanner)?

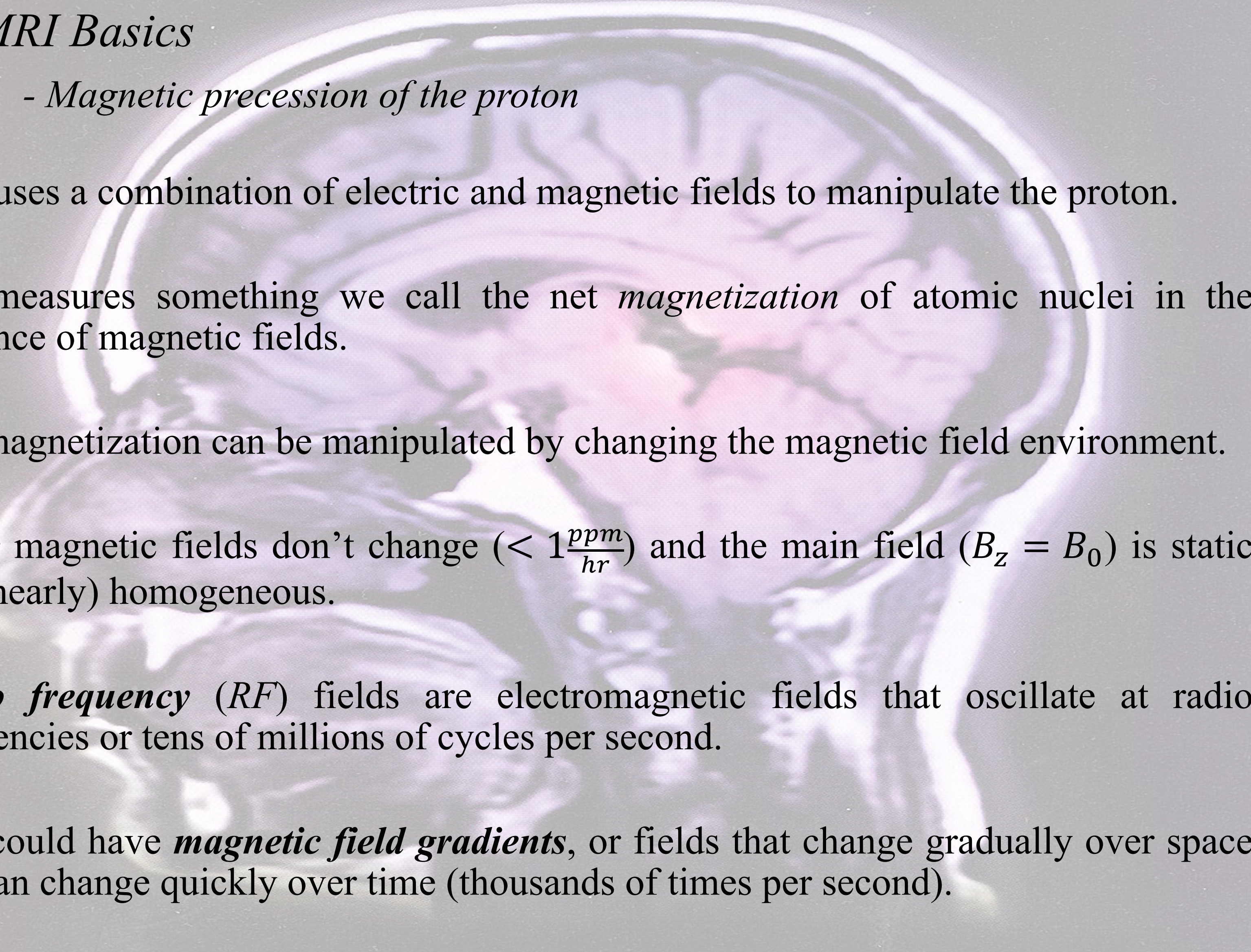


- This works ok for one proton, say. But how do you track a collection of protons?

# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- Magnetic precession of the proton*

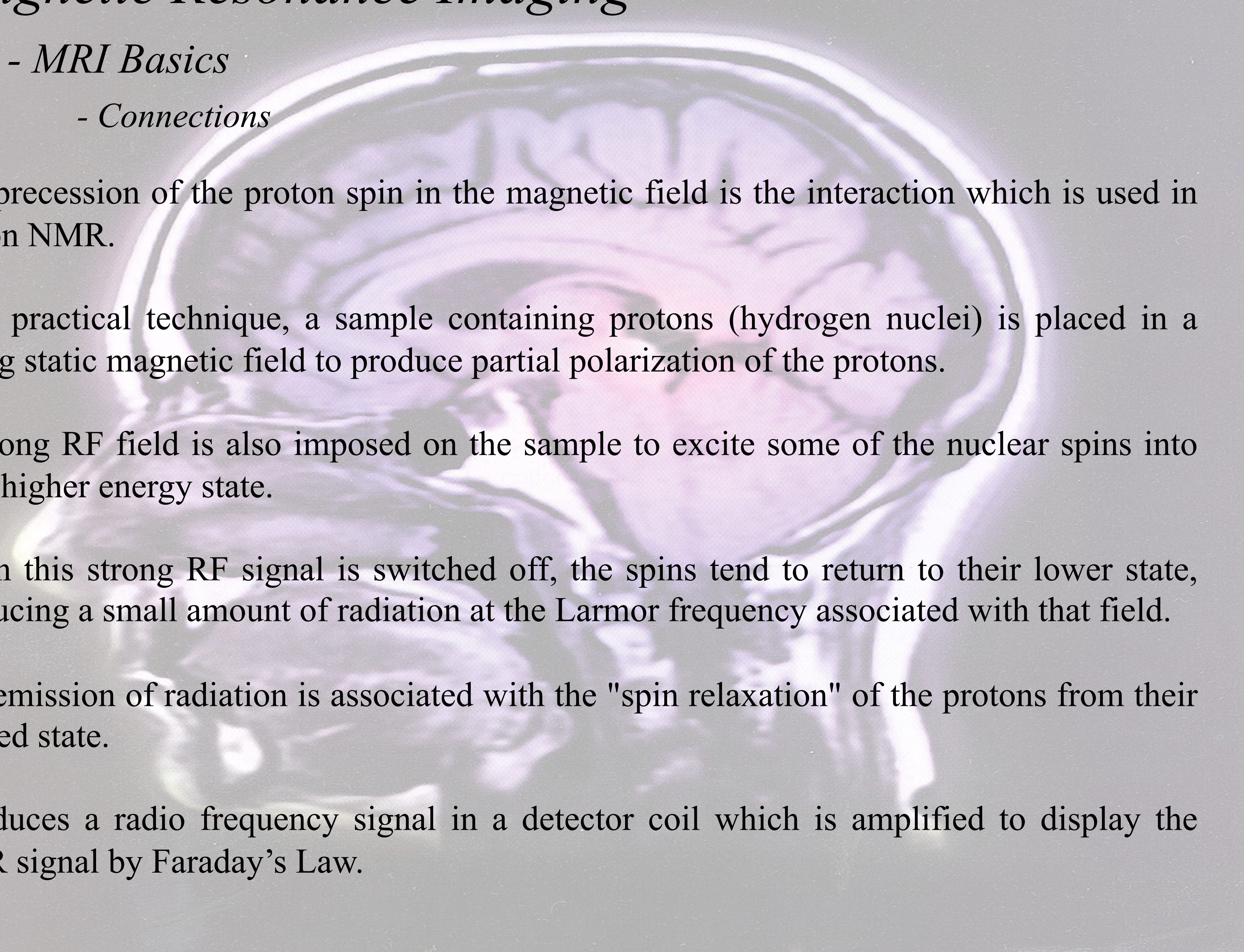


- MRI uses a combination of electric and magnetic fields to manipulate the proton.
- MR measures something we call the net *magnetization* of atomic nuclei in the presence of magnetic fields.
- The magnetization can be manipulated by changing the magnetic field environment.
- ***Static*** magnetic fields don't change ($< 1\frac{ppm}{hr}$) and the main field ($B_z = B_0$) is static and (nearly) homogeneous.
- ***Radio frequency*** (*RF*) fields are electromagnetic fields that oscillate at radio frequencies or tens of millions of cycles per second.
- You could have ***magnetic field gradients***, or fields that change gradually over space and can change quickly over time (thousands of times per second).

# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- Connections*



- The precession of the proton spin in the magnetic field is the interaction which is used in proton NMR.

- As a practical technique, a sample containing protons (hydrogen nuclei) is placed in a strong static magnetic field to produce partial polarization of the protons.

- A strong RF field is also imposed on the sample to excite some of the nuclear spins into their higher energy state.

- When this strong RF signal is switched off, the spins tend to return to their lower state, producing a small amount of radiation at the Larmor frequency associated with that field.

- The emission of radiation is associated with the "spin relaxation" of the protons from their excited state.

- It induces a radio frequency signal in a detector coil which is amplified to display the NMR signal by Faraday’s Law.

