# Physics 210
# Medical Physics
# Midterm Exam
Spring 2021
May 7, 2021

Name___________________________

| | |
|---|---|
| Problem 1 | /35 |
| Problem 2 | /35 |
| Total | /70 |

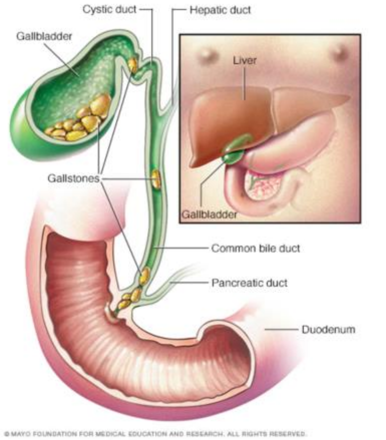
For the exam, you may use your in-class notes, any Power Point slides you'd like, your textbook (Kane), and the readings from Wolbarst. You may not use old exams, homework solutions or use worked out solutions to problems.

*I affirm that I have carried out my academic endeavors with full academic honesty.*

______________________________
Signature

## 1. *Gallstone removal*

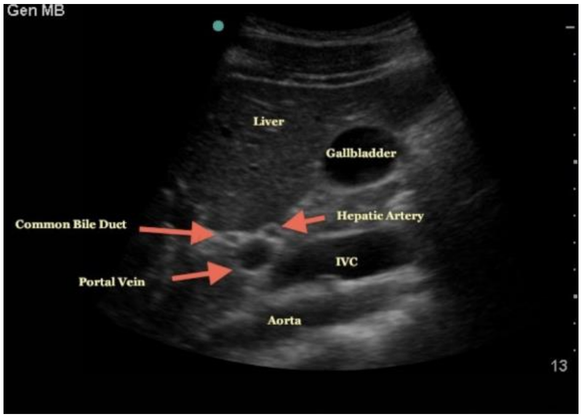
Your gallbladder is a small, pear-shaped organ on the right side of your abdomen, just beneath your liver. The gallbladder holds a digestive fluid called bile that's released into your small intestine that helps to further digest food. Gallstones are hardened deposits of that digestive fluid that can form in your gallbladder. The gallstones (*Cholelithiasis*) can make their way from the gallbladder to the cystic duct and ultimately to the common bile duct. Anywhere along this path, the gallstones may become lodged. The general anatomy is shown in Figure 1 on the right. As much as 6% percent of adult men and 10% of adult women are affected and the cause of gallstone formation is not completely understood.



https://www.mayoclinic.org/diseases-conditions/gallstones/symptoms-causes/syc-20354214#dialogId5183802

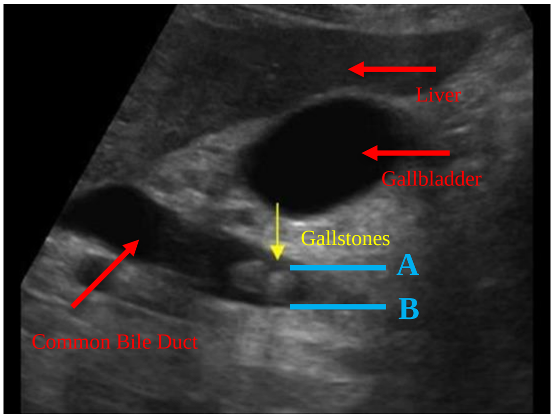
Figure 1: Anatomy of the upper right abdomen showing the location of the liver, stomach, and gall bladder.

Consider the ultrasound images shown below (Figures 2 & 3) taken of a patient who presented in the ER complaining of pain in the upper right side of his abdomen using a $5MHz$ ultrasound beam. The first ultrasound image (Figure 2) shows the basic anatomy from the ultrasound while Figure 3 shows an ultrasound image of two gallstones that are lodged in the common bile duct.



https://coreem.net/core/biliary-ultrasound/

Figure 2: US scan of the upper right abdomen showing the major anatomical features.



https://www.ultrasoundcases.info/gallstones-in-the-extrahepatic-bile-ducts-4760/#gallery-1

Figure 3: US scan of the upper right abdomen showing gallstones lodged in the common bile duct.

