Physics 111
Fall 2007
Magnetism Solutions

1. How fast must an electron travel in an extremely large magnetic field (30 T) so that the force on it will be as large as the force on a single myosin muscle protein from the chemical energy of one ATP molecule, 3 pN or 3 x 10^{-12} N? This should indicate to you that even large constant magnetic fields can exert only negligible forces on the atoms in our body. In fact, people are routinely exposed to such large DC magnetic fields in Magnetic Resonance Imaging without any ill effects.

   The speed is given from the magnetic force equation

   $$F = qvB \rightarrow v = \frac{F}{qB} = \frac{3.0\times10^{-12}\,N}{1.6\times10^{-19}\,C\times30T} = 6.25\times10^{5}\,\tfrac{m}{s}$$

2. A mass spectrometer can detect the different isotopes of an ionized element. If Zn^{2+} ions are accelerated through a 10 kV potential and enter a 10 T magnetic field region, calculate the different radii that the isotopes ^{64}Zn and ^{66}Zn, where the numbers refer to the atomic weight in atomic mass units.

   Here we'll assume that 1 atomic mass unit is approximately the mass of 1 proton. Therefore the mass of ^{64}Zn is $64\times1.67\times10^{-27}\,kg = 1.0688\times10^{-25}\,kg$ and the mass of ^{66}Zn is $66\times1.67\times10^{-27}\,kg = 1.1022\times10^{-25}\,kg$.

   To calculate the radii of each isotope we equate the magnetic force to the centripetal force experienced by each isotope. This give for the radius of a particle of mass $m, r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$ having used the work-kinetic energy theorem to replace the speed of the particle in terms of its mass and the potential difference it has been accelerated through.

   Therefore the radii of ^{64}Zn and ^{66}Zn are

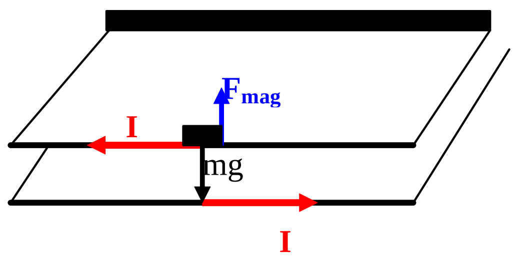
   $$r_{64_{Zn}} = \frac{1}{B}\sqrt{\frac{2m_{64_{Zn}}\Delta V}{q}} = \frac{1}{10T}\sqrt{\frac{2\times1.0688\times10^{-25}\,kg\times10000V}{2\times1.6\times10^{-19}\,C}} = 0.008173m = 8.173mm$$

   and

   $$r_{66_{Zn}} = \frac{1}{B}\sqrt{\frac{2m_{66_{Zn}}\Delta V}{q}} = \frac{1}{10T}\sqrt{\frac{2\times1.1022\times10^{-25}\,kg\times10000V}{2\times1.6\times10^{-19}\,C}} = 0.00830m = 8.30mm$$

   respectively.

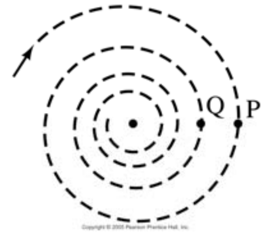
3. A current balance is a device that has two parallel rigid wires carrying the same current in opposite directions. One of the wires is fixed while the other one is attached in such a way that it can pivot in response to a force from the second wire (see figure). First the pivot is adjusted to the top wire is in equilibrium with no current flowing, and then the current is turned on. By adding external weight to the top wire it can be kept at its equilibrium separation distance and the magnetic force between the wires can be determined. This device can be used to calibrate current by direct force measurement.
   a. Write down the *B* field produced by the bottom fixed wire (assuming it to be infinite) and determine that it will produce an upward force on the top current carrying wire.
   b. Compute the force on the 40 cm long top wire if both currents are equal to 10 A and the separation distance is 0.5 cm, and thereby determine the mass needed to be added to the top rod to keep it at the 1 cm distance. Note that these forces are not large.