# *Magnetic Resonance Imaging*

## *- MRI Basics*

### *- RF Photon Energy, Absorption, Emission and Spin*

- Quantum Mechanics governs state transitions
  - Energy of transition

$$\Delta E = \hbar\omega$$

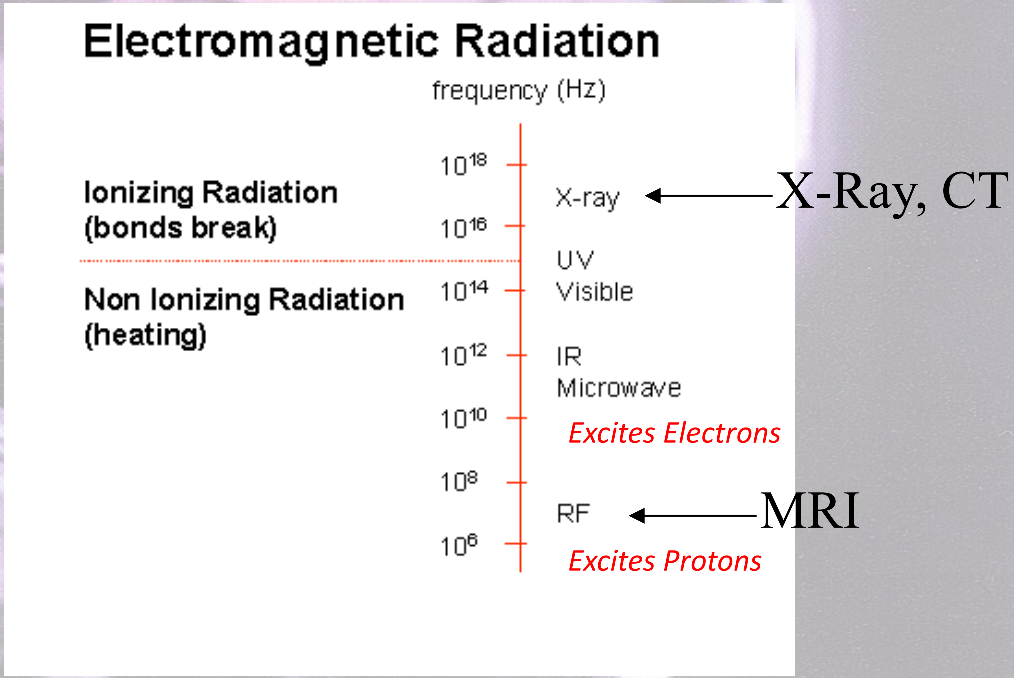
  - Planck's constant

$$h = 4.1357{\times}10^{15} eV \cdot s$$

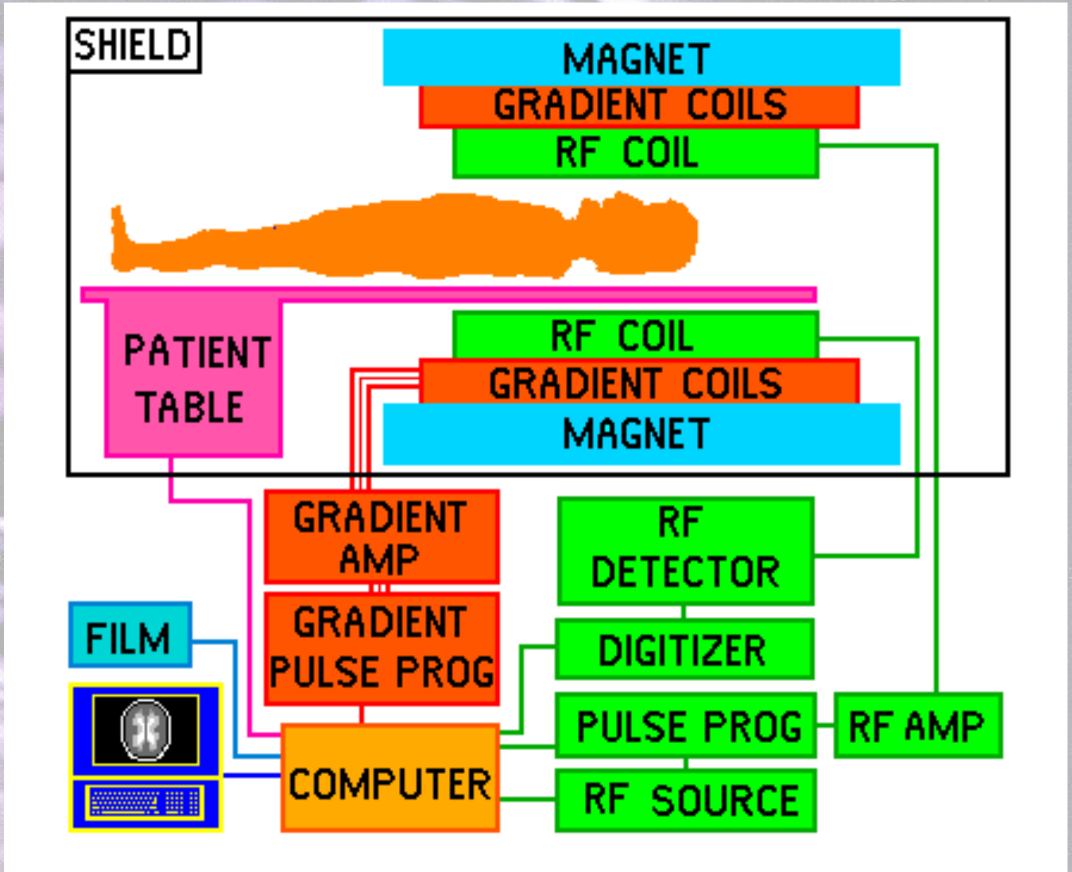
  - Energy values

$$E_{x-ray} > 100's\ eV$$

$$E_{MRI} \sim \frac{1}{10}\mu eV$$

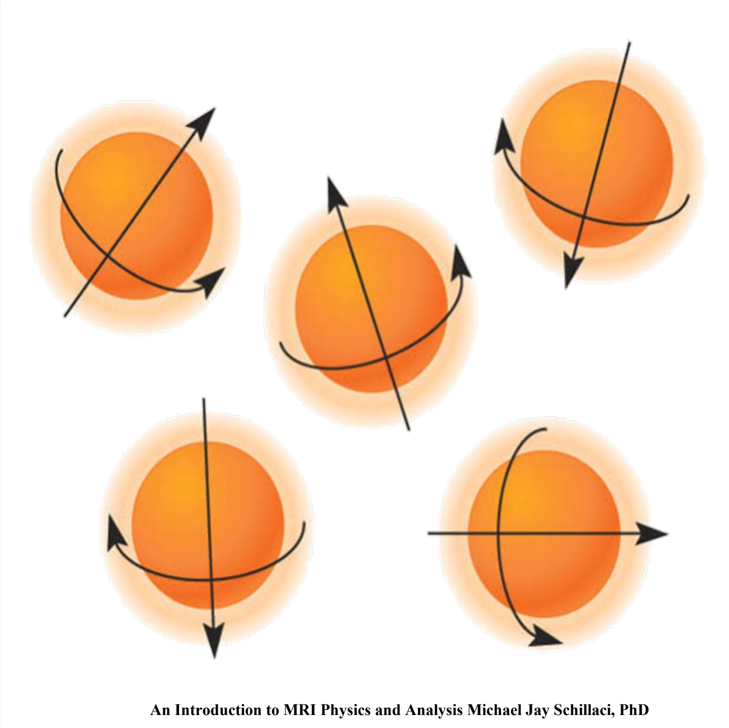


# *MRI Scanner*



An Introduction to MRI Physics and Analysis Michael Jay Schillaci, PhD

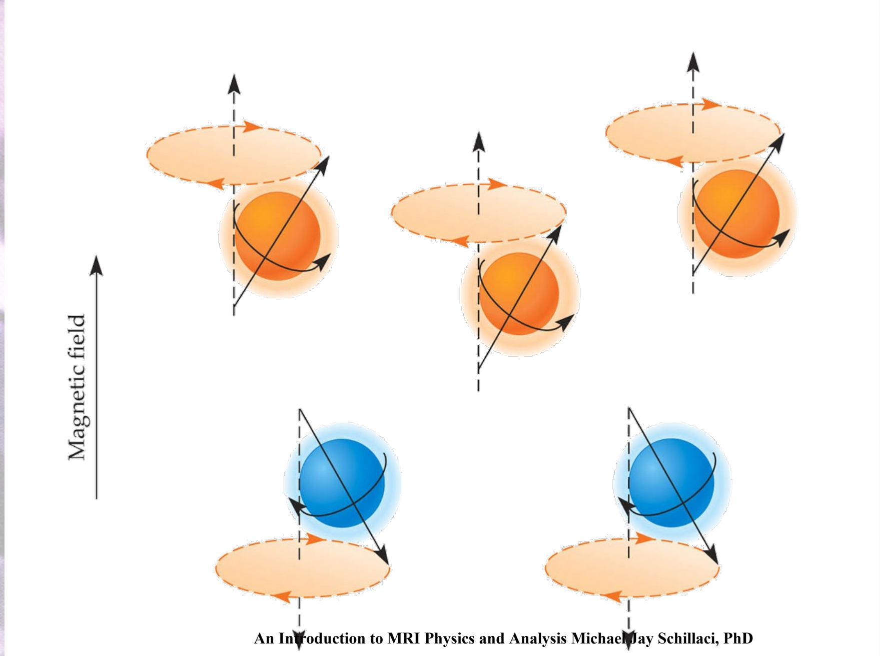
# *Magnetization of a Collection of Protons*



An Introduction to MRI Physics and Analysis Michael Jay Schillaci, PhD

- In the absence of a strong magnetic field, the spins are oriented randomly.
- Thus, there is no net magnetization ($M$).

In a magnetic field, protons can take either *high*- or low-energy states.

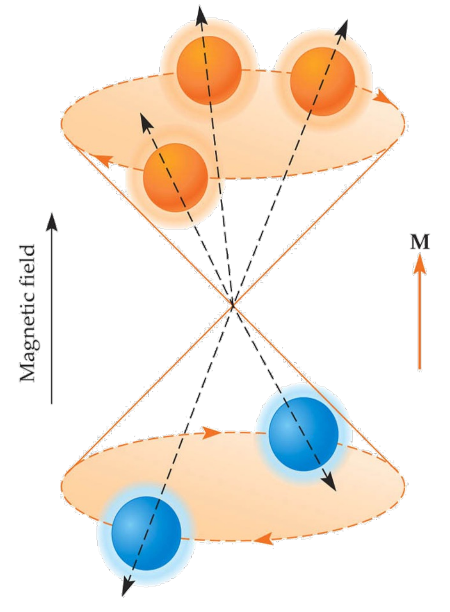


An Introduction to MRI Physics and Analysis Michael Jay Schillaci, PhD

FUNCTIONAL MAGNETIC RESONANCE IMAGING, Figure 3.5 © 2004 Sinauer Associates, Inc.

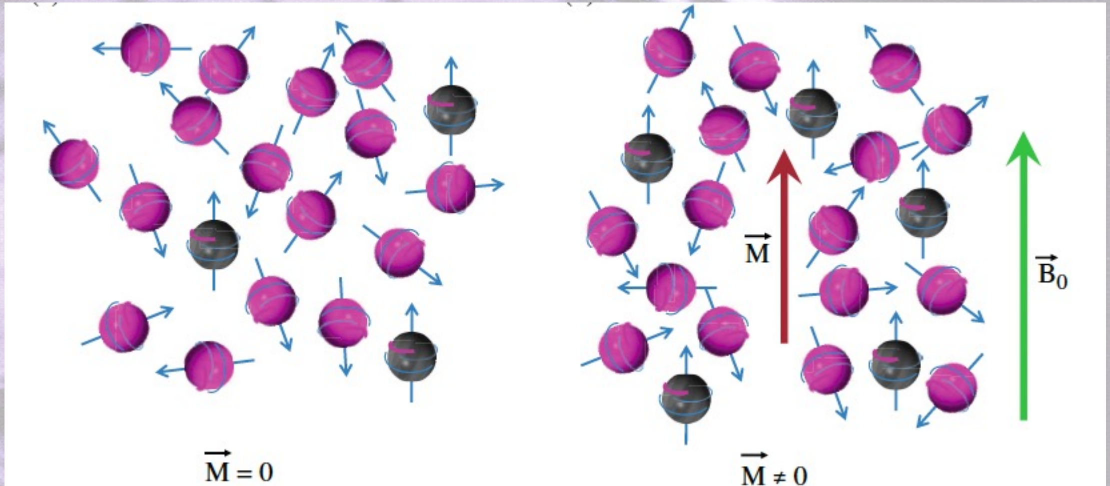
- There is now a net magnetization ($M$).
- Protons all precessing out of phase.

# *Magnetization of a Collection of Protons*



FUNCTIONAL MAGNETIC RESONANCE IMAGING, Figure 3.7 © 2004 Sinauer Associates, Inc.

**An Introduction to MRI Physics and Analysis Michael Jay Schillaci, PhD**

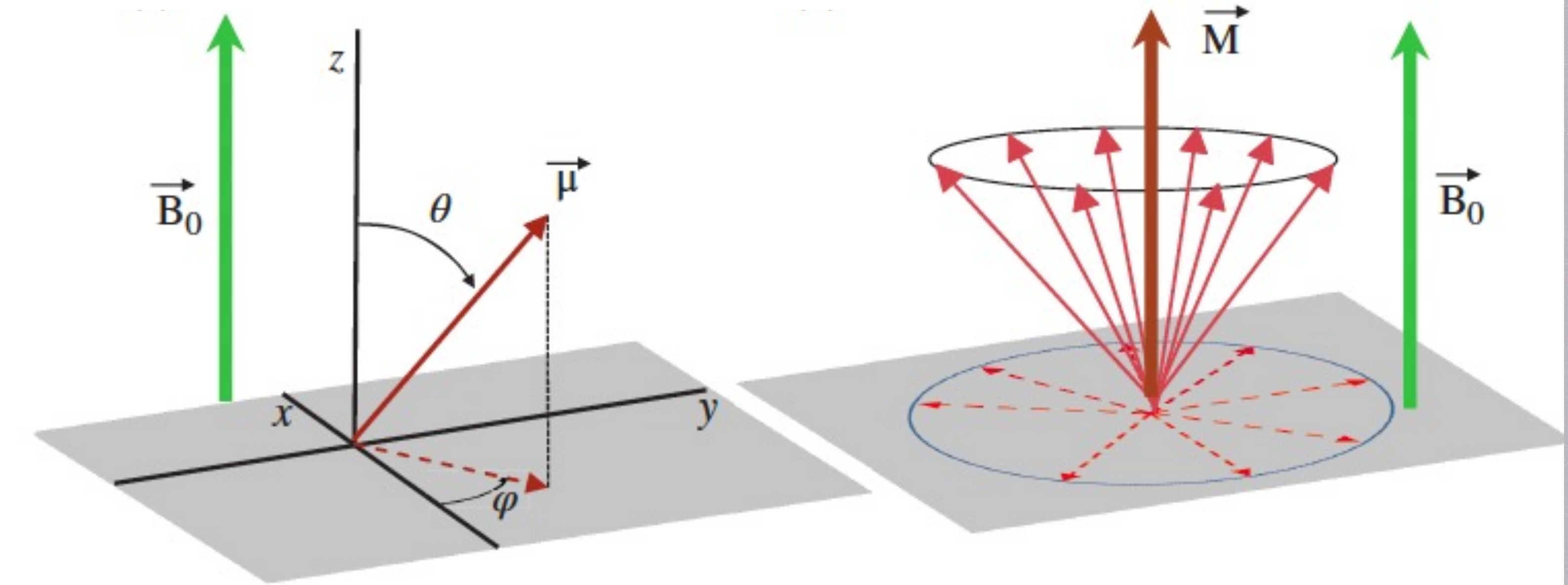


Hendee's Physics of Medical Imaging, Peck D., Samei, E., 5h Edition, John Wiley & Sons, 2019.

$$\omega = 2\pi f = \gamma B_{ext}$$

- The difference between the numbers of protons in the high-energy and low-energy states results in a *net magnetization* ($\vec{M}$) and the system preceses at the Lamour frequency.
- The net magnetization points along the static magnetic field and is defined as the magnetic moment per unit volume of space.

# *Magnetization of a Collection of Protons*



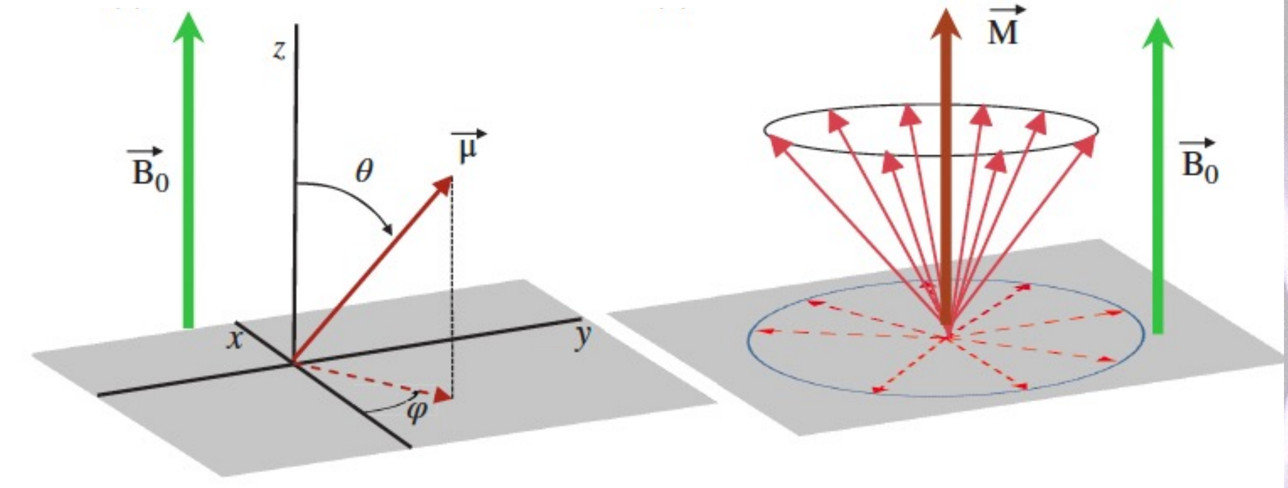
Hendee's Physics of Medical Imaging, Peck D., Samei, E., 5h Edition, John Wiley & Sons, 2019.

- We define the net magnetization to point in the direction where there are more proton spins pointing.

- This is in the direction of the low energy state.

- Since most of the protons want to align with the external magnetic field, The magnetization will initially point in this direction, aligned with the external magnetic field.

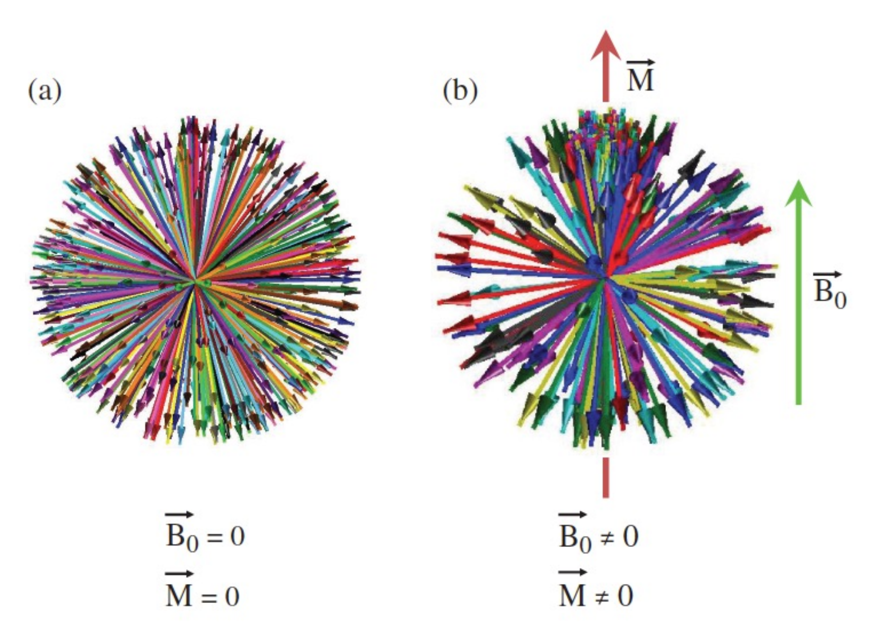
# *Magnetization of a Collection of Protons and the Origin of the MR Signal*



Hendee's Physics of Medical Imaging, Peck D., Samei, E., 5h Edition, John Wiley & Sons, 2019.

