Physics 210
Medical Physics
Midterm Exam
Winter 2019
February 15, 2019

Name__________________________

Problem 1	/24
Problem 2	/24
Problem 3	/24
Total	/72

For the exam, you may use your notes, any Power Point slides you'd like and your textbook. You may not use old exams, homework solutions or use worked out solutions to problems.

I affirm that I have carried out my academic endeavors with full academic honesty.

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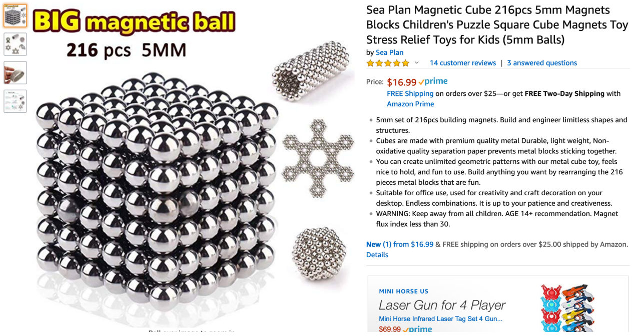
Signature

1. *X-ray imaging of small objects*

Accidental ingestion of foreign bodies is a common problem in children. One such recent hazard is the ingestion of small rare earth magnets from toys. When ingested, the magnets can have potentially lethal consequences if not immediately treated. The magnets may lodge in the esophagus, the stomach, or in any segment of the bowel. In the stomach or bowel, the magnets can bind together to form obstructions, which can lead to severe complications if left untreated. According to the American Academy of Pediatrics 100's of cases of magnet ingestion are reported annually by emergency rooms across the US. These toys sold could contain hundreds of small magnets and it's hard to tell if a few have gone missing by say a parent. An image of such a toy is shown below in Figure 1.



Sea Plan Magnetic Cube 216pcs 5mm Magnets Blocks Children's Puzzle Square Cube Magnets Toy Stress Relief Toys for Kids (5mm Balls)

by Sea Plan

★★★★★ 14 customer reviews | 3 answered questions

Price: $16.99 ✓prime

FREE Shipping on orders over $25—or get **FREE Two-Day Shipping** with Amazon Prime

- 5mm set of 216pcs building magnets. Build and engineer limitless shapes and structures.
- Cubes are made with premium quality metal Durable, light weight, Non-oxidative quality separation paper prevents metal blocks sticking together.
- You can create unlimited geometric patterns with our metal cube toy, feels nice to hold, and fun to use. Build anything you want by rearranging the 216 pieces metal blocks that are fun.
- Suitable for office use, used for creativity and craft decoration on your desktop. Endless combinations. It is up to your patience and creativeness.
- WARNING: Keep away from all children. AGE 14+ recommendation. Magnet flux index less than 30.

New (1) from $16.99 & FREE shipping on orders over $25.00 shipped by Amazon. Details

MINI HORSE US

Laser Gun for 4 Player

Mini Horse Infrared Laser Tag Set 4 Gun...

$69.99 ✓prime

Figure 1: An ad from Amazon.com showing a set of 216 5-mm spherical magnets that you can by as a toy for children.

Consider the film x-ray image shown below (Figure 2) taken of a 3-year-old boy in the ER showing seven magnets lodged in his lower esophagus and upper stomach. The upper two magnets are in the esophagus while the remaining lower five are in the upper stomach.

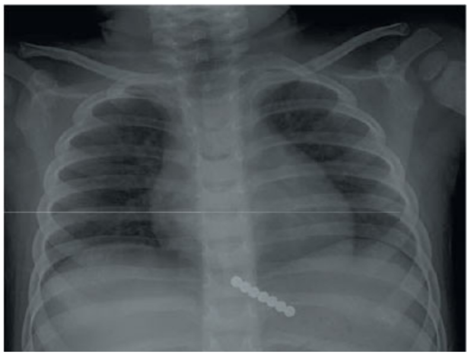


Figure 2: X-ray image of a 3-year-old child with a set of magnets lodged in his gastro-intestinal tract.
http://www.scielo.org.za/scielo.php?script=sci_arttext&pid=S0256-95742014000400017#f1

a. Figure 3 below shows the x-ray energy spectrum for the x-rays that were used to produce the film image shown in Figure 2. Explain how is this energy spectrum generated? In particular, address what the operating potential of the x-ray tube is and what material the anode is made?

