Physics 110
Spring 2006
Momentum Problems – Their Solutions

1. A karate expert strikes downward with his fist of mass m_{fist} = 0.7kg breaking a 0.14kg board. He then does the same to a 3.2kg concrete block. The spring constant for the board and block are 4.1x10^4 N/m and 2.6x10^6 N/m respectively. The board breaks at a deflection d = 16mm and while for the block the deflection is d = 1.1mm.
    a. Just before each object breaks, what is the energy stored in the object?
    b. What is the lowest fist speed required to break the objects? (Hint: The collision is completely inelastic and bending begins just after the collision. Mechanical energy is conserved from the beginning of the bending until just before the object breaks. The speed of the fist and the object are negligible at the breaking point.)

    a. For the board we have $PE_{s,board} = \frac{1}{2}kx^2 = \frac{1}{2}\times 4.1\times 10^4 \frac{N}{m}\times(0.016m)^2 = 5.23J$ while for the block $PE_{s,block} = \frac{1}{2}kx^2 = \frac{1}{2}\times 2.6\times 10^6 \frac{N}{m}\times(0.0011m)^2 = 1.57J$.
    b. If mechanical energy is conserved in this particular inelastic collision, we have for the board

$$\Delta KE_{hand+board} = -\Delta PE_{board} \rightarrow -\tfrac{1}{2}(m_{hand}+m_{board})v^2_{hand+board} = -\tfrac{1}{2}k_{board}x^2_{board}$$

$$\therefore v_{hand+board} = \sqrt{\frac{k_{board}x^2_{board}}{(m_{hand}+m_{board})}} = \sqrt{\frac{4.1\times 10^4 \frac{N}{m}\times(0.016m)^2}{0.70kg+0.14kg}} = 3.53\tfrac{m}{s}$$

and for the block

$$\Delta KE_{hand+block} = -\Delta PE_{block} \rightarrow -\tfrac{1}{2}(m_{hand}+m_{block})v^2_{hand+block} = -\tfrac{1}{2}k_{block}x^2_{block}$$

$$\therefore v_{hand+block} = \sqrt{\frac{k_{block}x^2_{block}}{(m_{hand}+m_{block})}} = \sqrt{\frac{2.6\times 10^6 \frac{N}{m}\times(0.0011m)^2}{0.70kg+3.2kg}} = 0.898\tfrac{m}{s}.$$

In order to determine the speed of the hand we use conservation of momentum. For the inelastic collision described we have $m_{hand}v_{hand} = (m_{hand}+m_{object})v_{hand+object} \rightarrow v_{hand} = \frac{m_{hand}+m_{object}}{m_{hand}}\times v_{hand+object}$.

Thus for the board v_{hand} = *4.24 m/s* and for the block v_{hand} = *5.00 m/s*.

2. The National Transportation Safety Board is testing the crash worthiness of new cars. A 2300kg car, moving at 15m/s is allowed to collide with a stationary concrete pier which brings the car to rest in 0.56s. What is the magnitude of the force that acts on the car during the impact?

The magnitude of the average force is
$$F_{avg} = \frac{|\Delta p|}{\Delta t} = \frac{m|\Delta v|}{\Delta t} = \frac{2300kg \times 15\frac{m}{s}}{0.56s} = 6.2 \times 10^4 N .$$

3. A paratrooper jumps from an airplane and tries to open her parachute. It fails to open and she falls 370m and happens to land in some snow, where she suffers only minor injuries. If her velocity at impact was 56m/s (due to air resistance, she reaches a terminal velocity and her speed, due to gravity, doesn't continue to increase.) Suppose further that her mass (including gear) is 85kg and that the force on her from the snow was 1.2×10^5N.
    a. What is the minimum depth of snow that would have stopped her safely?
    b. What is the impulse on her from the snow?