- Although out of phase, most proton spins are aligned with the magnetic field in their lowest energy state.
- The x- and y-components of the angular momenta of a large quantity of protons tends to cancel and the only remining component ($L_z$ or $\mu_z$) points along the external magnetic field.
- Since one proton's magnetic moment precesses about the magnetic field, the net number of proton spins in a particular direction will also precess about the external field.

# *Magnetization of a Collection of Protons and the Origin of the MR Signal*



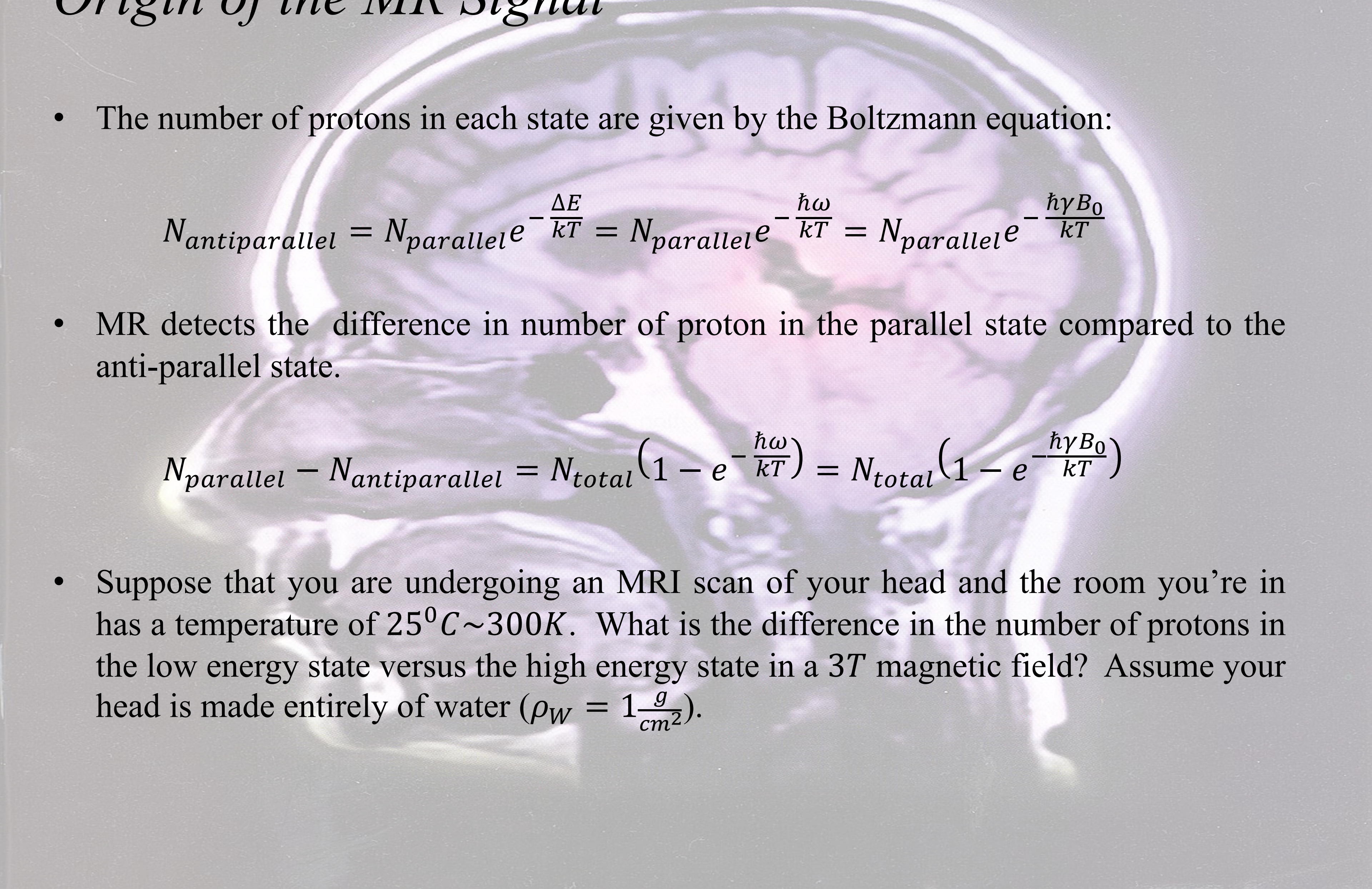
Hendee’s Physics of Medical Imaging, Peck D., Samei, E., 5h Edition, John Wiley & Sons, 2019.

- In the absence of any external magnetic field the magnetization is zero.

- When the static external magnetic field is turned on, the magnetization grows along the external field and becomes fixed.

- Since the external magnetic field is static, the magnetization is static.

# *Magnetization of a Collection of Protons and the Origin of the MR Signal*



- The number of protons in each state are given by the Boltzmann equation:

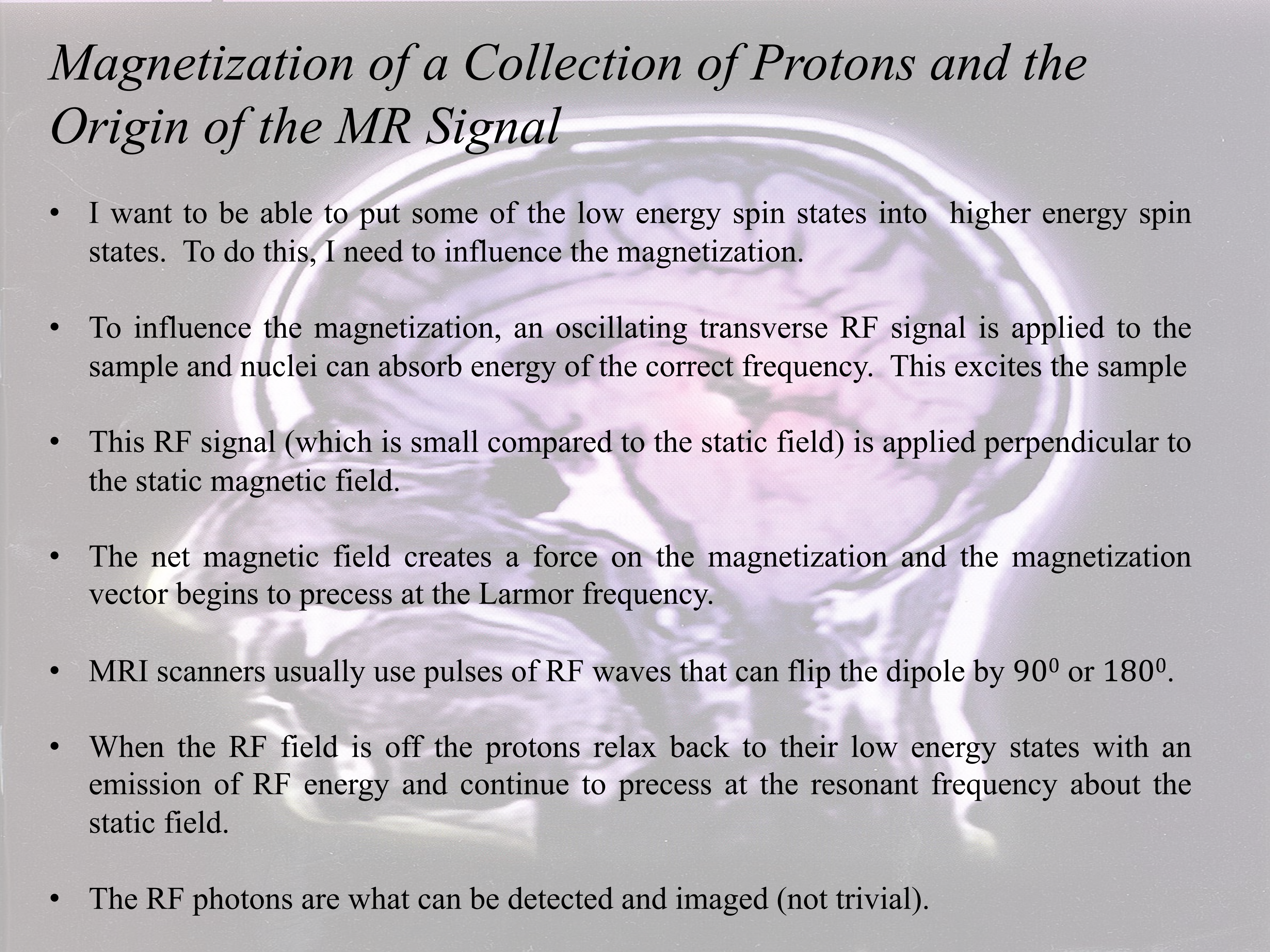
$$N_{antiparallel} = N_{parallel}e^{-\frac{\Delta E}{kT}} = N_{parallel}e^{-\frac{\hbar\omega}{kT}} = N_{parallel}e^{-\frac{\hbar\gamma B_0}{kT}}$$

- MR detects the difference in number of proton in the parallel state compared to the anti-parallel state.

$$N_{parallel} - N_{antiparallel} = N_{total}\left(1 - e^{-\frac{\hbar\omega}{kT}}\right) = N_{total}\left(1 - e^{-\frac{\hbar\gamma B_0}{kT}}\right)$$

- Suppose that you are undergoing an MRI scan of your head and the room you're in has a temperature of $25^0C \sim 300K$. What is the difference in the number of protons in the low energy state versus the high energy state in a $3T$ magnetic field? Assume your head is made entirely of water ($\rho_W = 1\frac{g}{cm^2}$).

# *Magnetization of a Collection of Protons and the Origin of the MR Signal*



- I want to be able to put some of the low energy spin states into higher energy spin states. To do this, I need to influence the magnetization.

- To influence the magnetization, an oscillating transverse RF signal is applied to the sample and nuclei can absorb energy of the correct frequency. This excites the sample

- This RF signal (which is small compared to the static field) is applied perpendicular to the static magnetic field.

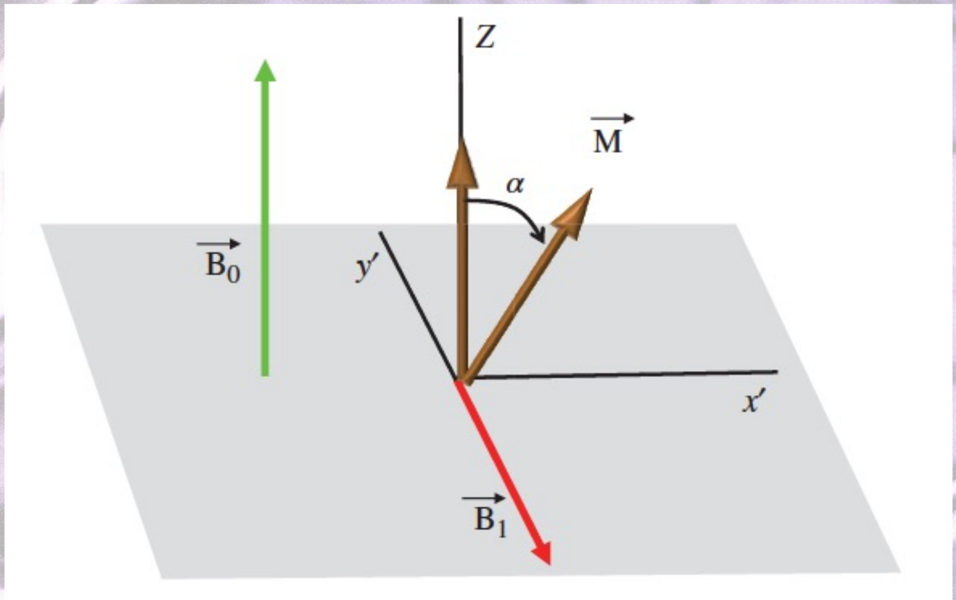
- The net magnetic field creates a force on the magnetization and the magnetization vector begins to precess at the Larmor frequency.

- MRI scanners usually use pulses of RF waves that can flip the dipole by $90^0$ or $180^0$.

- When the RF field is off the protons relax back to their low energy states with an emission of RF energy and continue to precess at the resonant frequency about the static field.

- The RF photons are what can be detected and imaged (not trivial).

# *Magnetization of a Collection of Protons and the Origin of the MR Signal*

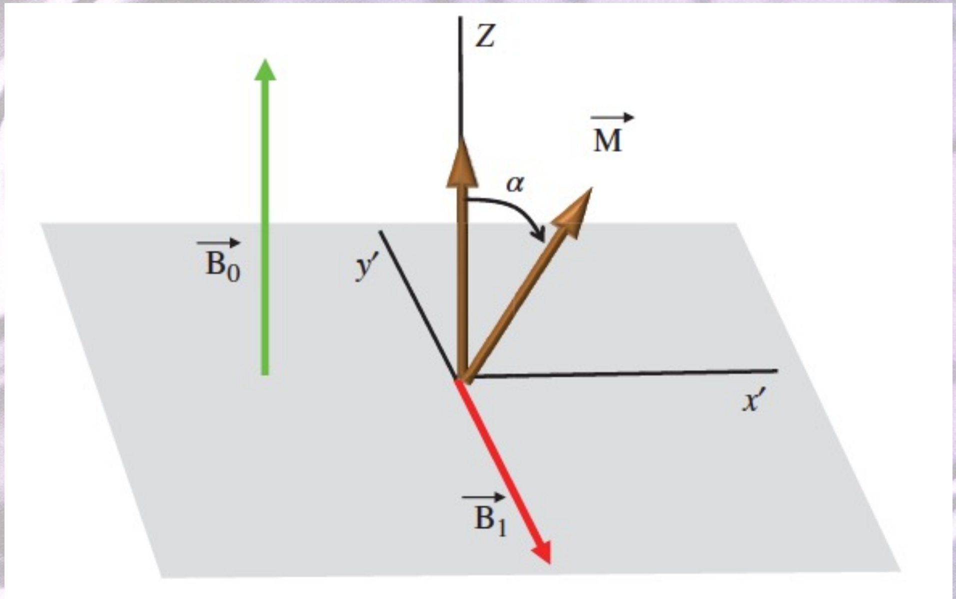


Hendee’s Physics of Medical Imaging, Peck D., Samei, E., 5h Edition, John Wiley & Sons, 2019.