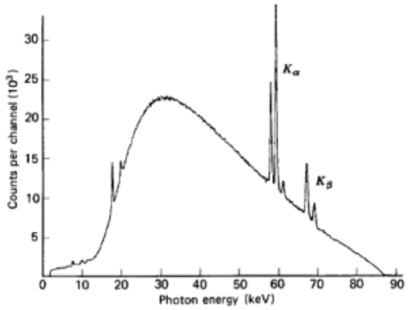


Figure 3: X-ray energy spectrum for an unknown anode material showing the intensity (number of x-rays) as a function of their energy.

Element	$K_\alpha(keV)$	$K_\beta(keV)$
Ni	7	8
Cu	8	9
Mo	18	20
W	60	67
Pt	67	76
Au	69	78

Table 1: Some useful data for determining the anode material.

Electrons are produced at the cathode of the x-ray tube and are accelerated from rest through a potential difference. The potential difference above is about 87kV. The electrons interact with atoms in the anode material and are decelerated. This produces the continuous background called bremsstrahlung. A small fraction of the time the incident electron ejects electrons from the atoms in the target material and this creates vacancies in the target material's atoms. The vacancies are filled with electrons from a higher orbital and in the de-excitation produce x-ray photons of a characteristic energy. These are the sharp peaks on top of the bremsstrahlung background. By examining the x-ray energies we suggest that the anode material is tungsten based on the $60keV$ and $66keV$ x-rays emitted. The number of photons in the spectrum is due the current on the x-ray tube.

b. Suppose that the beam of x-rays were directed through the child from front to back as in the image shown in Figure 2. In particular, let the cartoon diagram, shown in Figure 4 represent the structures in the body that the x-ray beam passed through. Further, let the intensity of the emerging x-ray beam be 0.04% of the incident beam intensity, or $I_{out} = 0.0004I_0$. From the information in Table 2 and using Figure 4, how thick was the magnet the child ingested? The magnet is colored blue in Figure 4. Assume that the x-ray beam goes through $0.5cm$ of fat on the belly, $9.0cm$ of liver tissue, $0.4cm$ of stomach wall muscle , $5.2cm$ of air in the stomach, another $0.4cm$ of stomach wall muscle, $4.0cm$ of the vertabra of the spine, $0.5cm$ of fat on the back, and of course the magnet in the stomach. Absorption coefficients and densities of the various structures in the body are given in Table 2.

Structure	$\mu_m\left(\frac{cm^2}{g}\right)$	$\rho\left(\frac{g}{cm^3}\right)$
Magnet	10.3	6.9
Fat/Liver	0.1974	0.95
Stomach/Muscle	0.2048	1.05
Bone	0.3148	1.92
Air	0.1875	0.0012

Table 2: X-ray mass attenuation coefficients and densities of various materials.

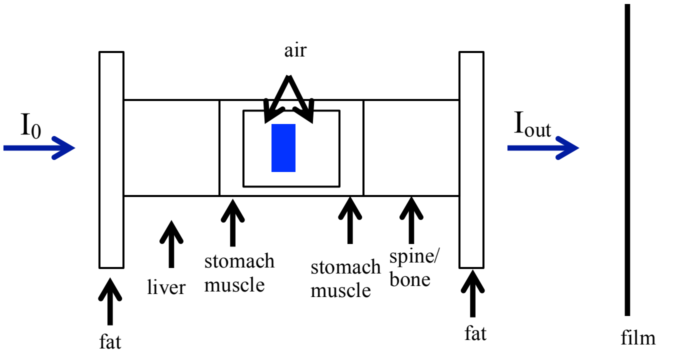


Figure 4: Cartoon version of the path that the x-ray beam takes through the child.

$\mu\left(cm^{-1}\right)$
71.1
0.1875
0.2150
0.6044
0.0012

The attenuation coefficients used in this problem are calculated using $\mu = \mu_m\rho$ and are shown under Table 2.