We choose +y upward, which implies a>0 (the acceleration is upward since it represents a deceleration of his downward motion through the snow).

a. The maximum deceleration a_{max} of the paratrooper (of mass m and initial speed v=56 m/s) is found from Newton's second law $F_{snow}-mg=ma_{\max}$, where $F_{snow} = 1.2\times10^5$N. Using $v^2=2a_{max}d$, we find the minimum depth of snow for the woman to survive: $d = \frac{v^2}{2a_{\max}} = \frac{mv^2}{2(F_{snow} - mg)} = \frac{85kg \times (56\frac{m}{s})^2}{2 \times 1.2 \times 10^5 N} = 1.1m$.

b. Her short trip through the snow involves a change in momentum $p_{f,y} - p_{i,y} = 0 - 85kg \times (-56\frac{m}{s}) = 4.8 \times 10^3 \frac{kgm}{s}$ (the negative value of the initial velocity is due to the fact that downward is the negative direction) which yields 4.8×10^3 kg·m/s. By the impulse-momentum theorem, this equals the impulse due to the net force F_{snow}-mg, but since $F_{snow} \gg mg$ we can approximate this as the impulse on her just from the snow.

4. A meteor impact crater is formed by the impact of a meteor with the earth. If the mass of a meteor is 5×10^{10}kg and has a speed of 7200m/s, what speed would this meteor give the earth in a direct head on collision?

Let m_m be the mass of the meteor and m_e be the mass of Earth. Let v_m be the velocity of the meteor just before the collision and let v be the velocity of Earth (with the meteor) just after the collision. The momentum of the Earth-meteor system is conserved during the collision. Thus, for this inelastic collision we have,
$$m_m v_m = (m_m + m_e)v \rightarrow v = \frac{m_m v_m}{m_m + m_e} = \frac{5\times10^5 kg \times 7200\frac{m}{s}}{5\times10^5 kg + 5.98\times10^{24} kg} = 6\times10^{-11}\frac{m}{s}$$, which converts to approximately 2mm per year!

5. Two cars A and B slide on an icy road as they attempt to stop at a traffic light. The mass of car A is 1100kg and that of B is 1400kg. The coefficient of kinetic friction between the locked wheels of either car and the road is 0.13. Car A

succeeds in stopping but car B does not and subsequently collides with car A. After the collision, car A stops 8.2m ahead of its position at impact and car B 6.1m ahead. Both drivers had their brakes locked throughout the incident.

a. What are the speeds of car A and car B immediately after the impact?
b. What was the speed of car B when it struck car A?

a. After the collision each car moves with some speed and friction does work and brings each car to rest after a certain distance.

Thus $W = -\frac{1}{2}mv^2_{\text{after collision}} = -\mu_k mgd \rightarrow v = \sqrt{2\mu_k gd}$. For car A, the velocity after collision is 4.57 m/s while for car B we have 3.94 m/s.

b. Using conservation of momentum we have

$$m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf} \rightarrow v_{Bi} = \frac{m_A v_{Af} + m_B v_{Bf}}{m_B} =$$

$$\frac{(1100kg \times 4.57\tfrac{m}{s}) + (1400kg \times 3.94\tfrac{m}{s})}{1400kg} = 7.53\tfrac{m}{s}$$

6. A railroad freight car of mass 3.2x10^{4}kg collides with a stationary caboose. After the collision the cars are coupled together and 27% of the initial kinetic energy has been lost during the collision. What is the mass of the caboose?