- To influence the collection of proton spins and to get the low energy proton spin state to move to the high energy spin state we need to apply an external magnetic field.

- This rotates the magnetization vector away from the external magnetic field direction by an amount ∝.

# *Magnetization of a Collection of Protons and the Origin of the MR Signal*

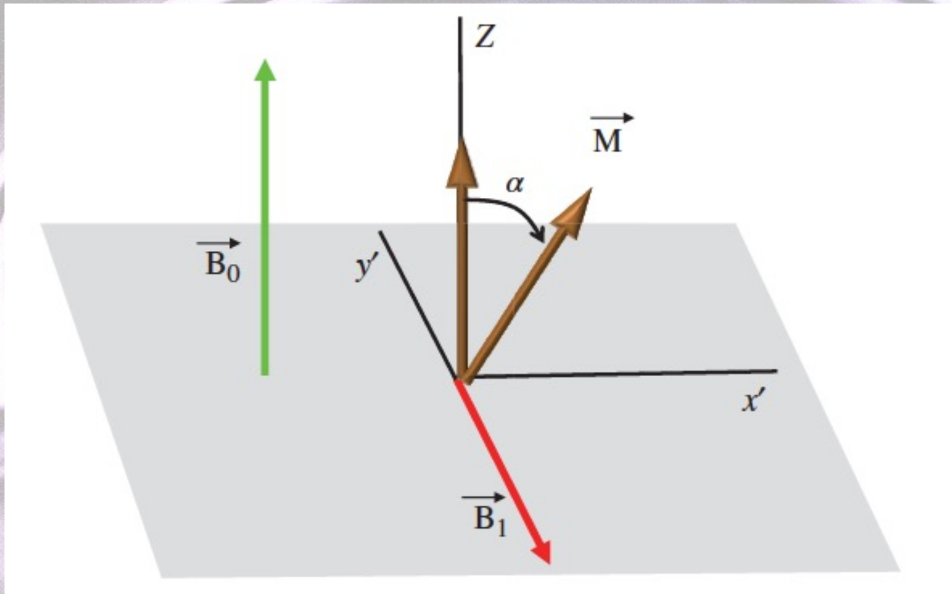


Hendee's Physics of Medical Imaging, Peck D., Samei, E., 5h Edition, John Wiley & Sons, 2019.

- Since the magnetization represents the net collection of magnetic moments and we've tipped the magnetization away from the z-axis, this reintroduces an x- and y-component to the magnetic moments of the precessing protons.

- The net effect is to make the magnetization vector precess about the external magnetic field at the Larmor frequency. You can think of the magnetization vector as being the net magnetic moment of the collection of spins.

# *Magnetization of a Collection of Protons and the Origin of the MR Signal*



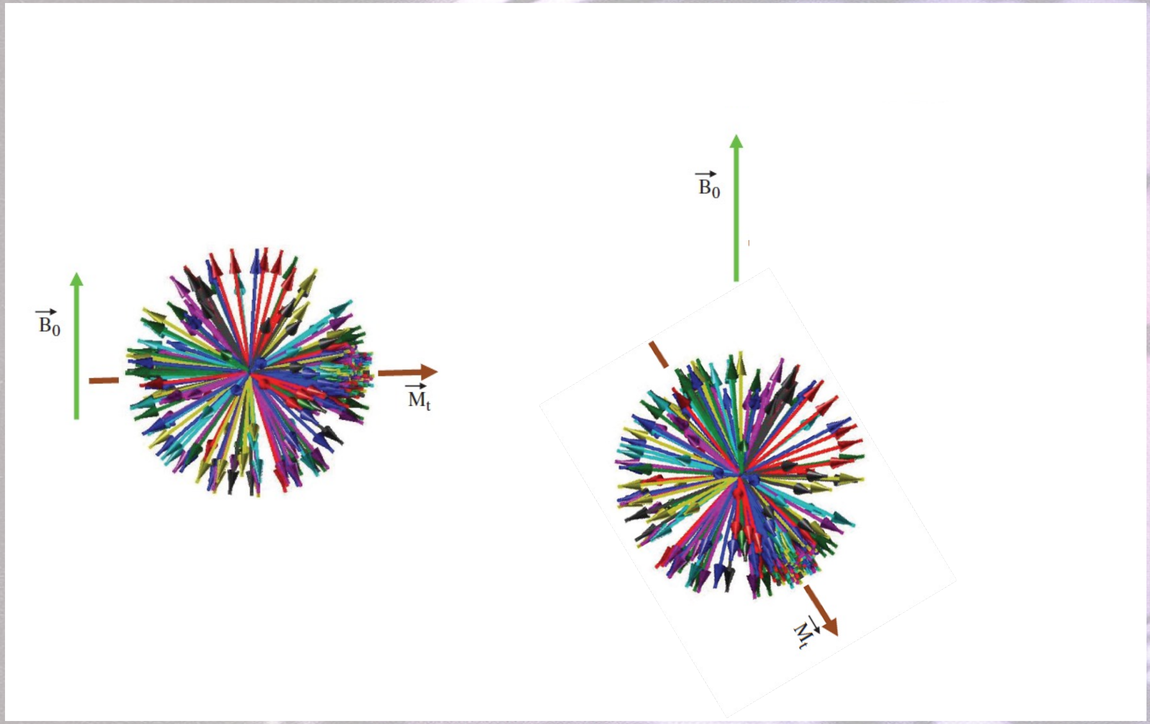
Hendee's Physics of Medical Imaging, Peck D., Samei, E., 5h Edition, John Wiley & Sons, 2019.

- Since the magnetization represents the net collection of magnetic moments in the presence of the external magnetic field, we've caused the magnetization vector to precess about the external magnetic field *AND* we've tipped it over by, say, $90^0$ or even by $180^0$.

- We've taken the low energy spin states and put them into the high energy spin states.

- The tip angle $\alpha$, is defined by $\alpha = \gamma B_1 \tau_{B_1}$, where $\tau_{B_1}$ is the time the RF field is applied.

# *Magnetization of a Collection of Protons and the Origin of the MR Signal*

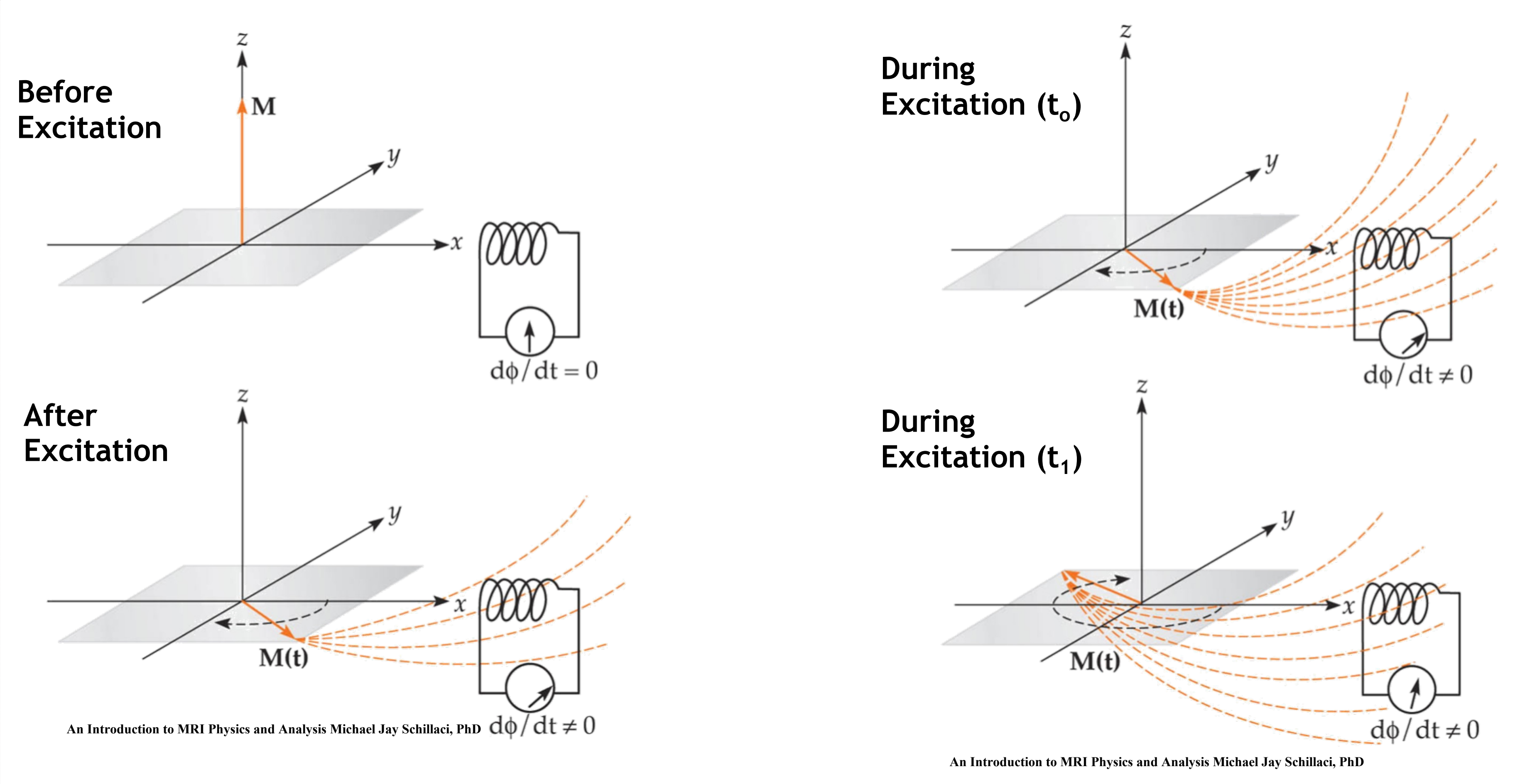


Hendee’s Physics of Medical Imaging, Peck D., Samei, E., 5h Edition, John Wiley & Sons, 2019.

- Rotation of the magnetization by $90^0$ or even by $180^0$.
- We’ve taken the low energy spin states and put them into the high energy spin state.

- Suppose that a typical value of the RF magnetic field were $B_1 = 30\mu T$. How long would you have to apply this field to rotate the magnetization by $90^0 = \frac{\pi}{2} rad$?

# *Origin of the MR Signal*

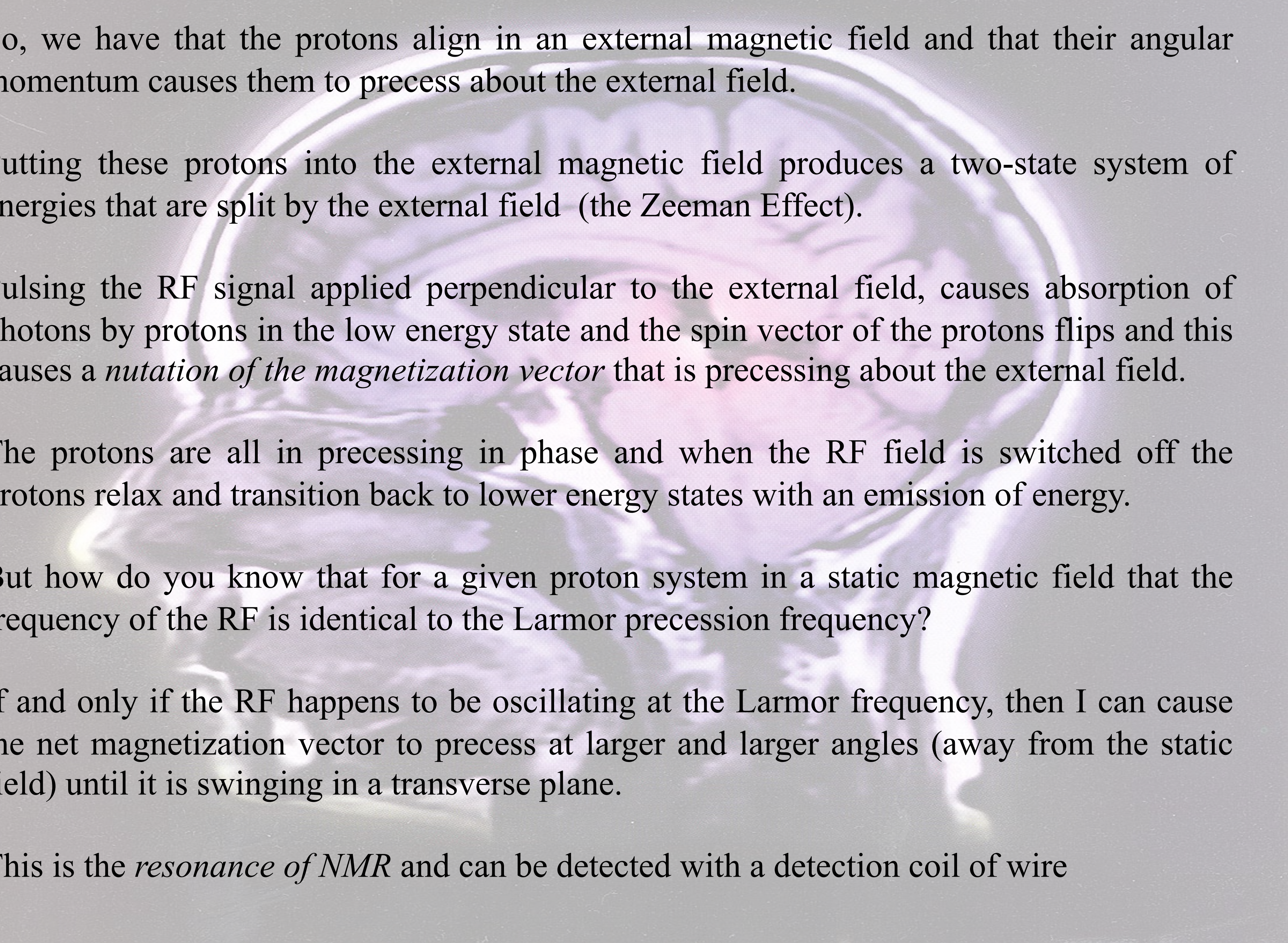


An Introduction to MRI Physics and Analysis Michael Jay Schillaci, PhD

An Introduction to MRI Physics and Analysis Michael Jay Schillaci, PhD

- Excitation (or the absorption of a RF photon) tips the net magnetization (M) down into the transverse plane, where it can generate current in detector coils (i.e., via induction).
- The amount of current oscillates at the (Larmor) frequency of the net magnetization.