a. Suppose that it takes a time $t_A = 51.5\mu s$ for an ultrasound pulse to reflect from the upper surface of the gallstone (labeled point A in Figure 3) and return to the transducer and a time $t_B = 54.0\mu s$ for the ultrasound pulse to reflect from the bottom surface of the gallstone (labeled point B in Figure 3) and return to the transducer. What is the approximate size of the gallstone if we model the gallstone as a calcium deposit surrounded by fat? In addition, what is the wavelength of the US in fat and in the gallstone and based on the values of the wavelength what can you conclude about the ability to see the gallstone surrounded by fat? Hint: table #2 at the end of the exam may be helpful.

$$\Delta t = \frac{t_A}{2} - \frac{t_B}{2} = \frac{x}{v_{s,CD}} \rightarrow x = v_{s,CD}\left(\frac{t_A}{2} - \frac{t_B}{2}\right) = 4080\frac{m}{s}\left(\frac{54}{2} - \frac{51.5}{2}\right) \times 10^{-6}s = 0.0051m$$

$$x = thickness = 5.1mm$$

$$v = f\lambda \rightarrow \lambda = \frac{v_s}{f}$$

$$\lambda_{s,CD} = \frac{v_{s,CD}}{f} = \frac{4080\frac{m}{s}}{5 \times 10^6 s^{-1}} = 0.00082 = 0.82mm$$

$$\lambda_{s,fat} = \frac{v_{s,fat}}{f} = \frac{1460\frac{m}{s}}{5 \times 10^6 s^{-1}} = 0.00029 = 0.29mm$$

Since the wavelength gives the smallest feature size, we can easily see the gallstone in the fat medium since it is an order of magnitude larger than the minimum feature size

b. What is the reflection coefficient for ultrasound waves reflected from the upper surface of the gallstone (labeled as point A in Figure 3) and from the lower surface of the gallstone (labeled as point B in Figure 3)? In addition, how many decibels of intensity loss does the ultrasound wave lose in crossing the gallstone? Assume that sound with intensity $I_0$ strikes the upper surface of the gallstone. Table 2 at the end of the exam may be helpful and you can assume that the gallstones are surrounded by fat.

$R = \left(\frac{z_1 - z_2}{z_1 + z_2}\right)^2 = \left(\frac{z_{CD} - z_m}{z_{CD} + z_m}\right)^2 = \left(\frac{4.7 - 1.4}{4.7 + 1.4}\right)^2 = 0.29 \rightarrow 29\%$ of $I_0$ reflected and thus $71\%$ of $I_0$ transmitted. Of the 71% transmitted, the beam is then attenuated by the gallstone itself. Using table 2, we see that there is $20\frac{dB}{cm}$ worth of intensity loss. For our $0.51cm$ gallstone, we lose about $10.2dB$ of ultrasound intensity. From this we can calculate the attenuation coefficient for sound in the gallstone. From the lecture notes: $dB = 4.3\mu_s x \rightarrow \mu_s = \frac{dB}{4.3x} = \frac{10.2dB}{4.3 \times 0.51cm} = 4.65cm^{-1}$. The US is attenuated according to $I = I_{incident}e^{-\mu_s x}$. Thus, the intensity at the bottom of the gallstone is $I = (0.71I_o)e^{-4.65cm^{-1} \times 0.51cm} = 0.066I_0 \rightarrow 6.6\%$ of $I_0$.

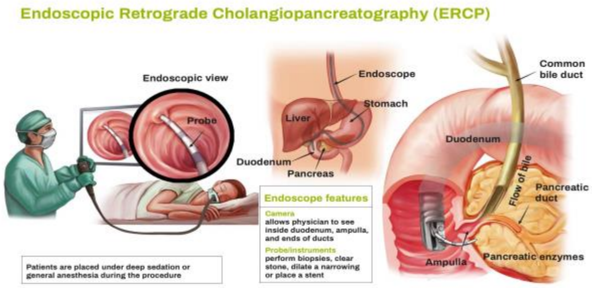
The reflection coefficient at point B: $R = \left(\frac{z_1 - z_2}{z_1 + z_2}\right)^2 = \left(\frac{z_{CD} - z_m}{z_{CD} + z_m}\right)^2 = \left(\frac{4.7 - 1.4}{4.7 + 1.4}\right)^2 = 0.29 \rightarrow$ $29\%$ of $0.066I_0 = 0.019I_0 \rightarrow 1.9\%$ of $I_0$.