From conservation of momentum and total energy we have $m_{car}v_{icar} = (m_{car} + m_{cab})V_f$ and $\frac{1}{2}m_{car}v^2_{icar} = \frac{1}{2}(m_{car} + m_{cab})v^2_f + E_{\text{loss due to coupling}}$ where the energy loss due to coupling is 27% if the initial kinetic energy. Thus we have incorporating the V_f from momentum and the loss to coupling

$$\tfrac{1}{2}m_{car}v^2_{icar} - 0.27\left(\tfrac{1}{2}m_{car}v^2_{icar}\right) = \tfrac{1}{2}(m_{car} + m_{cab})v^2_f = \tfrac{1}{2}(m_{car} + m_{cab})\frac{m^2_{car}v^2_{icar}}{(m_{car} + m_{cab})^2}$$

$$(1-0.27)\tfrac{1}{2}m_{car}v^2_{icar} = \tfrac{1}{2}m_{car}v^2_{icar}\left(\frac{m_{car}}{m_{car} + m_{cab}}\right) \rightarrow 0.73 = \left(\frac{m_{car}}{m_{car} + m_{cab}}\right)$$

$$\therefore m_{cab} = (1.37 - 1)m_{car} = 0.37 \times 3.2 \times 10^4 kg = 1.18 \times 10^4 kg$$

7. In the basement of Science and Engineering we routinely accelerate alpha particles into targets of various elements. A famous experiment called Rutherford's experiment fires alpha particles at a target of gold. An alpha particle (a helium nucleus) is accelerated to a certain speed and makes an elastic head-on collision with a stationary gold nucleus. What percentage of its original kinetic energy is transferred to the gold nucleus?

To calculate the percentage transferred we need to calculate the final kinetic energy of the alpha particle. From class we have $v_\alpha = \frac{m_\alpha - m_{Au}}{m_\alpha + m_{Au}} v_{\alpha i} = \frac{4-197}{4+197} v_{\alpha i} = -0.960 v_{\alpha i}$.

Thus we have

$$KE_{\alpha i} = KE_{\alpha f} + KE_{Auf} \rightarrow KE_{Auf} = KE_{\alpha i} - KE_{\alpha f} = KE_{\alpha i}\left(1 - (-0.96)^2\right) = 0.0784 KE_{\alpha i}$$

$\therefore 7.84\%$ is transferred.

8. Suppose that a block of mass m_1 traveling to the right with speed v_{1i} collides with another block of mass m_2 traveling to the left (at block 1) with speed v_{2i}. If the blocks collide, what are the final velocities of the two blocks? (Hint: make sure that you explore all cases for the masses of the blocks to see if your solutions make sense.)

   Applying conservation of momentum and kinetic energy we find

   $$m_1 v_{1i} - m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

   $$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 .$$

   The next step is to rewrite the equation for conservation of momentum as $m_1(v_{1i} - v_{1f}) = -m_2(v_{2i} - v_{2f})$ and kinetic energy as

   $$m_1(v_{1i}^2 - v_{1f}^2) = -m_2(v_{2i}^2 - v_{2f}^2) \rightarrow m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2(v_{2i} - v_{2f})(v_{2i} + v_{2f})$$

   Dividing theses last two expressions and solving for (say) v_{1f} we get $v_{1f} = v_{2i} + v_{2f} - v_{1i}$. Substituting this into the equation for conservation of momentum and solving for v_{2f} we obtain $v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$. Using this in the expression we derived for v_{1f} we have $v_{1f} = \frac{2m_2}{m_1 + m_2} v_{2i} + \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$.

   Special cases: if $v_{2i} = 0$ then the results reduce to what we say in class.

   If both blocks are moving with positive initial velocities and $m_2 >> m_1$, $v_{2f} \sim v_{2i}$ and $v_{1f} \sim 2v_{2i} - v_{1i}$

   If both blocks are moving with positive initial velocities and $m_1 >> m_2$, $v_{1f} \sim v_{1i}$ and $v_{2f} \sim 2v_{1i} - v_{2i}$

   If $m_1 = m_2$ and both are moving with positive initial velocities then $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$ and the two objects exchange velocities.

   If $m_1 = m_2$ and both are moving oppositely directed initial velocities then $v_{1f} = -v_{2i}$ and $v_{2f} = -v_{1i}$ and the two objects exchange velocities and move off opposite to their original directions.

