**Physics in Modern Medicine** **Fall 201o0**

**Homework #6**

**Chapter 5 – X-Ray Vision – The problems**

**Problems**

P5.2 a) What fraction of 140 keV x-rays incident upon a 0.5 mm thick lead apron will be transmitted?

The mass absorption coefficient of lead for 140 keV x-rays is: $\mu_m = 2.0\ cm^2\ g^{-1}$

The density of lead is: $\rho = 11.3\ g/cm^3$.

We can compute the attenuation coefficient for 140 keV x-rays from:

$$\mu = \mu_m \times \rho = 2.0\ cm^2\ g^{-1} \times 11.3\ g/cm^3 = 23\ cm^{-1}$$

We then can use the exponential fall-off of the x-ray intensity to find out how many x-rays will be transmitted through the apron. Here, $x = 0.5\ mm = 5 \times 10^{-2}$ cm.

$$\begin{aligned} I &= I_o\ e^{-\mu x} \\ &= I_o\ e^{-(23 cm^{-1}) \times 5\times10^{-2} cm} \\ &= I_o\ e^{-1.15} \\ &= 0.3\ I_o \\ &= 30\%\ I_o \end{aligned}$$

So, 30% of the original x-ray intensity is transmitted through the lead apron. See comments following part b).

b) How thick would the lead shielding in a wall of an x-ray experimental laboratory have to be to reduce the intensity of 8.0 keV x-rays to 1% of its original value?

The mass absorption coefficient of lead for 8.0 keV x-rays is: $\mu_m = 232\ cm^2\ g^{-1}$

Since we know the density of lead from part a), we can compute the attenuation coefficient of lead for 8.0 keV x-rays:

$$\mu = \mu_m \times \rho = 232\ cm^2\ g^{-1} \times 11.3\ g/cm^3 = 2620\ cm^{-1}$$

We can now compute the relationship between the desired x-ray transmission and the thickness of lead, x, required:

$$\begin{aligned} I &= 1\% I_o = 0.01 I_o \\ &= I_o\ e^{-\mu x} \\ &= I_o\ e^{-(2620 cm^{-1})x} \\ & or: \end{aligned}$$

$$0.01 = e^{-(2620 cm^{-1})x}$$

We can use the natural logarithm function (ln) to extract out the value of x:

$$0.01 = e^{-\left(2620cm^{-1}\right)x}$$

$$\ln(0.01) = \ln\left(e^{-\left(2620cm^{-1}\right)x}\right)$$

$$-4.6 = -\left(2620cm^{-1}\right)x$$

$$x = \frac{4.6}{2620cm^{-1}} = 1.8 \times 10^{-3}\, cm$$

(Note: you can always check this answer by using this value of x in the equation for the transmitted intensity, to make sure it really does give you 1% transmission.) This is a very, very thin piece of lead indeed. Now, we return to the results of part a). Note that shielding low energy x-rays is relatively easy. It requires only a very thin lead foil. The apron from part a) would give excellent shielding of these x-rays. To shield higher energy x-rays--energies corresponding to the high end of those used in diagnostic imaging--requires much thicker lead shielding. A lead apron with the thickness given in part a) would be adequate to shield lower energy x-rays commonly used in imaging, for example, but a thicker layer of lead would be necessary to shield higher energy x-rays. (As noted in the previous problem, 1 mm of lead absorbs all but 0.016% of 50 keV x-rays, so the lead apron described here would actually provide quite good shielding for the x-rays used in, say, mammography.)