c. The emergency room physician has determined from the ultrasound scan that the gallstone is indeed lodged in the common bile duct and will not pass on its own. The ER physician decides that the patient needs surgery to remove the stones and relieve the pain. The surgery, called *Endoscopic Retrograde Choloangio-Pancreatography* (ERCP) is an endoscopic procedure where a scope is directed down the esophagus, through the stomach and upper duodenum to the common bile duct. The procedure is cartooned in Figure 4 below. Once in the common bile duct the surgeon can assess the type of technique to employ to remove the stone. One common way to remove the stone is to use a laser to shock the stone and break the stone apart into smaller pieces that can then be removed by the surgeon. Suppose that the surgeon elects to use a pulsed Holmium:YAG laser with a $2100nm$ wavelength, a pulse rate of $20Hz$, and a pulse energy of $0.5\frac{J}{pulse}$. What treatment time would you need to break up the gallstone if you need to deliver approximately $3kJ$ of energy to the stone? Does the time you calculate seem reasonable?

$$P = \frac{E}{t} \rightarrow t = \frac{E}{P} = \frac{3000J}{0.5\frac{J}{pulse}\times 20\frac{pulse}{s}} = 300s$$

This corresponds to about 5 minutes.

Probably reasonable for a treatment time. The procedure to shatter the stone and its subsequent removal is probably on the order of about 15 minutes total. This seems ok. What takes a long time would be prepping the patient and waiting for the patient to recover from the anesthesia.



https://www.gutworks.com.au/endoscopy-ercp-eus-procedure-murdoch-perth/

Figure 4: Endoscopic Retrograde Choloangio_Pancreatogrphy procedure to remove a gallstone.

d. Suppose that during the ERCP procedure and subsequent removal of gallstone pieces (shown in Figure 5 below) the stone accidentally makes some small tears in the common bile duct that bleed. Of the possible lasers available for laser surgery, which one (or ones) would the surgeon want to use to cauterize the bleeds produced by the stone. Justify your answer as to why you chose this type of laser.

Since there is blood involved, we want a laser that does not emit in the red portion of the electromagnetic spectrum. From the Kane text, suitable choices are perhaps an argon ion (514nm), Excimer (190nm – 250nm in the UV and depends on the lasing material) or a copper vapor (510nm).

e. In Figure 5, we see an image of a gallstone piece being extracted using a tool fed through the endoscope. The procedure is viewed on a television screen in the operating room. Explain (in sufficient detail) how the image as created.

http://www.edoriumjournalofsurgery.com/archive/2017-archive/2017-images-archive/100023S05MB2017-baghdadi/figure1.gif

Figure 5: Endoscopic image of a gallstone being removed from the common bile duct.

This image is made by total internal reflection in the fiber optics of the endoscope. Light in the fiber optic is channeled (with some of the light exceeding the critical angle of the scope) and thus is transmitted down the scope. This light is just to illuminate the target. The light that returns to the observer exceeds the critical angle of the fiber optic and if we have a collection of ordered fibers then we get a high-resolution image produced.

## 2. X-ray Imaging

a. Figure 6 below shows the x-ray energy spectrum for x-rays generated from an unknown anode. Explain how is this energy spectrum generated for the upper non-shaded spectrum? In particular, what is the operating potential of the x-ray tube, and the material the anode is made out of? For the lower shaded spectrum, what would explain the difference in the spectrum height from the upper unshaded spectrum?