$$I = I_0 e^{-\mu_{effective}x_{effective}} \rightarrow 0.0004I_o = I_0 e^{-\mu_{effective}x_{effective}} \rightarrow \mu_{effective}x_{effective} = -\ln\left(\frac{0.0004I_o}{I_0}\right) = 7.82$$

$$\mu_{effective}x_{effective} = 2\mu_{fat}x_{fat} + \mu_{liver}x_{liver} + 2\mu_{stomach}x_{stomach} + \mu_{air}\left(x_{air} - x_{magnet}\right) + \mu_{bone}x_{bone} + \mu_{magnet}x_{magnet}$$

$$7.82 = \left(2\times0.5\times0.1875\right) + \left(9cm\times0.1875\right) + \left(2\times0.4\times0.2150\right) + 0.00025\left(5.2 - x_{magnet}\right) + \left(4\times0.6044\right) + \left(x_{magnet}\times71.1\right)$$

$$7.82 = 0.1875 + 1.6875 + 0.172 + 0.0012 - 0.00023x + 2.4176 + 71.1x$$

$$\therefore x = 0.047cm \sim 0.5mm$$

c. Instead of magnet, suppose that a patient presents in the ER complaining of chronic (happening for a long time) stomach pain. It is believed by the ER physician that the patient may be suffering from a stomach ulcer. Stomach ulcers, are painful sores that develop in the stomach lining. The ER physician would like to image the ulcer using an x-ray scan. If the ulcer can be modeled by blood ($\mu_{m,blood} = 0.2057 \frac{cm^2}{g};\ \rho = 1.06 \frac{g}{cm^3}$) and the physician wanted to see a $\frac{1}{4} cm$ thick ulcer in the lining of the stomach wall, what is the contrast between the ulcer and the stomach wall? Assume that the lining of the stomach can be modeled by muscle and use Table 2. Comment on the result that you get using x-rays to image an ulcer.

$$C = \left(1 - e^{-(\mu_m - \mu_{blood})x}\right) \times 100\% = \left(1 - e^{-\left(\left[0.2057 \times 1.06\right] cm^{-1} - 0.2150 cm^{-1}\right) \times 0.25 cm}\right) \times 100\%$$

$$C = 0.08\%$$

Since the contrast between the ulcer in the stomach lining and the stomach lining is 0.08%, these would be almost impossible to distinguish on an x-ray. There are other methods that can be used to image the ulcer. See for example problem 2.

2. *Endoscopy of Stomach Ulcers*

Stomach ulcers, also known as gastric ulcers, are painful sores in the stomach lining. Stomach ulcers occur when the thick layer of mucus that protects your stomach from digestive juices is reduced and this allows the digestive acids to eat away at the tissues that line the stomach, causing the ulcer. Treatment of the ulcer can be done with medicines or with laser surgery. In Figure 5 on the right we see, in the left pane an image of the healthy lining of the stomach while in the right pane an image of the ulcerous lesions in the stomach lining.

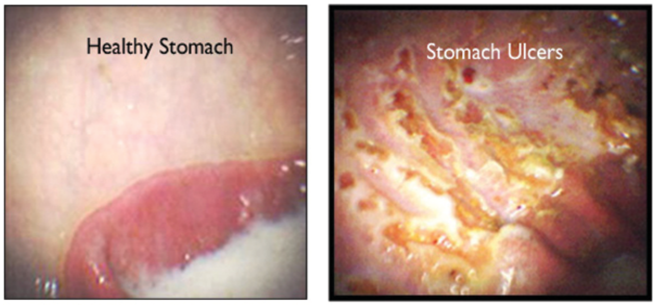


Figure 5: Endoscopic image of the normal lining of the stomach on the left and of ulcerations of the stomach lining on the right.
http://swfhealthandwellness.com/stomach-ulcer-causes-treatment-prevention/

a. To treat the ulcer, we will use an endoscopic procedure that involves use of a laser to cauterize the ulcer. To look into the stomach and subsequently to see the presence or absence of an ulcer, we use the idea of total internal reflection of light. To make use of total internal reflection of light, how should the endoscope be constructed? That is, should $n_{core} < n_{cladding}$ or $n_{core} > n_{cladding}$? Then from Table 3 below, select a core and cladding combination and from the combination determine the critical angle for total internal reflection. If light is incident on the front of the endoscope at an angle of $\theta = 30^0$ with respect to the normal to the front surface, as shown in Figure 6, is the light internally reflected?

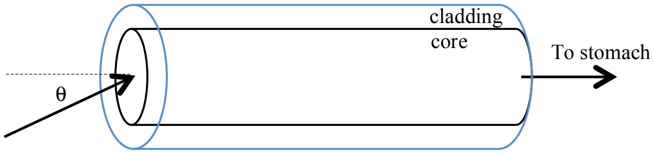


Figure 6: Schematic for an endoscope showing the basic setup and geometry.