P5.3 a) For the case of the 1 cm thick rib, we have x = 1 cm and for 20 keV $\mu_{bone} = 4.8\ cm^{-1}$

$$\text{Percent transmission} = 100\% \times e^{-\mu x}$$

$$= 100\% \times e^{-4.8 cm^{-1} \times 1\, cm}$$

$$= 0.82\%$$

Note that if we always use the lower value for For 60 keV, $\mu_{bone} = 0.55\ cm^{-1}$

$$\text{Percent transmission} = 100\% \times e^{-0.55 cm^{-1} \times 1\, cm}$$

$$= 58\%$$

From the text, we learned that $2.1 \times 10^{-7}$ of the original beam is transmitted by 20 cm of soft tissue alone for 20 keV x-rays; for 60 keV x-rays, $1.5 \times 10^{-2}$ of the original x-rays are transmitted. We multiply this factor by the two results above to get a total transmission at 20 keV of $0.82\ \% \times 2.1 \times 10^{-7} = 1.7 \times 10^{-7}\ \%$ and at 60 keV of 0.87 %.

(b) For the 4 cm region of breast tissue, we have transmissions of:

20 keV: $\frac{I_{trans}}{I_o} = e^{-\mu x} = e^{-0.76 cm^{-1} \times 4 cm} = 0.048$

60 keV: $\frac{I_{trans}}{I_o} = e^{-\mu x} = e^{-0.20 cm^{-1} \times 4 cm} = 0.43$

As pointed out in the text, more higher energy x-ray photons are transmitted through breast tissue to develop the image receptor. However, as the following problem shows, raising the energy degrades the contrast.

P5.4 a) For the case of the 1 cm thick rib, we have x = 1cm and for 20 keV: $\mu_2$ = 0.76 $cm^{-1}$ and $\mu_1$= 4.8 $cm^{-1}$

$$\text{Percent difference} = 100\% \times \frac{1 - e^{-(\mu_1 - \mu_2)x}}{1 + R}$$

$$= 100\% \times \frac{1 - e^{-(4.8 - 0.76)\,\text{cm}^{-1} \times 1\,\text{cm}}}{1 + 0}$$

$$= 92\%$$

Note that we have chosen our values of absorption coefficient such that $\mu_2$ is always the lower value. This ensures that the contrast varies between 100% (for very different values of transmission) to 0% (for identical transmission). This choice of values means that the rib is less transmitting than the soft tissue. For 60 keV: $\mu_2$ = 0.20 $cm^{-1}$ and $\mu_1$= 0.55 $cm^{-1}$

$$\text{Percent difference} = 100 \times \frac{1 - e^{-(0.55 - 0.20)\,\text{cm}^{-1} \times 1\,\text{cm}}}{1 + 0}$$

$$= 30\%$$

Again, the ordering of absorption coefficients reflects the fact that the bone is less transmitting/more absorbing than the soft tissue.

b) Now the thickness, x, is still 1 cm and for air, $\mu_2$=0 always. For 20 keV: $\mu_1$ = 0.76 $cm^{-1}$

$$\text{percent difference} = 100 \times \frac{1 - e^{-(0.76 - 0)\,\text{cm}^{-1} \times 1\,\text{cm}}}{1 + 0}$$

$$= +53\%$$

For 60 keV: $\mu_1$ = 0.20 $cm^{-1}$

$$\text{Percent difference} = 100 \times \frac{1 - e^{-(0.20 - 0)\,\text{cm}^{-1} \times 1\,\text{cm}}}{1 + 0}$$

$$= +18\%$$

Both contrast values indicate that more transmission occurs for the air sac. For 20 keV a greater increase is seen than at 60 keV, again indicating the increased contrast at lower energies.

P5.5 a) For the case of the microcalcification, we have x = 0.1 mm = $10^{-2}$ cm and for 20 keV: $\mu_1$ = 0.6 $cm^{-1}$ and $\mu_2$= 4.8 $cm^{-1}$

$$
\begin{aligned}
C &= \frac{I_1 - I_2}{I_1} \\
&= 100\% \times \left(1 - e^{-(\mu_2 - \mu_1)x}\right) \\
&= 100\% \times \left(1 - e^{-(4.8-0.5)\mathrm{cm}^{-1} \times 10^{-2}\ \mathrm{cm}}\right) \\
&= 100\% \times \left(1 - e^{-0.043}\right) \\
&= 4\%
\end{aligned}
$$