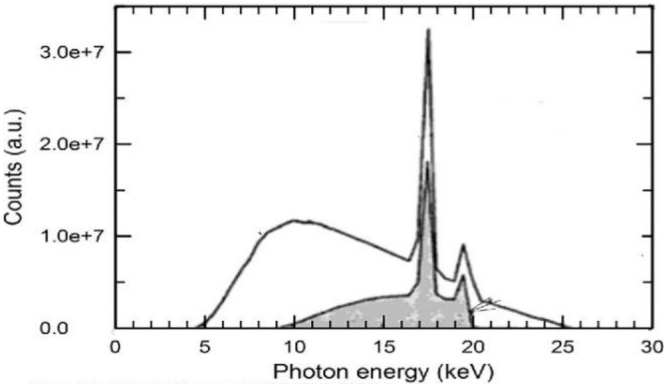


Figure 6: X-ray energy spectrum for an unknown anode material showing the intensity (number of x-rays) as a function of their energy.

| Z | Element | $K_{\alpha}(keV)$ | $K_{\beta}(keV)$ |
|---|---|---|---|
| 28 | Ni | 7 | 8 |
| 29 | Cu | 8 | 9 |
| 42 | Mo | 18 | 20 |
| 74 | W | 60 | 67 |
| 78 | Pt | 67 | 76 |
| 79 | Au | 69 | 78 |

Table 1: Some useful data for determining the anode material.

This shaded and non-shaded spectrua ire made by accelerating electrons from a filament toward an anode made from some material. The material seems to be Molybdenum (based on the location of the characteristic x-rays produced) and why explained below. When the electrons interact with the anode they decelerate and radiate. This produces the background called bremsstrahlung. The characteristic x-ray peaks are due to ejection of electrons from the inner shells of the anode material. When electrons in the anode material transition from higher energy states to fill the vacancy, they release a photon of light. This photon of light is in the x-ray portion of the electromagnetic spectrum and the x-rays that are produced are characteristic of the material out of which the anode is made. Since the x-rays have energies approximately 17.5keV and 19.5keV we can identify the target as Molybdenum. The shaded spectrum is due to some filter material placed in front of the emerging x-ray beam between the beam and the target. The filter lowers the background and the number of characteristic x-rays produced. The filter does not change the energy of the characteristic x-rays.

b. If the x-ray tube in Figure 6 above produced a current of $I = 250mA$, what is the power delivered to the anode, the power released in the form of x-rays and the efficiency of x-ray production? Use the non-shaded curve to answer this.

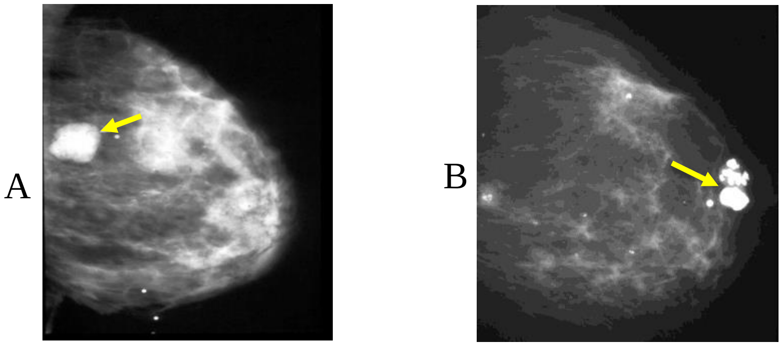
$$P_d = IV = 250 \times 10^{-3}A \times 2.5 \times 10^{4}V = 6250W$$

$$P_r = 0.9 \times 10^{-9}\tfrac{C}{J} \times ZIV^2 = 0.9 \times 10^{-9}\tfrac{C}{J} \times (42) \times 250 \times 10^{-3}A \times (2.5 \times 10^{4}V)^2$$

$$\rightarrow P_r = 5.9W$$

$\epsilon = \frac{P_r}{P_d} = \frac{5.9W}{6250W} = 0.0095 \rightarrow 0.95\%$ of the incident electrons are converted to x-rays.

c. Breast cancer is a disease in which malignant (cancer) cells form in the tissues of the breast. The damaged cells can invade surrounding tissue, but with early detection and treatment, most people continue a normal life. One in eight women will be diagnosed with breast cancer in her lifetime. Breast cancer is the most diagnosed cancer in women and is the second leading cause of cancer death among women. Each year it is estimated that over 275,000 women in the United States will be diagnosed with breast cancer and more than 40,000 will die. Although breast cancer in men is rare, an estimated 2,600 men will be diagnosed with breast cancer and approximately 500 will die each year. Over 3.5 million breast cancer survivors are alive in the United States today. (Facts from http://www.nationalbreastcancer.org/breast-cancer-facts). To that end, women with a family history of breast cancer or women over the age of 45 are encouraged to get a mammogram yearly. A mammogram is an x-ray image of the breast. Suppose you have the two images of the breast shown below in Figure 7. For x-rays (which is what we approximately have produced in Figure 7 below), the mass attenuation coefficients for soft tissue, bone, and breast tissue are $\mu_{m,ST} = 0.7616\frac{cm^2}{g}$ , $\mu_{m,CD} = 4.00\frac{cm^2}{g}$ , and $\mu_{m,BT} = 0.6889\frac{cm^2}{g}$ , respectively. What is the contrast for a $0.5mm$ microcalcification (modeled as a calcium deposit) in breast tissue and for a $0.5mm$ soft tissue lump in breast tissue?