a. The magnetic force on a long current-carrying wire is given as $F_{upper,lower} = I_{upper} L_{upper} B_{upper} = I_{upper} L_{upper} \left(\frac{\mu_o I_{lower}}{2\pi d} \right) = \frac{\mu_o I_{lower}^2 L_{upper}}{2\pi d}$ where the magnetic field produced by the lower wire at the location of the upper wire is $B_{upper} = \left(\frac{\mu_o I_{lower}}{2\pi d} \right)$. By the right-hand rule, the magnetic field at the upper wire produced by the current in the lower wire is pointing out of the page (along the plane of the wires toward the viewer.) This produces a force, by the right-hand rule on the upper wire that is directed upward as indicated in the diagram.

b. The magnetic force felt by the upper wire is therefore $F_{upper,lower} = \frac{\mu_o I_{lower}^2 L_{upper}}{2\pi d} = \frac{4\pi \times 10^{-7} \frac{Tm}{A} \times (10A)^2 \times 0.4m}{2\pi \times 0.5 \times 10^{-2} m} = 1.6 \times 10^{-3} N$ and the mass needed to balance this upward force is given by Newton's 2nd Law $F = mg \rightarrow m = \frac{F}{g} = \frac{1.6 \times 10^{-3} N}{9.8 \frac{m}{s^2}} = 1.63 \times 10^{-4} kg = 160 mg$

4. A proton follows a spiral path through a gas in a magnetic field of 0.010 T, perpendicular to the plane of the spiral, as shown below. In two successive loops, at points P and Q, the radii are 10.0 mm and 8.5 mm, respectively. Calculate the change in the kinetic energy of the proton as it travels from P to Q.



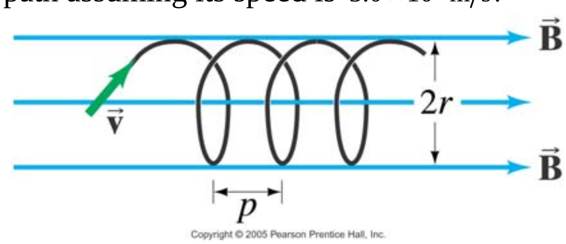
The speed of the proton can be calculated based on the radius of curvature of the (almost) circular motion. From that the kinetic energy can be calculated.

$$qvB = \frac{mv^2}{r} \quad \rightarrow \quad v = \frac{qBr}{m} \qquad \mathrm{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m}$$

$$\Delta\mathrm{KE} = \frac{q^2B^2}{2m}\left(r_2^2 - r_1^2\right) = \frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)^2\left(0.010\,\mathrm{T}\right)^2}{2\left(1.67\times10^{-27}\ \mathrm{kg}\right)}\left[\left(8.5\times10^{-3}\ \mathrm{m}\right)^2 - \left(10.0\times10^{-3}\ \mathrm{m}\right)^2\right]$$

$$= \boxed{-2.1\times10^{-20}\ \mathrm{J}}\ \text{or } -0.13\,\mathrm{eV}$$

5. An electron enters a uniform magnetic field $B = 0.23\,\mathrm{T}$ at a 45° angle to $\vec{\mathbf{B}}$. Determine the radius r and pitch p (distance between loops as shown below) of the electron's helical path assuming its speed is $3.0\times10^6\ \mathrm{m/s}$.



The centripetal force is caused by the magnetic field, and is given by Eq. 20-3.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \quad \rightarrow$$

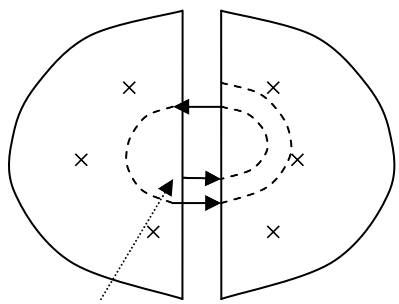
$$r = \frac{mv_\perp}{qB} = \frac{\left(9.11\times10^{-31}\ \mathrm{kg}\right)\left(3.0\times10^6\ \mathrm{m/s}\right)\sin 45^\circ}{\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(0.23\,\mathrm{T}\right)} = 5.251\times10^{-5}\ \mathrm{m} \approx \boxed{5.3\times10^{-5}\ \mathrm{m}}$$