# *Origin of the MRI Signal*



- So, we have that the protons align in an external magnetic field and that their angular momentum causes them to precess about the external field.

- Putting these protons into the external magnetic field produces a two-state system of energies that are split by the external field (the Zeeman Effect).

- Pulsing the RF signal applied perpendicular to the external field, causes absorption of photons by protons in the low energy state and the spin vector of the protons flips and this causes a *nutation of the magnetization vector* that is precessing about the external field.

- The protons are all in precessing in phase and when the RF field is switched off the protons relax and transition back to lower energy states with an emission of energy.

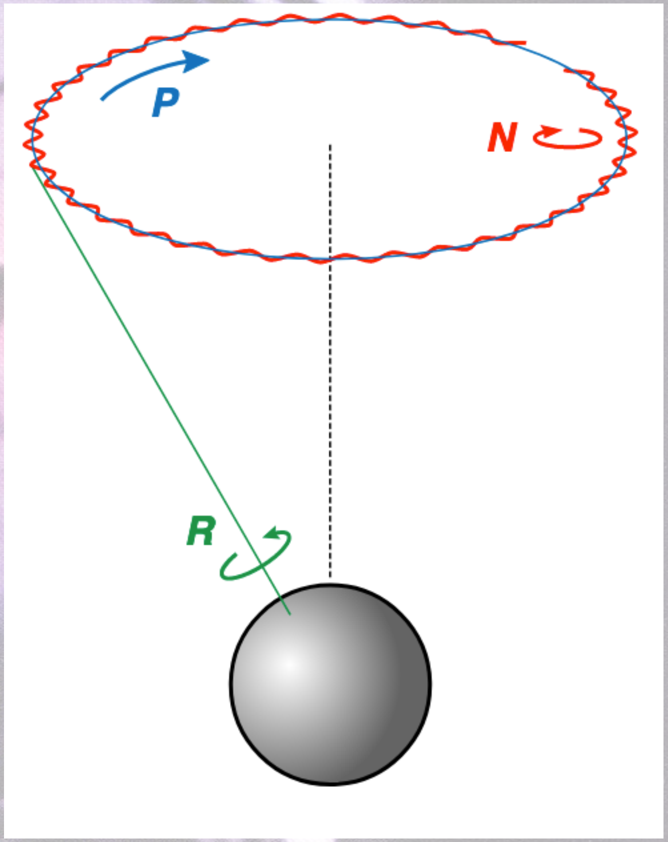
- But how do you know that for a given proton system in a static magnetic field that the frequency of the RF is identical to the Larmor precession frequency?

- If and only if the RF happens to be oscillating at the Larmor frequency, then I can cause the net magnetization vector to precess at larger and larger angles (away from the static field) until it is swinging in a transverse plane.

- This is the *resonance of NMR* and can be detected with a detection coil of wire

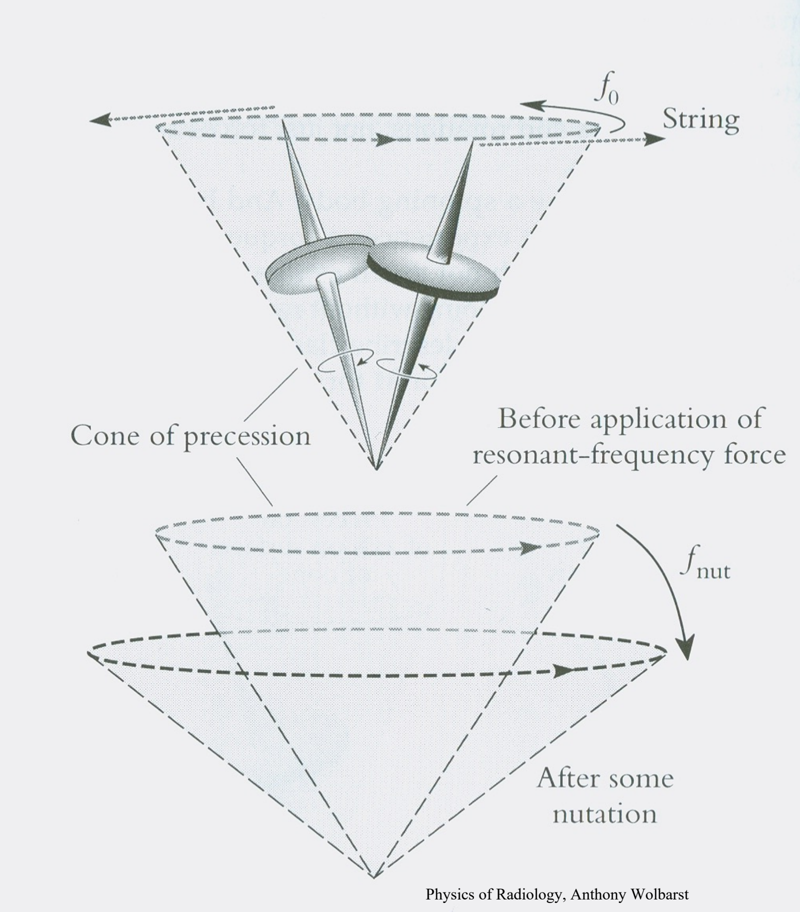
# *Nutation – The Resonance in NMR*

- The pulse perpendicular RF field creates a force perpendicular to the main magnetic field.
- This causes the magnetization vector to oscillate back and forth (away and toward the static magnetic field.
- We call this a nutation.
- If the timing is just right, we can tip the magnetization completely into the transverse plane.



https://en.wikipedia.org/wiki/Nutation

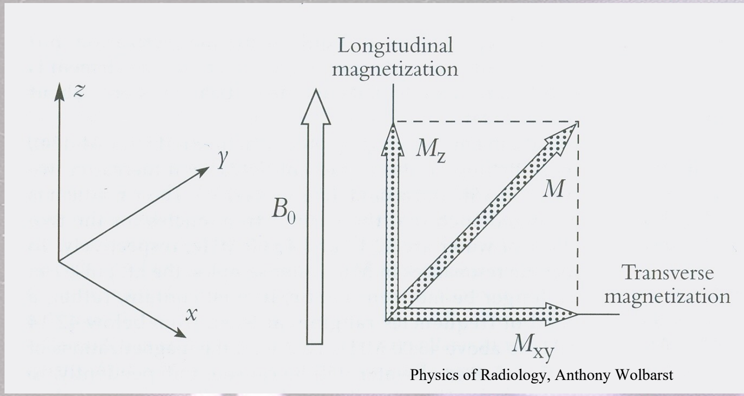
# *Nutation – The Resonance in NMR*



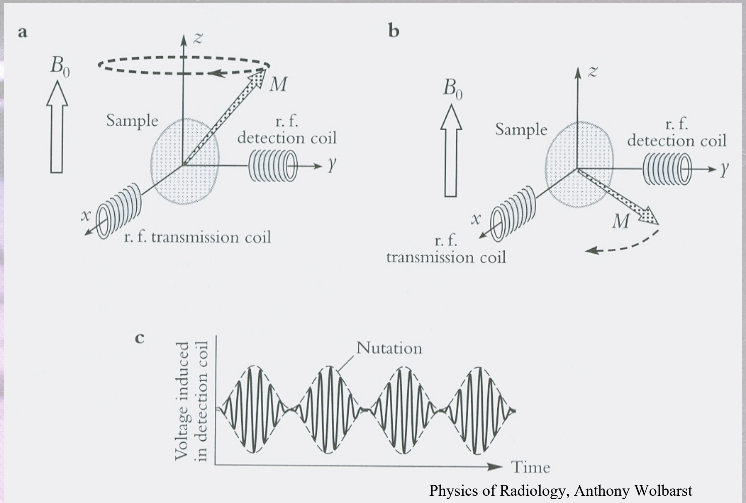
Physics of Radiology, Anthony Wolbarst

- A tangentially applied force (one that is $90^o$ to the external force) causes nutation and the object precesses at a larger angle with respect to the applied force.
- Here for the top, the external force is gravity, and the tangential force is say, tension in an attached string.
- For the proton precessing in a magnetic field, the external force is due to the magnetic field and the tangential force is a $90^o$ RF pulse.
- I'm applying a perpendicular B field to the static field in NMR.
- If the frequency of this field $B_{RF}$ is above or below the Larmor frequency not much happens. If it is at the Lamor frequency, then I cause a resonance in the system.

# Origin of the MR Signal



Physics of Radiology, Anthony Wolbarst



Physics of Radiology, Anthony Wolbarst

- The magnetization vector $\vec{M}$ is a vector meaning that you can change its magnitude and direction.

- The magnetization vector $\vec{M}$ precessing about the external magnetic field has both a longitudinal component (parallel to the applied field $M_z$) and a transverse component (perpendicular to the applied field $M_{xy}$).

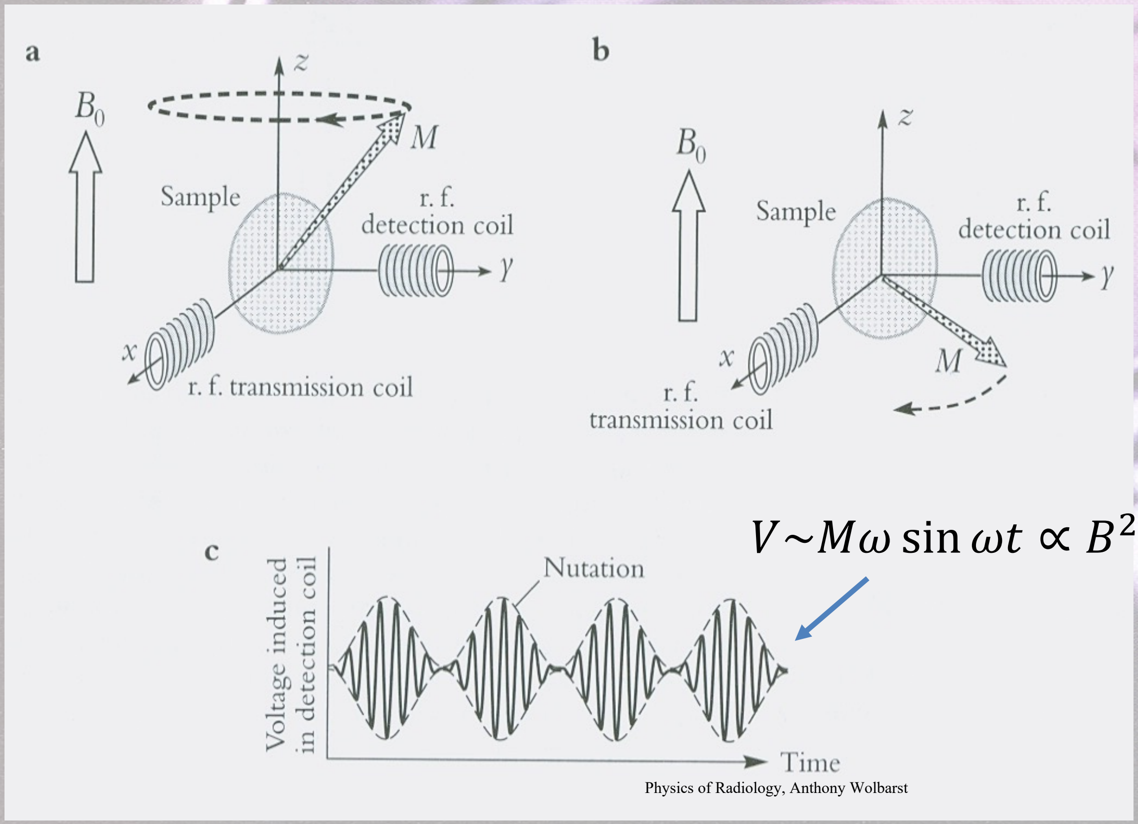
- For the component of $\vec{M}$along the z-axis, $M_z$, there is no change in voltage (or induced current) in the detection coil because none of $M_z$ passes through the coil.

- For the component of $\vec{M}$ precessing *in or near* the transverse plane, $M_{xy}$, the changing magnetic flux through the coils produces a voltage across (and a current through) the detection coil that changes in time.

- The shape is periodic and the amplitude of the voltage (and current) change as $\vec{M}$ moves in and out of the transverse plane (relaxations).

# *Origin of the MR Signal*

- This only happens if you hit the proton sample at the correct frequency.
- The system is tunable, and the system is said to be in *resonance*!