http://www.meddean.luc.edu/lumen/meded/radio/curriculum/surgery/mammography1.htm

http://www.meddean.luc.edu/lumen/meded/radio/curriculum/surgery/mammography1.htm

Figure 7: X-ray images of the breast.

$$\mu_{CD} = \mu_{m,CD}\rho_{CD} = 4.00\frac{cm^2}{g} \times 1.550\frac{g}{cm^3} = 6.2cm^{-1}$$

$$\mu_{ST} = \mu_{m,ST}\rho_{ST} = 0.7616\frac{cm^2}{g} \times 0.901\frac{g}{cm^3} = 0.686cm^{-1}$$

$$\mu_{BT} = \mu_{m,BT}\rho_{BT} = 0.6889\frac{cm^2}{g} \times 0.901\frac{g}{cm^3} = 0.621cm^{-1}$$

$$C_{CD,BT} = 1 - e^{-(\mu_2-\mu_1)x_2} = 1 - e^{-(6.2cm^{-1}-0.621cm^{-1})0.05cm} = 0.24 \rightarrow C_{CD,BT} = 24\%$$

$$C_{ST,BT} = 1 - e^{-(\mu_2-\mu_1)x_2} = 1 - e^{-(0.686cm^{-1}-0.621cm^{-1})0.05cm} = 0.003 \rightarrow C_{ST,BT} = 3\%$$

d. Examining the two CT images (A and B) above which image do you think corresponds to the micro-calcification and which corresponds to the soft tissue lump? The arrows in the images point to the suspects. Justify your answer.

   Based on the results from part c and in light of the definition of the Houndsfield unit, we can state the image in Figure 7B is the macrocalcification and the image in Figure 7A is the soft tissue lump. We make this claim since the contrast between the microcalcification and the surr0unding breast tissue is so large and this can be seen easily on Figure 7B. In addition, using the definition of the Houndsfield unit, the macrocalcification should be more absorbing (it has a higher density) and thus will appear brighter white on the CT scan.

e. For a mammogram, the breast is usually compressed before the x-ray image is taken. What would the advantages and/or disadvantages of breast compression in taking the x-ray images above?

   Advantages:
      Thinner less photoelectric absorption.
      Less of a dose of x-rays to the patient.
      Better spatial resolution.

   Disadvantages:
      Uncomfortable to the patient.
      Location of the suspicious lesion/lump moves
      Still a radiation dose of x-rays to the patient.

Some useful data:

| Material | Speed (m/s) | Z ($\times 10^6$kg/m$^2$s) | $\rho\left(\frac{g}{cm^3}\right)$ | US intensity loss (dB/cm) |
|---|---|---|---|---|
| Air | 343 | 0.0004 | 0.0013 | 12 |
| Blood | 1570 | 1.6 | 1.057 | 0.15 |
| Bone | 3500 | 7.8 | 1.900 | 14.2 |
| Calcium Deposit | 4080 | 4.7 | 1.550 | 20 |
| Fat/Soft Tissue | 1460 | 1.4 | 0.901 | 0.6 |
| Water | 1480 | 1.5 | 1.000 | 0.0022 |
| Muscle | 1580 | 2.2 | 1.060 | 1.4 |
| Breast Tissue | 1547 | 1.6 | 0.901 | 8 |

Table 2: Speeds of sound, acoustic impedance, density of common materials, and US intensity loss in various materials. Values are taken from http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html, http://www.nist.gov/pml/data/xraycoef/, and Physics of Radiology, 2nd Ed., Anthony Wolbarst, p120.