The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi\frac{mv_\perp}{qB}}{v_\perp} = \frac{2\pi m}{qB}$$

$$p = v_{\parallel}T = v\cos 45^\circ\left(\frac{2\pi m}{qB}\right) = \left(3.0\times10^6\ \mathrm{m/s}\right)\cos 45^\circ\,\frac{2\pi\left(9.11\times10^{-31}\ \mathrm{kg}\right)}{\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(0.23\,\mathrm{T}\right)} = \boxed{3.3\times10^{-4}\ \mathrm{m}}$$

6. A **cyclotron** consists of large magnets (called dees since they are in the shape of the letter D) with a small gap between them as shown in the figure. An accelerating voltage is applied across the gap and charged particles, such as protons, are accelerated across the gap and then enter a region where a uniform magnetic field steers them in a semi-circle to return to the gap. The accelerating voltage polarity is reversed and so the particle accelerates further returning across the gap and entering the opposite region of magnet field, where it is steered around in a semi-circle again. This process is repeated many times to accelerate the particle to high speeds in a relatively small region of space.



a) First show that if the particle of mass m and charge q has a speed v and the uniform magnetic field is B, then it will travel in a semi-circle of radius $r = \frac{m\text{v}}{qB}$.

b) Then show that the particle will travel in the semi-circle in a time $t = \frac{\pi m}{eB}$ that is independent of the radius of the orbit. This allows the cyclotron to have a constant frequency of oscillation of the accelerating voltage given by f = 1/(2t) so long as the particle energy is below about 50 MeV. Beyond this relativity effects occur and the time does vary with particle velocity or radius of orbit.

a. To calculate the radii of a particle of mass *m* and charge *q*, we equate the magnetic force to the centripetal force experienced by the mass. This give for the radius of a particle of mass *m*, $F_B = F_C \rightarrow qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB}$.

b. We want the particle to travel a semi-circle of distance πr and calculate the amount of time that this takes. To do this we need to know that velocity of the particle. Again we equate the centripetal force experienced by the particle to the magnetic force and this time solve for the velocity. Doing this we find $F_B = F_C \rightarrow qvB = \frac{mv^2}{r} \rightarrow v = \frac{qrB}{m}$. The velocity, a constant, is the ratio of the distance traveled by the time it takes to travel this distance. Thus the time is $v = \frac{\pi r}{t} \rightarrow t = \frac{\pi r}{v} = \frac{\pi r m}{qrB} = \frac{\pi m}{qB}$.

7. A cyclotron is sometimes used for carbon dating as will be described in Chapter 30. Carbon-14 and carbon-12 ions are obtained from a sample of the material to be dated and are accelerated in the cyclotron. If the cyclotron has a magnetic field of magnitude 2.40 T, what is the difference in cyclotron frequencies for the two ions?

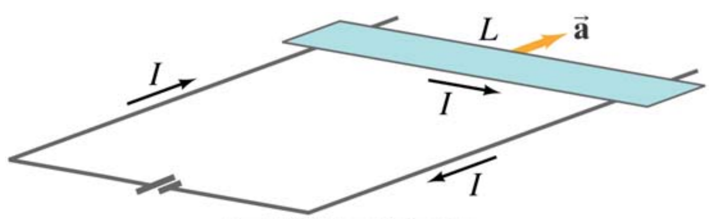
$\sum F = ma$ or $qvB\sin 90.0° = \frac{mv^2}{r}$

$\therefore$ the angular frequency for each ion is $\frac{v}{r} = \omega = \frac{qB}{m}$ and

$$\Delta\omega = \omega_{12} - \omega_{14} = qB\left(\frac{1}{m_{12}} - \frac{1}{m_{14}}\right) = \frac{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.40\ \text{T}\right)}{\left(1.66 \times 10^{-27}\ \text{kg/u}\right)}\left(\frac{1}{12.0\ \text{u}} - \frac{1}{14.0\ \text{u}}\right)$$

$$\Delta\omega = \omega_{12} - \omega_{14} = 2.75 \times 10^{6}\ \text{s}^{-1} = \boxed{2.75\ \text{Mrad/s}}$$