Material	n
Silica	1.46
Glass	1.52
Amber	1.55
Sapphire	1.76
Diamond	2.42

Table 3: Various materials and their respective indices of refraction.

To make use out of total internal reflection we require $n_{core} > n_{cladding}$, so that as the light enters the lower index of refraction material the light ray bends away from the normal. As the light in the higher refractive index approaches the critical angle, the angle of refraction in the higher refractive index material will be 90^0. This allows us to define the critical angle as:

$$n_{core}\sin\theta_c = n_{cladding}\sin 90 \rightarrow \theta_c = \sin^{-1}\left(\frac{n_{cladding}}{n_{core}}\right).$$

There are many combinations we could make the endoscope out of. I've picked the one I'd like to make my endoscope out of by madding the cladding out of glass and the core out of sapphire. This, by the way, would be a very expensive endoscope using sapphire.

$$n_{core}\sin\theta_c = n_{cladding}\sin 90 \rightarrow \theta_c = \sin^{-1}\left(\frac{n_{glass}}{n_{saphire}}\right) = \sin^{-1}\left(\frac{1.52}{1.76}\right) = 59.7^0 \; .$$

If light is incident on the front surface at $\theta_{air} = 30^0$, the angle of refraction on the glass surface is:

$$n_{air} \sin\theta_{air} = n_{sapphire} \sin\theta_{sapphire} \rightarrow \theta_{sapphire} = \sin^{-1}\left(\frac{n_{air} \sin\theta_{air}}{n_{sapphire}}\right) = \sin^{-1}\left(\frac{1\times \sin 30}{1.76}\right)$$

$$\theta_{sapphire} = 16.5^0$$

Using the geometry of the system, we have the light incident on the upper surface, say, of the sapphire/glass interface as $\theta_{sapphire} + \theta_{sapphire/glass} = 90^0 \rightarrow \theta_{sapphire/glass} = 73.5^0$. Since this is greater than the critical angle between the sapphire/glass interface, the light will be totally internally reflected.

b. Ulcers have a tendency to bleed if left untreated. If you wanted to treat the ulcer by say using a laser to cauterize the wound, which type of laser would you choose for the procedure and why?

Since the ulcer is mostly blood we require a laser that will be absorbed by hemoglobin. This means that we need a laser that operates in the green, blue, UV portion of the electromagnetic spectrum. The choices of laser could be an argon, eximer or perhaps a Nd-YAG.

c. To cauterize the ulcer you choose a laser that has a power output (per unit area) of $100\frac{W}{cm^2}$. Assuming that a circle of radius $1.25cm$ can describe the ulcer and that the smallest spot size you can focus your laser to is a circle of radius $1.0mm$. Since the laser spot is smaller than the size of the ulcer, you will have to scan the laser across the ulcer to treat the entire ulcer. Each spot you treat on the ulcer with the laser beam has to be irradiated for a time of $0.5s$. How long would it take to treat the entire ulcer and how much total energy is given to the ulcer during the treatment?

The number of laser pulses is given by the ratio of the area of the ulcer to the area of the laser beam. Thus we have:

$$\#_{pulses} = \frac{A_{ulcer}}{A_{laser}} = \frac{\pi r_{ulcer}^2}{\pi r_{laser}^2} = \left(\frac{r_{ulcer}}{r_{laser}}\right)^2 = \left(\frac{12.5mm}{1mm}\right)^2 = 156.25 \sim 157 .$$

Thus the treatment time is $t = \#_{pulses} \times t_{pulse} = 157 \times 0.5s = 78s = 1.3\min$.

The total energy delivered to the ulcer is

$$I = \frac{Power}{Area} = \frac{Energy}{time \times Area} \rightarrow Energy = \left(\frac{Power}{Area}\right) \times time \times Area .$$

$$Energy = 100\frac{W}{cm^2} \times 78s \times \pi\left(1.25cm\right)^2 = 38288J \sim 38kJ$$

3. *Ultrasonography*

Ultrasonography is ideally suited to soft tissue imaging and especially suited to imaging the abdomen. Figure 7 below is normal ultrasound image of the abdomen showing the major structures of the abdomen.