9. Suppose that we again accelerate an alpha particle at an oxygen nucleus this time. Suppose that the alpha particle is incident along the x-axis and makes a grazing collision with a stationary oxygen atom. Suppose further that the alpha particle is scattered through an angle of 64° with respect to its initial direction and that the oxygen nucleus recoils at an angle of 51° below the +x-axis. What are the initial and final speeds of the alpha particle if the speed of the oxygen atom is 1.20×10^5 m/s?

Setting the problem up as we did in class we have for conservation of momentum in the x and y directions

$$p_x: \quad m_\alpha v_{i\alpha} = m_\alpha v_{\alpha f}\cos 64 + m_o v_{of}\cos 51 \rightarrow 4v_{i\alpha} = 1.75v_{\alpha f} + 1.21\times10^6$$

$$p_y: \quad 0 = m_\alpha v_{\alpha f}\sin 64 - m_o v_{of}\sin 51 \rightarrow v_{\alpha f} = \frac{1.49\times10^6}{3.595} = 4.14\times10^5\ \tfrac{m}{s}$$

$$\therefore v_{i\alpha} = \frac{1.75v_{\alpha f} + 1.21\times10^6}{4} = 4.84\times10^5\ \tfrac{m}{s}$$

10. A shell is shot with an initial velocity of 20m/s at an angle of 60° with respect to the horizontal. At the top of the trajectory the shell explodes into two fragments of equal mass. Immediately after the explosion, one fragment whose speed is zero falls vertically to the ground. How far from the launcher does the other fragment land, assuming that the ground is level and that air drag is negligible?

We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the +x axis is rightward, and the +y direction is upward. The y component of the velocity is given by $v = v_{iy}$-gt and this is zero at time $t=v_{iy}/g=(v_i/g)\sin\theta_o$, where v_i is the initial speed and θ_o is the firing angle. The coordinates of the highest point on the trajectory are $x = v_{ix}t = v_i t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\text{m/s})^2}{9.8\text{m/s}^2}\sin60\cos60 = 17.7\text{ m}$ and $y = v_{iy}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\frac{m}{s})^2}{9.8\frac{m}{s^2}}\sin^2 60° = 15.3\text{ m}$. Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_o\cos\theta_o$, in the positive x direction. Let M be the mass of the shell and let V_o be the velocity of the fragment. Then $Mv_o\cos\theta_o=MV_o/2$, since the mass of the fragment is M/2. This means $V_0 = 2v_0\cos\theta_0 = 2(20\text{m/s})\cos60° = 20\text{ m/s}$. This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time t=0 with a speed of 20m/s from a location having coordinates x_o=17.7m, y_o=15.3m. Its y coordinate is given by $y=y_o-(1/2)gt^2$, and when it lands this is zero. The time of landing is $t=\sqrt{(2y_o/g)}$ and the x coordinate of the landing point is $x = x_i + V_i t = x_i + V_i\sqrt{\frac{2y_i}{g}} = 17.7\text{m} + 20\text{m/s}\sqrt{\frac{2\times15.3\text{m}}{9.8\frac{m}{s^2}}} = 53\text{ m}$

11. Three particles are located in an x-y coordinate system. Particle 1 is located at the origin (x,y) = (0m,0m) and has mass 3kg. Particle 2 is located at the position (2m,1m) and has mass 4kg while particle 3 is located at position (1m,2m) and has mass 8kg.
    a. What are the x- and y- coordinates of the center of mass of the three particle system?
    b. What happens to the center of mass as the mass of the top most particle gradually increases?

a.

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{0 + 8kg \times 1m + 4kg \times 2m}{3kg + 8kg + 4kg} = 1.1m$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{0 + 8kg \times 2m + 4kg \times 1m}{3kg + 8kg + 4kg} = 1.3m$$

b. As the mass of the topmost particle is increased, the center of mass shifts toward that particle. As we approach the limit as the topmost particle is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of that particle.