8. A sort of "projectile launcher" is shown below. A large current moves in a closed loop composed of fixed rails, a power supply, and a very light, almost frictionless bar touching the rails. A magnetic field is perpendicular to the plane of the circuit. If the bar has a length $L = 22\ \text{cm}$, a mass of 1.5 g, and is placed in a field of 1.7 T, what constant current flow is needed to accelerate the bar from rest to $28\ \text{m/s}$ in a distance of 1.0 m? In what direction must the magnetic field point?



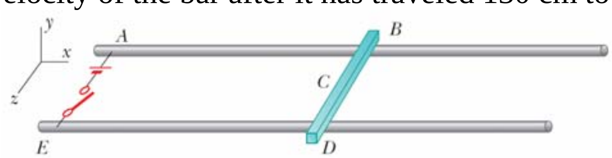
The accelerating force on the bar is due to the magnetic force on the current. If the current is constant, the magnetic force will be constant, and so constant acceleration kinematics can be used.

$$v^2 = v_0^2 + 2a\Delta x \quad \rightarrow \quad a = \frac{v^2 - 0}{2\Delta x} = \frac{v^2}{2\Delta x}$$

$$F_{\text{net}} = ma = ILB \quad \rightarrow \quad I = \frac{ma}{LB} = \frac{m\left(\frac{v^2}{2\Delta x}\right)}{LB} = \frac{mv^2}{2\Delta x LB} = \frac{\left(1.5 \times 10^{-3}\,\text{kg}\right)\left(28\,\text{m/s}\right)^2}{2\left(1.0\,\text{m}\right)\left(0.22\,\text{m}\right)\left(1.7\,\text{T}\right)} = \boxed{1.6\,\text{A}}$$

Using the right hand rule, for the force on the bar to be in the direction of the acceleration shown in the figure, the magnetic field must be $\boxed{\text{down}}$.

9. *Rail guns* have been suggested for launching projectiles into space without chemical rockets and for ground-to-air antimissile weapons of war. A tabletop model rail gun consists of two long, parallel, horizontal rails 3.50 cm apart, bridged by a bar *BD* of mass 3.00 g. The bar is originally at rest at the midpoint of the rails and is free to slide without friction. When the switch is closed, electric current is quickly established in the circuit *ABCDEA*. The rails and bar have low electric resistance, and the current is limited to a constant 24.0 A by the power supply.
   (a) Find the magnitude of the magnetic field 1.75 cm from a single very long, straight wire carrying current 24.0 A.
   (b) Find the magnitude and direction of the magnetic field at point *C* in the diagram, the midpoint of the bar, immediately after the switch is closed.
   (c) At other points along the bar *BD*, the field is in the same direction as at point *C* but is larger in magnitude. Assume that the average effective magnetic field along *BD* is five times larger than the field at *C*. With this assumption, find the magnitude and direction of the force on the bar.
   (d) Find the acceleration of the bar when it is in motion.
   (e) Does the bar move with constant acceleration?
   (f) Find the velocity of the bar after it has traveled 130 cm to the end of the rails.



(a) $$B=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(24.0\ \mathrm{A})}{2\pi(0.017\,5\ \mathrm{m})}=\boxed{2.74\times10^{-4}\ \mathrm{T}}$$

(b) At point *C*, conductor *AB* produces a field $\frac{1}{2}\left(2.74\times10^{-4}\ \mathrm{T}\right)\left(-\hat{\mathbf{j}}\right)$,  conductor *DE* produces a field of $\frac{1}{2}\left(2.74\times10^{-4}\ \mathrm{T}\right)\left(-\hat{\mathbf{j}}\right)$,  *BD* produces no field, and *AE* produces negligible field. The total field at *C* is $\boxed{2.74\times10^{-4}\ \mathrm{T}\left(-\hat{\mathbf{j}}\right)}$.