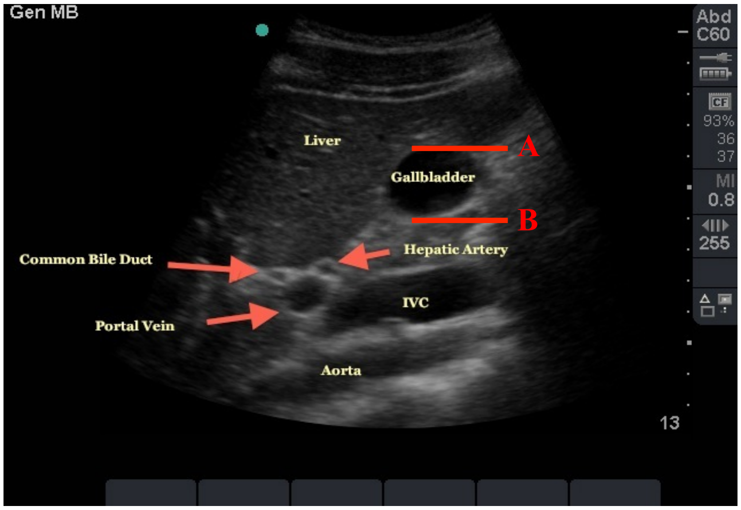


Figure 7: Normal ultrasound image of the abdomen showing the major structures.
http://www.em.emory.edu/ultrasound/ImageWeek/Abdominal/mickey_mouse.html

a. If the size of the normal gallbladder is approximately $4cm$ in diameter, what would the difference in arrival time be at the transducer (located at the top of the image) from US waves reflected from point A compared to those reflected from point B?

Let x be the distance from the transducer to point A. We have

$$\Delta t = t_B - t_a = \frac{2d_B}{v_s} - \frac{2d_A}{v_s} = \frac{2}{v_s}\left(\left[x + x_{gb}\right] - x\right) = \frac{2\times 0.04m}{1540\,\frac{m}{s}} = 5.2\times 10^{-5}s = 52\mu s\,.$$

b. If the intensity of the US waves that are transmitted into the gallbladder at point A is I_0, what is the intensity of the reflected ultrasound waves (as a percent of I_0) at point B? To answer this, model the gallbladder as being filled with gastric fluid that we can treat as if it were blood. The ultrasound attenuation coefficient for US waves in blood is $\mu_s = 0.233 cm^{-1}$ and the acoustic impedances for blood and muscle (the lining of the gallbladder walls) are $Z_{blood} = 1.48 \times 10^6\, Rayl$ and $Z_{muscle} = 1.65 \times 10^6\, Rayl$ respectively.

The intensity that gets to point B from point A has been attenuated. The intensity incident at point B is given by $I_B = I_A e^{-\mu_s x} = I_0 e^{-0.233 cm^{-1} \times 4 cm} = 0.394 I_0$. To calculate what gets reflected we use the refection coefficient given by

$$I_R = I_B \left(\frac{Z_{blood} - Z_{muscle}}{Z_{blood} + Z_{muscle}} \right)^2 = 0.394 I_0 \times \left(\frac{1.48 - 1.65}{1.48 + 1.65} \right)^2 = 0.0012 I_0 .$$

$$\frac{I_R}{I_0} = 0.12\%$$

c. When we talk about US imaging we usually assume that the speed of sound in all tissues is $v_s = 1540 \frac{m}{s}$. This speed of sound is generally programed into the software of the ultrasound scanner and this is used to help form the image of the structure in the body. Of course, in reality, the speed of sound is different in different materials and in some cases it can be drastically different from $v_s = 1540 \frac{m}{s}$. For example, the speed of sound in air is approximately $v_{air} = 340 \frac{m}{s}$ while for bone it is $v_{bone} = 3600 \frac{m}{s}$. What problems could arise in an ultrasound image from assuming that the speed of sound is $v_s = 1540 \frac{m}{s}$? Be as specific as possible.

One problem with all US is refraction of sound as the sound waves pass between two different materials of different densities. Assuming that the speed of sound is constant means that there is very little refraction as the sound waves pass between different structures. This could lead to misidentification of where an abnormality may actually lie. Another problem is with acoustic shadowing by a dense structure in front of a less dense material. We could miss some features of the structures behind the denser material. In addition we also have to worry about the line of sight distances being incorrect. With drastically different speeds the locations of objects could be distorted and not where the sonographer thinks they are. This could lead to incorrect locations/diagnosis of structures involved.