I'll comment on this result after doing the calculation for part b).

b) For 60 keV: $\mu_1$ = 0.17 cm$^{-1}$ and $\mu_2$= 0.55 cm$^{-1}$ for mineralized bone, used to model the microcalcification's absorption coefficient. (Here I've chosen the greatest difference in attenuation coefficients to give the best possible contrast. You could have chosen the other value of 0.47 to be conservative.) The contrast, C, or percentage difference in x-ray intensity transmitted, is given by:

$$
\begin{aligned}
C &= 100\% \times \left(1 - e^{-(\mu_2 - \mu_1)x}\right) \\
&= 100\% \times \left(1 - e^{-(0.55-0.17)\mathrm{cm}^{-1} \times 10^{-2}\ \mathrm{cm}}\right) \\
&= 100\% \times \left(1 - e^{-0.0038}\right) \\
&= 3.8 \times 10^{-3} = 0.38\%
\end{aligned}
$$

The positive sign in both cases means that the microcalcification is less transmitting/more absorbing than fat, so that $I_1$ (the transmission through fat alone) is greater than $I_2$ (the transmission through fat plus the microcalcification); for 20 keV, a much greater difference in transmission occurs for the two tissues than is the case for 60 keV x-rays. In fact, only an image made with 20 keV x-rays would be able to distinguish the microcalcification given an x-ray film/phosphor combination only sensitive to contrasts greater than about 1%.

c) Now the thickness, x= 0.1 cm. For 20 keV: $\mu_1$ = 0.5 cm$^{-1}$ and $\mu_2$= 0.76 cm$^{-1}$

$$
\begin{aligned}
C &= \frac{I_1 - I_2}{I_1} \\
&= 100\% \times \left(1 - e^{-(\mu_2 - \mu_1)x}\right) \\
&= 100\% \times \left(1 - e^{-(0.76-0.5)\mathrm{cm}^{-1} \times 10^{-1}\ \mathrm{cm}}\right) \\
&= 100\% \times \left(1 - e^{-0.026}\right) \\
&= 2.6\%
\end{aligned}
$$

See comments below after the calculation for part d)

d) For 60 keV: For 20 keV: $\mu_1$ = 0.17 cm$^{-1}$ and $\mu_2$= 0.20 cm$^{-1}$

$$
\begin{aligned}
C &= 100\% \times \left(1 - e^{-(\mu_2 - \mu_1)x}\right) \\
&= 100\% \times \left(1 - e^{-(0.20-0.17)\text{cm}^{-1} \times 10^{-1}\text{ cm}}\right) \\
&= 100\% \times \left(1 - e^{-0.0030}\right) \\
&= 3 \times 10^{-3} \\
&= 0.3\%
\end{aligned}
$$

For 20 keV the contrast is again significantly higher than for 60 keV, and only the 20 keV case would be detectable on the film/phosphor combination.

Thus, we see that for both of these cases, only the 20 keV case would correspond to a detectable image, even neglecting the effects of scattering, noise, etc. Thus, although the 20 keV x-rays provide a higher x-ray dose than would 60 keV x-rays, they are essential for imaging the possible signs of a tumor. In both cases, we see that these numbers come out close to the limits of detectability anyway, showing that microcalcifications smaller than 0.1 mm are considered to be undetectable with most current mammography setups, and that small solid tumors are also difficult to distinguish. Earlier mammography systems used in the early 1970's were actually unable to perform at this level, and consequently did not provide adequate mammograms for detecting early breast cancer. It is hoped that ongoing improvements in x-ray imaging will lead to even better detection rates, *improving* the rates of breast cancer detection and cures relative to those observed in the population studies to date. (In fact, these studies could not really assess improvements available since the mid-1980's, since not enough time has elapsed since then to evaluate their effectiveness.)