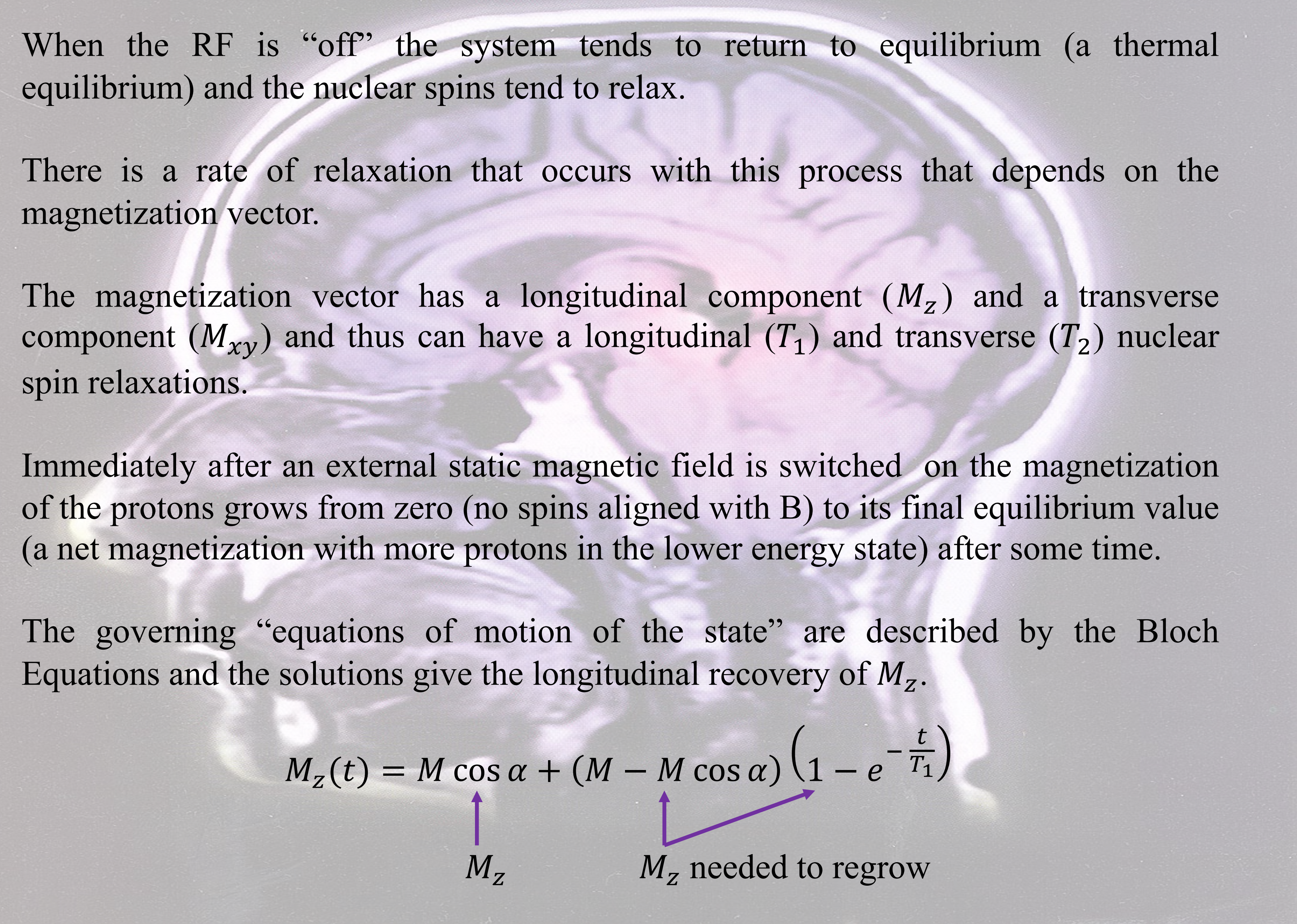
Physics of Radiology, Anthony Wolbarst

$$V \sim M\omega \sin \omega t \propto B^2$$

$$M \propto B$$

$$\omega \propto B$$

# *1 Dimensional Imaging: T1 Spin Relaxation*



- When the RF is "off" the system tends to return to equilibrium (a thermal equilibrium) and the nuclear spins tend to relax.

- There is a rate of relaxation that occurs with this process that depends on the magnetization vector.

- The magnetization vector has a longitudinal component ($M_z$) and a transverse component ($M_{xy}$) and thus can have a longitudinal ($T_1$) and transverse ($T_2$) nuclear spin relaxations.

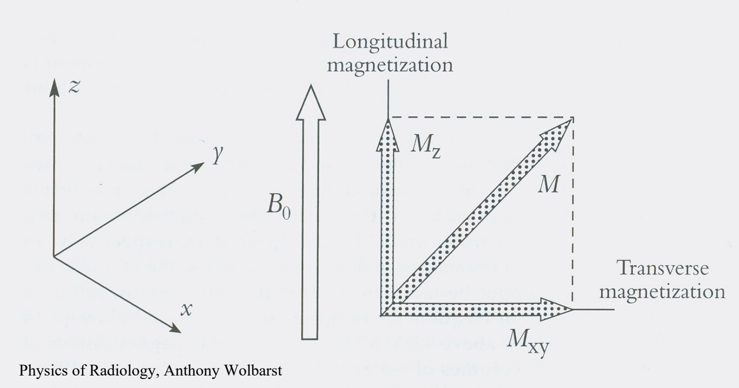
- Immediately after an external static magnetic field is switched on the magnetization of the protons grows from zero (no spins aligned with B) to its final equilibrium value (a net magnetization with more protons in the lower energy state) after some time.

- The governing "equations of motion of the state" are described by the Bloch Equations and the solutions give the longitudinal recovery of $M_z$.

$$M_z(t) = M\underbrace{\cos\alpha}_{M_z} + (M - \underbrace{M\cos\alpha)\left(1 - e^{-\frac{t}{T_1}}\right)}_{M_z \text{ needed to regrow}}$$

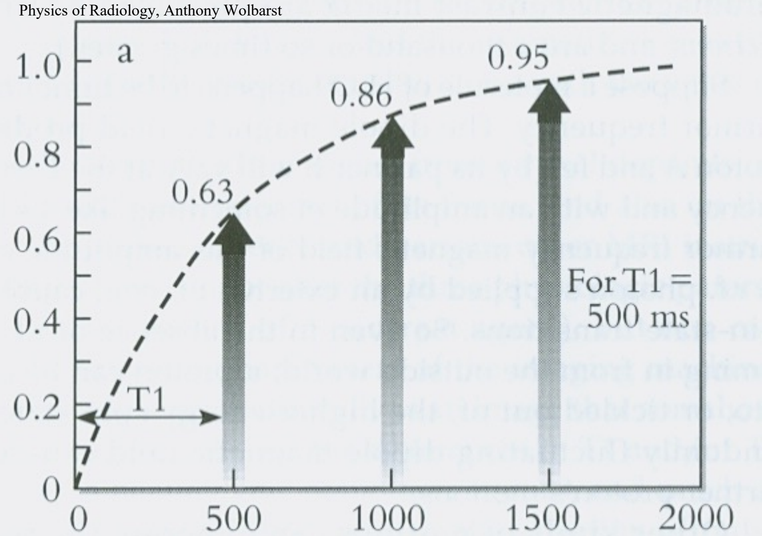
# *1 Dimensional Imaging: T1 Spin Relaxation*

- $T_1$ is called the *longitudinal relaxation time.*
- It characterizes the re-growth of the longitudinal component of the magnetization and involves transitions between the spin-up and spin-down states.
- $\frac{1}{T_1}$ is the rate at which the system comes into thermal equilibrium.
- The RF pulse disturbs the system and as the RF oscillates in time, $\vec{M}$ changes with time and we could use this to measure $T_1$.
- Suppose we applied an $\alpha = 90^0$ RF pulse. The longitudinal magnetization would vary in time according to:



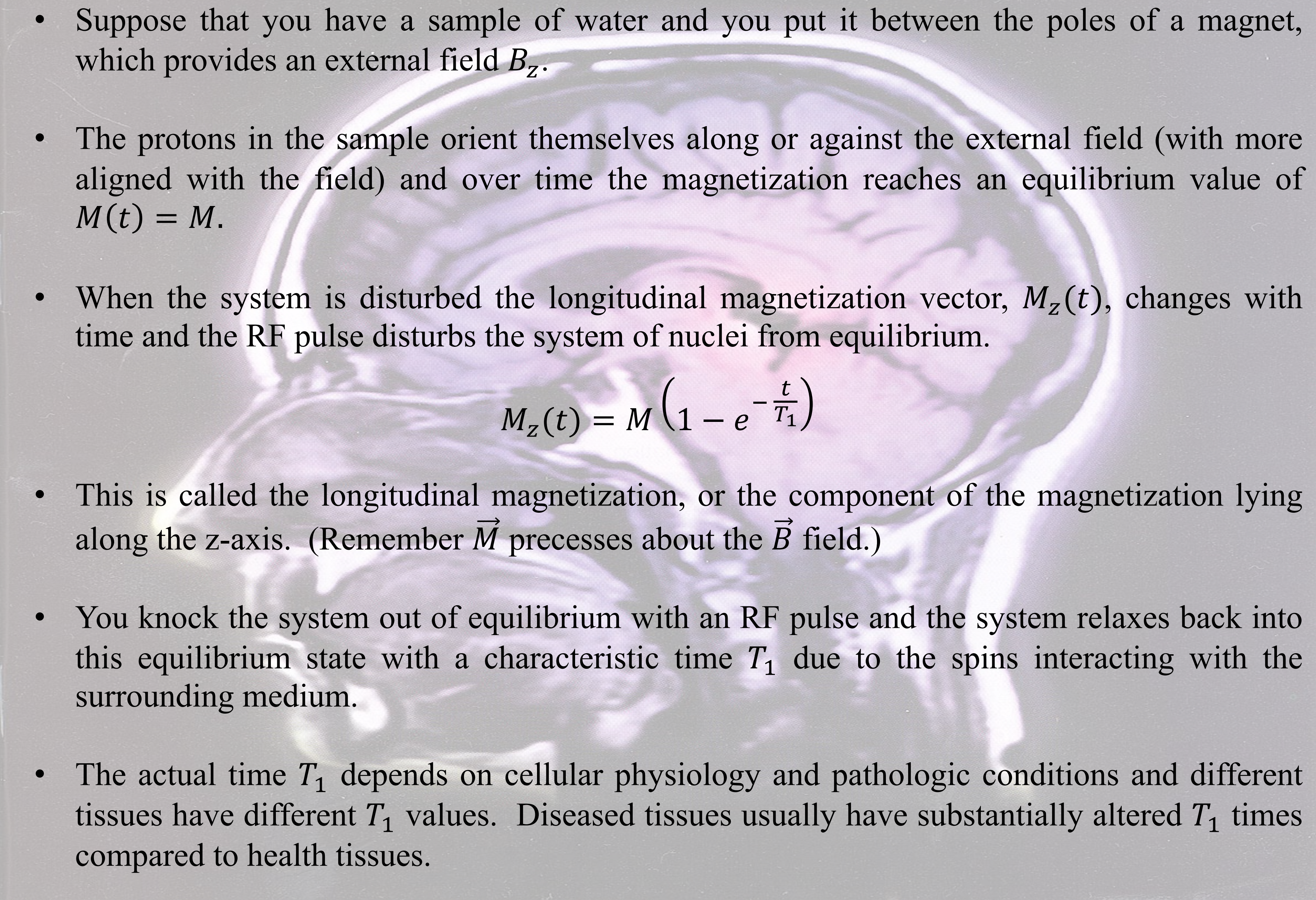
Physics of Radiology, Anthony Wolbarst

Physics of Radiology, Anthony Wolbarst



$$M_z(t) = M\cos\alpha + (M - M\cos\alpha)\left(1 - e^{-\frac{t}{T_1}}\right) \rightarrow M_z(t) = M\left(1 - e^{-\frac{t}{T_1}}\right)$$

# *1 Dimensional Imaging: T1 Spin Relaxation*



- Suppose that you have a sample of water and you put it between the poles of a magnet, which provides an external field $B_z$.

- The protons in the sample orient themselves along or against the external field (with more aligned with the field) and over time the magnetization reaches an equilibrium value of $M(t) = M$.

- When the system is disturbed the longitudinal magnetization vector, $M_z(t)$, changes with time and the RF pulse disturbs the system of nuclei from equilibrium.

$$M_z(t) = M\left(1 - e^{-\frac{t}{T_1}}\right)$$

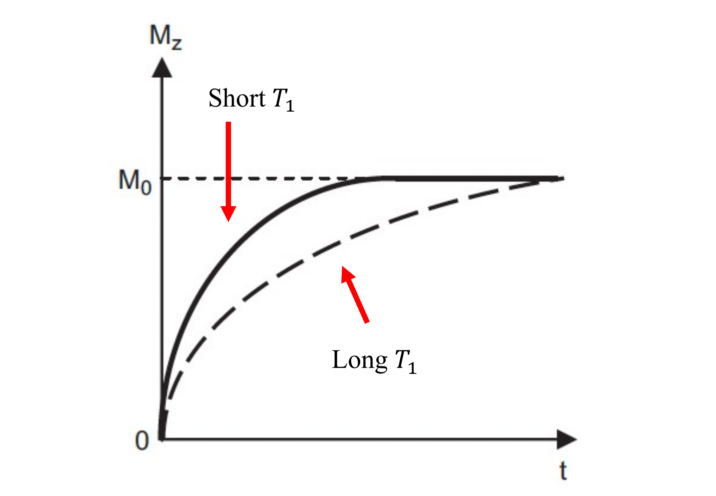
- This is called the longitudinal magnetization, or the component of the magnetization lying along the z-axis. (Remember $\vec{M}$ precesses about the $\vec{B}$ field.)

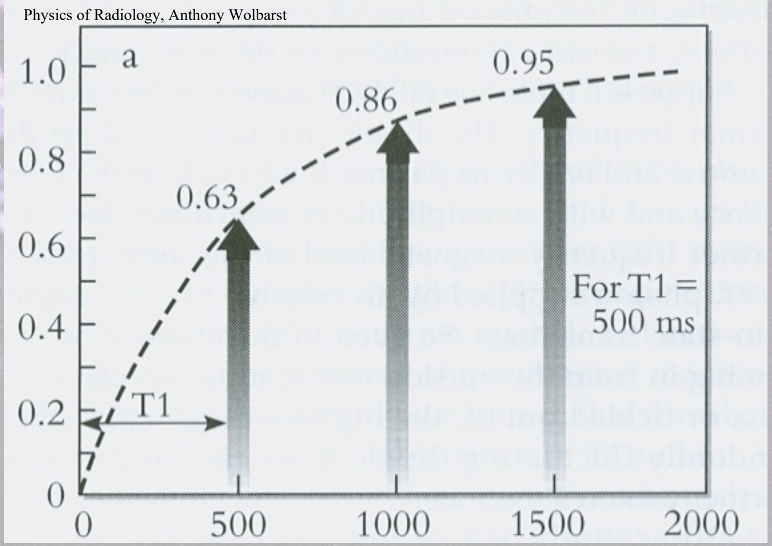
- You knock the system out of equilibrium with an RF pulse and the system relaxes back into this equilibrium state with a characteristic time $T_1$ due to the spins interacting with the surrounding medium.

- The actual time $T_1$ depends on cellular physiology and pathologic conditions and different tissues have different $T_1$ values. Diseased tissues usually have substantially altered $T_1$ times compared to health tissues.