(c)

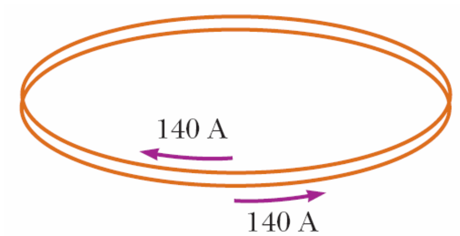
$$\vec{\mathbf{F}}_B=I\vec{\ell}\times\vec{\mathbf{B}}=(24.0\ \mathrm{A})\left(0.035\,0\ \mathrm{m}\hat{\mathbf{k}}\right)\times\left[5\left(2.74\times10^{-4}\ \mathrm{T}\right)\left(-\hat{\mathbf{j}}\right)\right]=\boxed{\left(1.15\times10^{-3}\ \mathrm{N}\right)\hat{\mathbf{i}}}$$

(d) $$\vec{\mathbf{a}}=\frac{\sum\vec{\mathbf{F}}}{m}=\frac{\left(1.15\times10^{-3}\ \mathrm{N}\right)\hat{\mathbf{i}}}{3.0\times10^{-3}\ \mathrm{kg}}=\boxed{\left(0.384\ \mathrm{m/s^2}\right)\hat{\mathbf{i}}}$$

(e) The bar is already so far from *AE* that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's $\boxed{\text{acceleration is constant}}$.

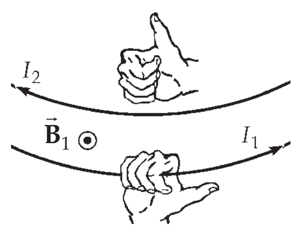
(f) $v_f^2 = v_i^2 + 2ax = 0 + 2\left(0.384\ \text{m/s}^2\right)\left(1.30\ \text{m}\right)$, so $\vec{\mathbf{v}}_f = \boxed{\left(0.999\ \text{m/s}\right)\hat{\mathbf{i}}}$

10. Two circular loops are parallel, coaxial, and almost in contact, 1.00 mm apart as shown in the figure below. Each loop is 10.0 cm in radius. The top loop carries a clockwise current of 140 A. The bottom loop carries a counterclockwise current of 140 A.
    (a) Calculate the magnetic force exerted by the bottom loop on the top loop.
    (b) The upper loop has a mass of 0.021 0 kg. Calculate its acceleration, assuming that the only forces acting on it are the force in part (a) and the gravitational force. (*Suggestion:* Think about how one loop looks to a bug perched on the other loop.)

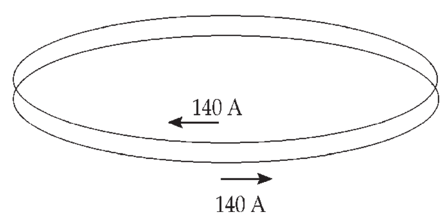


Model the two wires as straight parallel wires (!)

(a) $F_B = \dfrac{\mu_0 I^2 \ell}{2\pi a}$ (Equation 22.27)

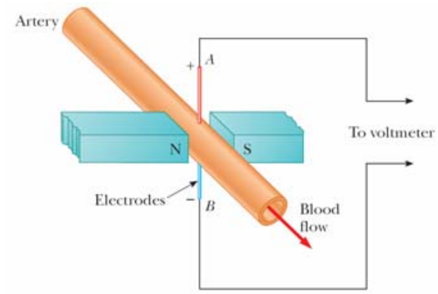


$$F_B = \frac{\left(4\pi \times 10^{-7}\right)(140)^2 (2\pi)(0.100)}{2\pi\left(1.00 \times 10^{-3}\right)}$$
$$= \boxed{2.46\ \text{N}}\ \text{upward}$$



(b) $a_{\text{loop}} = \dfrac{2.46\ \text{N} - m_{\text{loop}}\, g}{m_{\text{loop}}} = \boxed{107\ \text{m/s}^2}$ upward

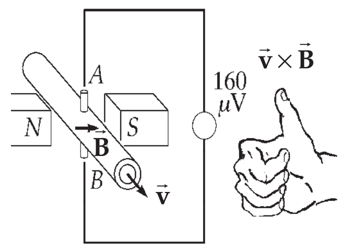
11. A heart surgeon monitors the flow rate of blood through an artery using an electromagnetic flowmeter, shown right. Electrodes A and B make contact with the outer surface of the blood vessel, which has interior diameter 3.00 mm.