# *1 Dimensional Imaging:  T1 Spin Relaxation*

- The differences in $T_1$ relaxation times can be used to introduce contrast into the scan.
- Or we could externally inject a contrast agent into the tissues to change their $T_1$ relaxation times.
- We arbitrarily define a time $T_1$ as the time required to get 63% of the way from zero longitudinal magnetization to full magnetization, or equilibrium, with 99% as full recovery



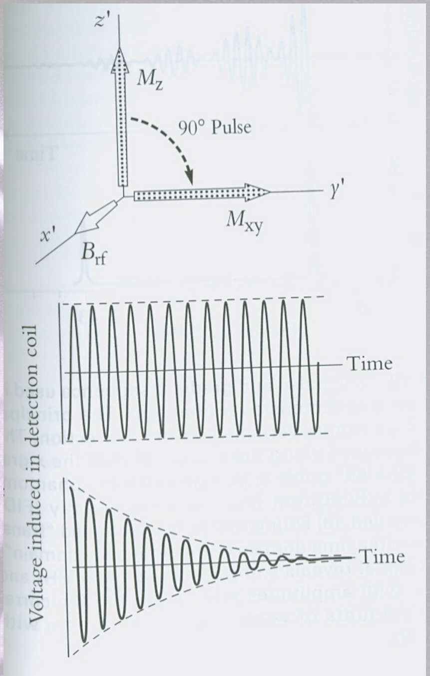

Physics of Radiology, Anthony Wolbarst

# 1 Dimensional Imaging:  T2 Spin Relaxation

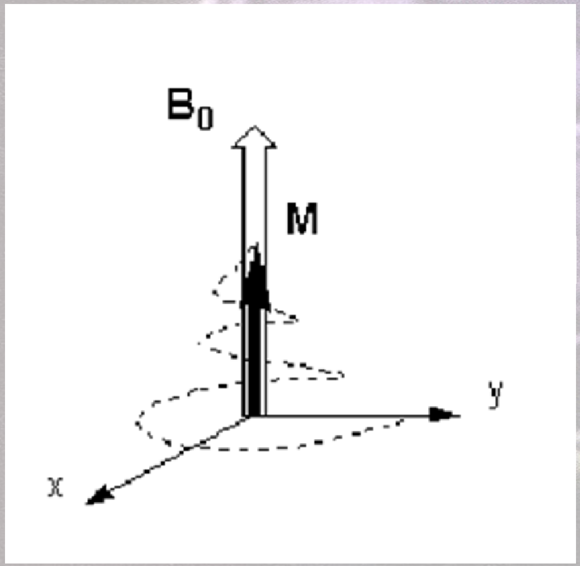
- As the longitudinal relaxation grows, the transverse magnetization shrinks.

- The magnetization still nutates, the but amplitude of the nutations are decreasing in time.

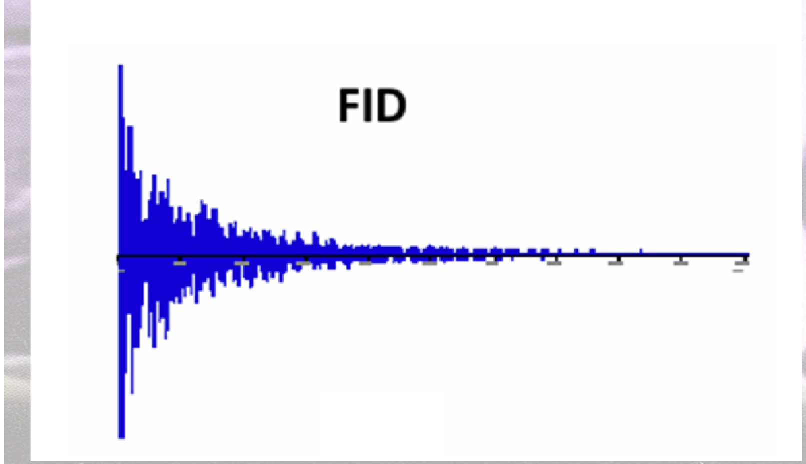
- This is called the free induction decay and is related in part to the nuclear spins interacting with each other and this acts like friction in the system and to inhomogeneities in the axial field.



Physics of Radiology, Anthony Wolbarst



Aljohani, Hind. (2016). Role of Carboxylate ligands in the Synthesis of AuNPs: Size Control, Molecular Interaction and Catalytic Activity. 10.13140/RG.2.1.4680.9208.



Aljohani, Hind. (2016). Role of Carboxylate ligands in the Synthesis of AuNPs: Size Control, Molecular Interaction and Catalytic Activity. 10.13140/RG.2.1.4680.9208.

# *1 Dimensional Imaging: T2 Spin Relaxation*

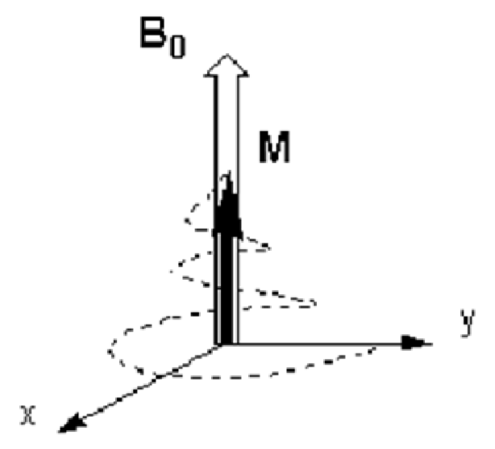
- We can model the transverse magnetization by fitting the FID curves and from using the Bloch equations.

$$M_{xy}(t) = (M \sin \alpha)e^{-\frac{t}{T_2}}$$

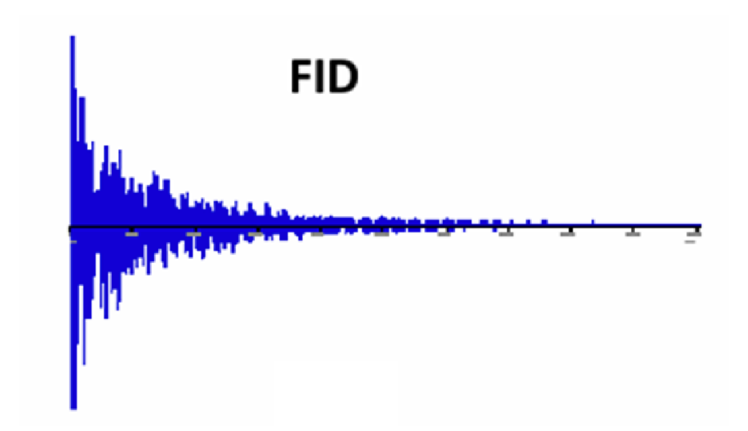
$M_{xy}$ needed to shrink

- As with $T_1$, tissues have different $T_2$ relaxation times which can be used to differentiate healthy from diseased tissue..

- For an $\alpha = 90^0$ RF pulse, we get $M_{xy} = Me^{-\frac{t}{T_2}}$

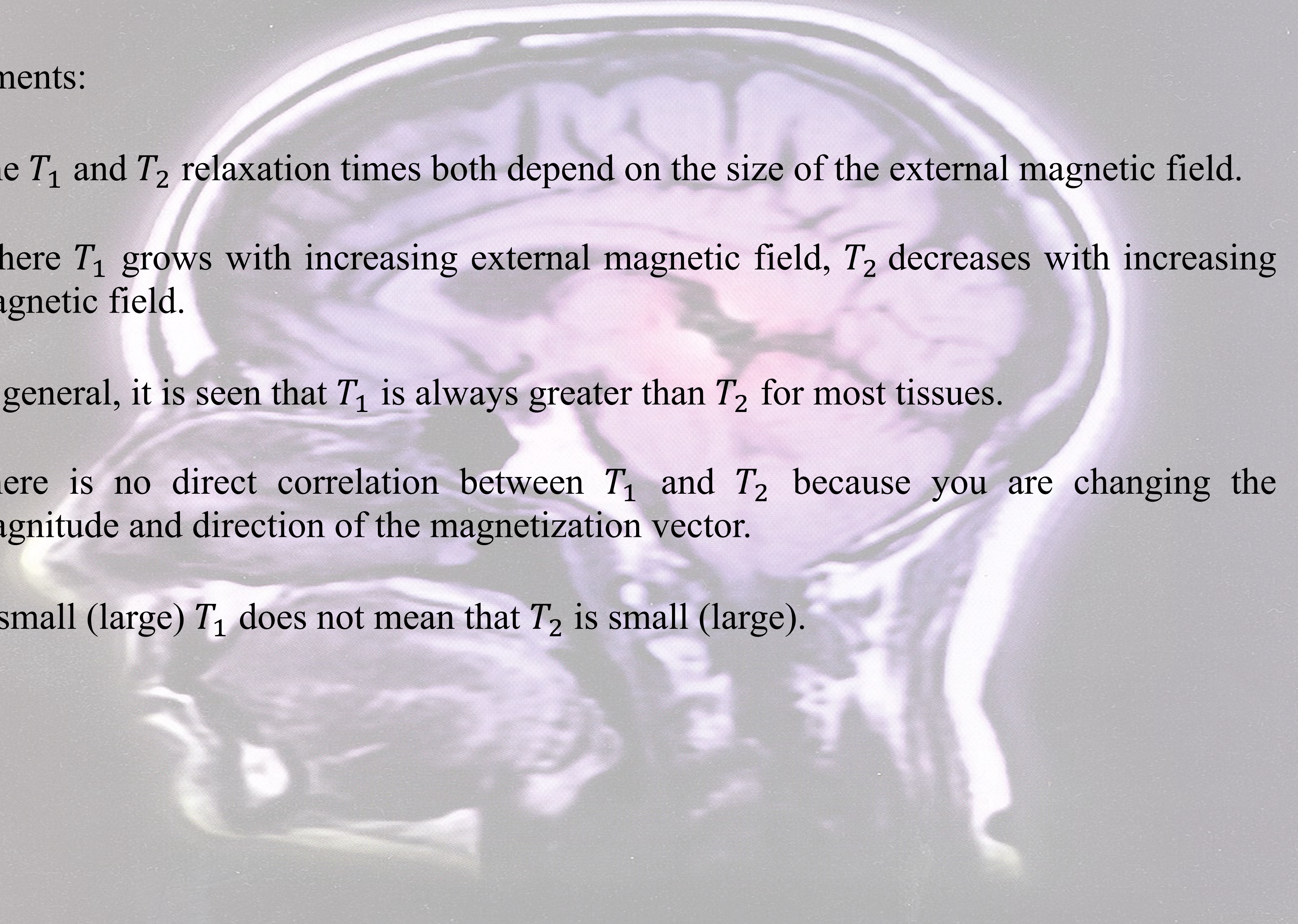


Aljohani, Hind. (2016). Role of Carboxylate ligands in the Synthesis of AuNPs: Size Control, Molecular Interaction and Catalytic Activity. 10.13140/RG.2.1.4680.9208.



Aljohani, Hind. (2016). Role of Carboxylate ligands in the Synthesis of AuNPs: Size Control, Molecular Interaction and Catalytic Activity. 10.13140/RG.2.1.4680.9208.

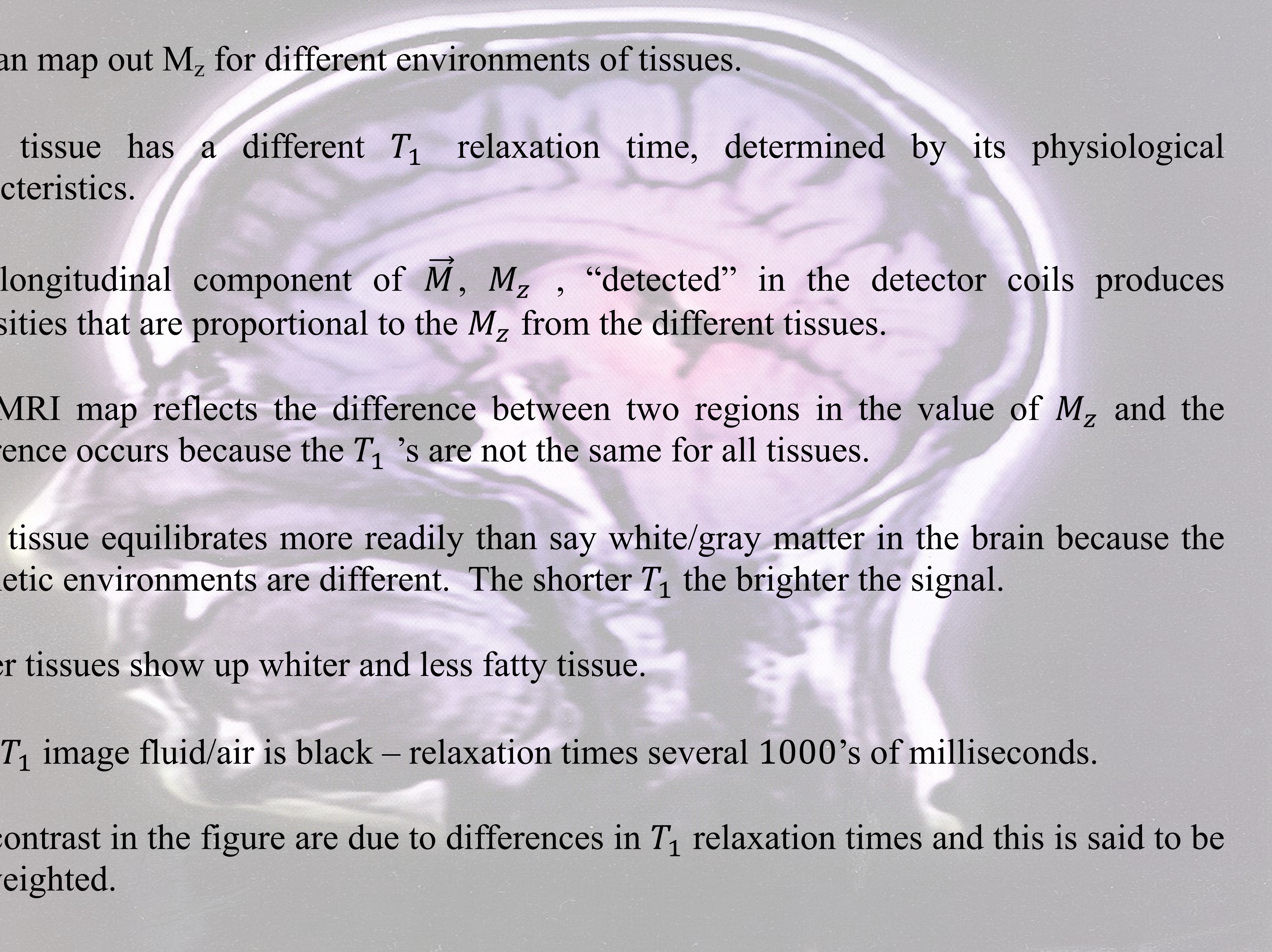
# *1 Dimensional Imaging: T2 Spin Relaxation*



Comments:

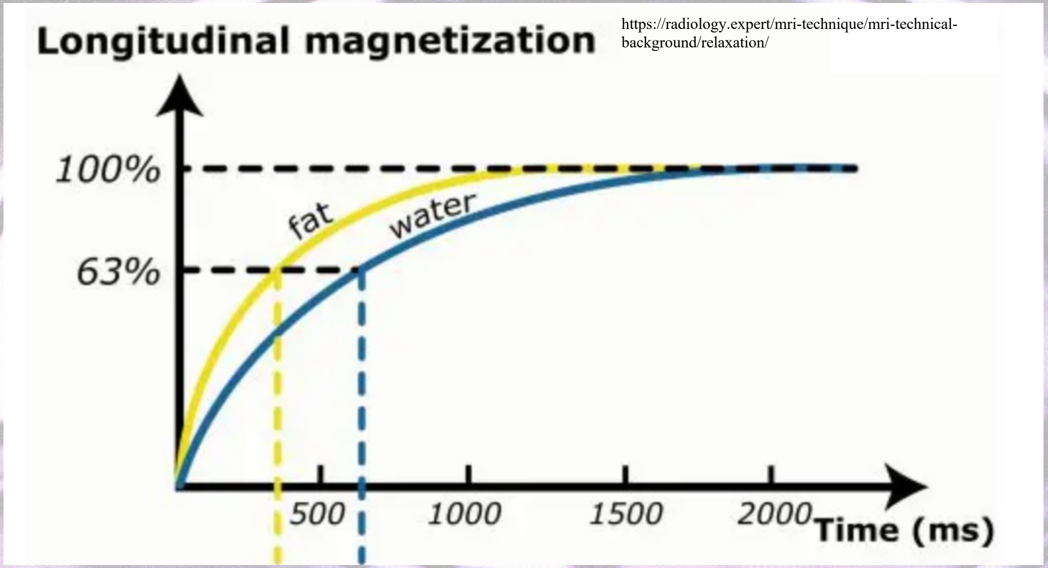
- The $T_1$ and $T_2$ relaxation times both depend on the size of the external magnetic field.
- Where $T_1$ grows with increasing external magnetic field, $T_2$ decreases with increasing magnetic field.
- In general, it is seen that $T_1$ is always greater than $T_2$ for most tissues.
- There is no direct correlation between $T_1$ and $T_2$ because you are changing the magnitude and direction of the magnetization vector.
- A small (large) $T_1$ does not mean that $T_2$ is small (large).

# *T1 Mapping*

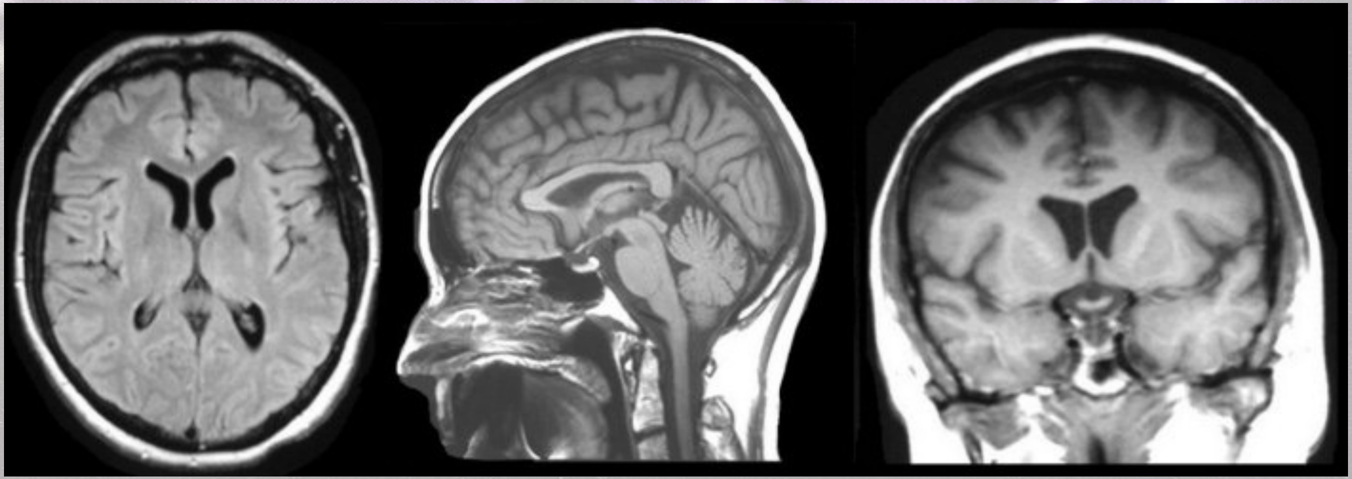


- We can map out $M_z$ for different environments of tissues.
- Each tissue has a different $T_1$ relaxation time, determined by its physiological characteristics.
- The longitudinal component of $\vec{M}$, $M_z$ , “detected” in the detector coils produces intensities that are proportional to the $M_z$ from the different tissues.
- The MRI map reflects the difference between two regions in the value of $M_z$ and the difference occurs because the $T_1$ ’s are not the same for all tissues.
- Fatty tissue equilibrates more readily than say white/gray matter in the brain because the magnetic environments are different. The shorter $T_1$ the brighter the signal.
- Fattier tissues show up whiter and less fatty tissue.
- On a $T_1$ image fluid/air is black – relaxation times several $1000$’s of milliseconds.
- The contrast in the figure are due to differences in $T_1$ relaxation times and this is said to be $T_1$ -weighted.

# *T1 Mapping*



https://radiology.expert/mri-technique/mri-technical-background/relaxation/



https://case.edu/med/neurology/NR/MRI%20Basics.htm

# *T2 Mapping*

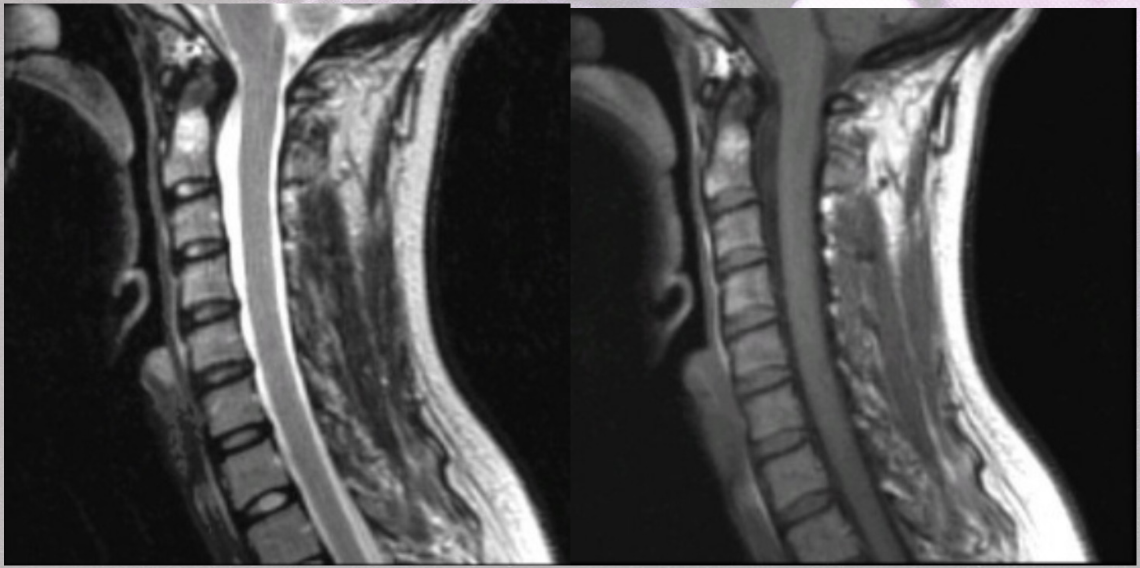
- $T_2$ weighted images are generated by an intensity mapping of the transverse relaxation times.

- Fatty tissues, compared to say water, equilibrate much faster and these $T_2$ signals we color black. Long $T_2$ times are colored bright white. This is opposite to the $T_1$ coloring scheme.

- $T_2$ weighted image accentuate water filled structures and show pathology while $T_1$ weighted images show anatomy "as is."

| Tissue | T1 (msec) | T2 (msec) |
|---|---|---|
| Water/CSF | 4000 | 2000 |
| Gray matter | 900 | 90 |
| Muscle | 900 | 50 |
| Liver | 500 | 40 |
| Fat | 250 | 70 |
| Tendon | 400 | 5 |
| Proteins | 250 | 0.1- 1.0 |
| Ice | 5000 | 0.001 |

https://mri-q.com/why-is-t1--t2.html



https://case.edu/med/neurology/NR/MRI%20Basics.htm

# *Producing a MRI Signal*

## *- Radiofrequency Coils*

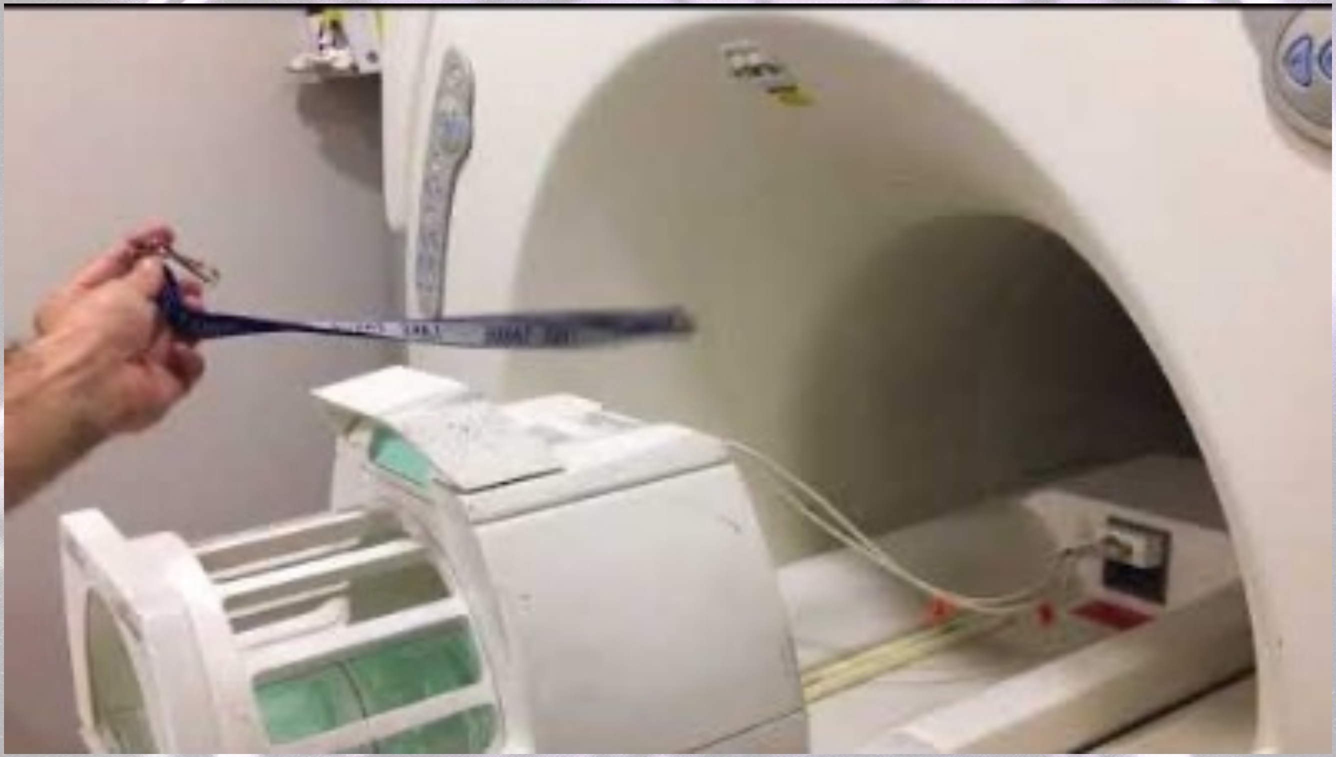
- Defined by their function:
  Transmit / receive coil (most common)
  Transmit only coil (can only excite the system)
  Receive only coil (can only receive MR signal)

- Defined by geometry
  Volume coil (low sensitivity but uniform coverage)
  Surface coil (High sensitivity but limited coverage)
  Phased-array coil (High sensitivity, near-uniform coverage)

- Operate based on Faraday’s Law of Induction

# *Producing a MRI Signal*

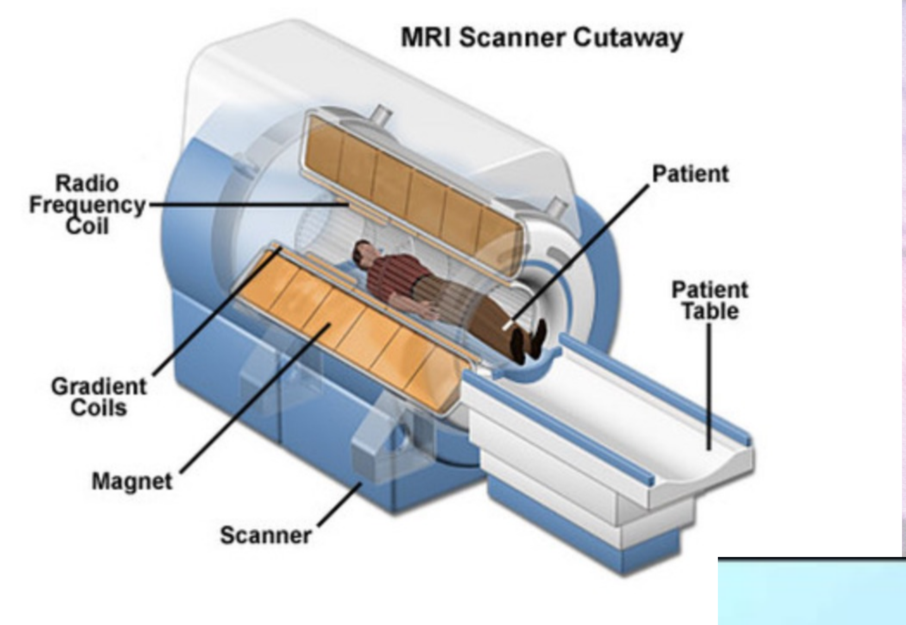
## *- The Magnetic Field*



https://www.youtube.com/watch?v=ug3e9W5H0jI

# Producing a MRI Signal

## - The MRI Scanner



https://snc2dmri.weebly.com/components--functions.html

https://www.youtube.com/watch?v=kmfmGhI8l9E