(a) For a magnetic field magnitude of 0.040 0 T, an emf of 160 μV appears between the electrodes. Calculate the speed of the blood.

(b) Verify that electrode A is positive as shown. Does the sign of the emf depend on whether the mobile ions in the blood are predominantly positively or negatively charged? Explain.

(a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point A and negative charges toward point B. This separation of charges produces an electric field directed from A toward B. At equilibrium, the electric force caused by this field must balance the magnetic force, so

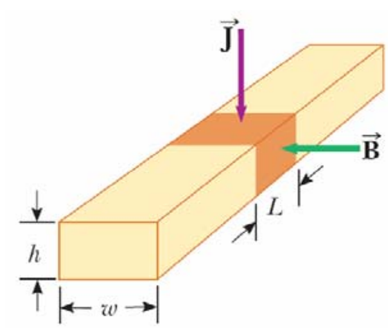


$$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

or $$v = \frac{\Delta V}{Bd} = \frac{\left(160 \times 10^{-6}\ \text{V}\right)}{(0.040\,0\ \text{T})\left(3.00 \times 10^{-3}\ \text{m}\right)} = \boxed{1.33\ \text{m/s}}.$$

(b) $\boxed{\text{No}}$. Negative ions moving in the direction of v would be deflected toward point B, giving A a higher potential than B. Positive ions moving in the direction of v would be deflected toward A, again giving A a higher potential than B. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

12. Heart–lung machines and artificial kidney machines employ blood pumps. A mechanical pump can mangle blood cells. The figure on the right represents an electromagnetic pump. The blood is confined to an electrically insulating tube, cylindrical in practice but represented as a rectangle of width w and height h. The simplicity of design makes the pump dependable. The blood is easily kept uncontaminated; the tube is simple to clean or cheap to replace. Two electrodes fit into the top and bottom of the tube. The potential difference between them establishes an electric current through the blood, with current density J over a section of length L. A perpendicular magnetic field exists in the same region.



(a) Explain why this arrangement produces on the liquid a force that is directed along the length of the pipe.

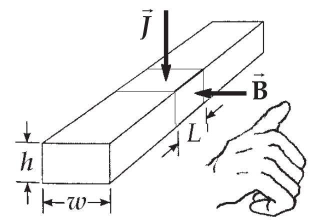
(b) Show that the section of liquid in the magnetic field experiences a pressure increase *JLB*.

(c) After the blood leaves the pump, is it charged? Is it current-carrying? Is it magnetized? The same magnetic pump can be used for any fluid that conducts electricity, such as liquid sodium in a nuclear reactor.

(a) Define vector $\vec{\mathbf{h}}$ to have the downward direction of the current, and vector $\vec{\mathbf{L}}$ to be along the pipe into the page as shown.

The $\boxed{\text{electric current experiences a magnetic force}}$.

$I\left(\vec{\mathbf{h}} \times \vec{\mathbf{B}}\right)$ in the direction of $\vec{\mathbf{L}}$.



(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length *L*, electrons drift upward to constitute downward electric current $J \times (\text{area}) = J\,Lw$.

The current then feels a magnetic force $I\left|\vec{\mathbf{h}} \times \vec{\mathbf{B}}\right| = JLwhB \sin 90°$ This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{JLwhB}{hw} = \boxed{JLB}.$$

(c) Charge moves within the fluid inside the length *L*, but charge does not accumulate: the fluid is not charged after it leaves the pump. It is not current-carrying and it is not magnetized.