Physics 210
Medical Physics
Midterm Exam
Winter 2015
February 13, 2015

Name___________________________

Problem 1	/24
Problem 2	/24
Problem 3	/24
Total	/76

I affirm that I have carried out my academic endeavors with full academic honesty.

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Signature

1. *Renal calculi* are composed of salts of inorganic or organic acids, and one common composition for the stone is calcium oxalate monohydrate ($C_2H_2CaO_5$). Renal calculi may form and become located in the calyces of the kidneys (as shown in figure 1 below), ureter, or bladder. A renal calculus (a kidney stone) may pass from the kidney into the renal pelvis and then into the ureter. The normal opening of the human ureter in the kidney is approximately $4mm$ in diameter and if the stone is sharp or larger than this opening, the loose stone may become lodged in the ureter. This causes a distension of the ureter and may cause severe intermittent pain (*ureteric colic*) as the stone is forced by contraction down the ureter. Consider a 46-year-old woman who presented for persistent recurring calculi in the renal pyramids. This condition is called *Nephrocalcinosis* and is shown in both the ultrasound image, figure 2 below. In figure 1, we can see the presence of the kidney stones in the renal calyces, while figure 2 shows an US image of the right kidney with several stones in the renal pyramid/calyx. Note: Some pertinent data for the problems in the exam that follows are given in table 1 at the end of the exam and various tables located in the questions throughout the exam.

a. Using the data in the table below and approximating the stone as uniform and spherical, if the stone is passed to the ureter, will it be able to be excreted in the urine stream or will it become lodged in the ureter? Explain your answer fully.

Location	Arrival time for sound (μs)
A → A	0
B → A	5.5
C → A	7

To determine the size of the stone we note that it takes $\Delta t = t_{C\to A} - t_{B\to A} = 7\mu s - 5.5\mu s = 1.5\mu s$ for the sound waves to cross the region between C and B. Therefore, we have

$$v = \frac{d}{t} \to d = vt = 4300\,\tfrac{m}{s} \times 1.5\times10^{-6}s = 6.45\times10^{-3}m = 6.45mm.$$

Since this is larger than the average size of the ureter, the stone most likely will not pass out of the urine stream.

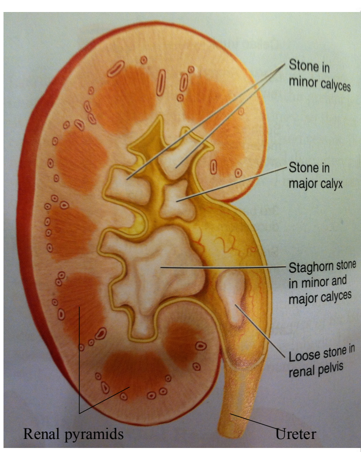

Figure 1: Schematic of the human kidney showing the major parts and several renal calculi. Figure from Clinically Oriented Anatomy, 6th Ed., Moore and Dalley, p. 300.

Figure 2: US scan of the right kidney showing renal calculi present throughout the kidney. http://ramb.elsevier.es/imatges/449/449v58n03/grande/449v58n03-90157901fig1.jpg

b. Suppose that $1MHz$ US waves were incident on the upper surface of the kidney stone (labeled B) with an intensity $I_{incident}$ what would the reflected and transmitted intensities be at point B and what intensity would the back of the stone located at point C be. Express all your answers in terms of $I_{incident}$?

The reflected intensity is given at point B by: $\frac{I_{ref}}{I_0} = \left|\frac{z_1 - z_2}{z_1 + z_2}\right|^2 = \left|\frac{2.2-4.7}{2.2+4.7}\right|^2 = 0.13 \rightarrow I_{ref} = 0.13 I_0$ or 13% reflected.

The transmitted intensity at point B is given by:
$I_0 = I_{ref} + I_{trans} \rightarrow I_{trans} = I_0 - I_{ref} = I_0(1-0.13) = 0.87 I_0$ or 87% transmitted.

The intensity of the sound that is incident at point C, assuming, as in part a, that the stone is uniform and spherical, we have from table 1: $@1MHz: dB_{loss} = 20\frac{dB}{cm} \times 0.645cm = 12.9dB$. Then the intensity at C is: $dB_{loss} = -10\log\left(\frac{I_C}{I_B}\right) \rightarrow I_C = I_B 10^{-\frac{dB_{loss}}{10}} = (0.87 I_0)10^{-\frac{12.9}{10}} = 0.045 I_0$ or 4.5% transmitted to point C.

c. Suppose that the calculus labeled as D becomes lodged in the ureter and that you could focus a $100\frac{W}{cm^2}$ ultrasound beam down to a spot approximately $10\mu m$ in diameter and that you have this US pulse incident on the stone lodged in the ureter what power would be incident on this kidney stone lodged in the ureter? (Coincidentally if the power in the ultrasound beam could be focused to a small spot, shock waves could be launched into the stone and this could shatter the stone, breaking it into smaller pieces and thus allowing it to pass. This is called shock wave lithotripsy.)

The power is given from the intensity and the area over which the sound waves are incident. Thus we have: $I = \frac{P}{A} \rightarrow P = IA = \left(100\frac{W}{cm^2} \times \left(\frac{100cm}{1m}\right)^2\right) \times \pi(5\times10^{-6}m)^2 = 7.9\times10^{-5}W$

2. Suppose that you have the CT scan of the lower abdomen as shown in figure 3 below with the basic anatomy labeled in the diagram.

   a. What is the approximate mass attenuation coefficient for a kidney stone located in the right kidney if the kidney stone's density is $2.12\frac{g}{cm^3}$ (the density of calcium oxalate monohydrate)? Assume that your x-ray beam is monoenergetic, the kidney stone is the size you calculated in question 1a, and that the incident intensity of the x-ray beam is $I_{incident}$, while the intensity of the exiting beam out of the back toward the detector is $0.0084\% I_{incident}$. What type of material does the kidney stone most closely resemble? Explain your answer fully.

The intensity is given as:

$$I_{out} = I_{in} e^{-\sum_i \mu_i x_i} = I_{in} e^{-\left[\mu_{fat} x_{fat} + \mu_{air} x_{air} + \mu_{muscle} x_{muscle} + \mu_{ks} x_{ks}\right]}$$

$$\mu_{ks} x_{ks} = -\ln\left(\frac{I_{out}}{I_{in}}\right) - \mu_{fat} x_{fat} - \mu_{air} x_{air} - \mu_{muscle} x_{muscle}$$

$$\mu_{ks} x_{ks} = -\ln\left(\frac{0.000084 I_{in}}{I_{in}}\right) - \left(0.179 cm^{-1} \times 5.8 cm\right) - \left(3.7 \times 10^{-5} cm^{-1} \times 14.5 cm\right) - \left(0.217 cm^{-1}\left(5.755 cm\right)\right)$$

$$\mu_{ks} x_{ks} = 7.098 \rightarrow \mu_{ks} = \frac{7.098}{0.645 cm} = 11 cm^{-1}$$

$$\mu_{m,ks} = \frac{\mu_{ks}}{\rho} = \frac{11 cm^{-1}}{2.12\frac{g}{cm^3}} = 5.2\frac{cm^2}{g}$$

and the closest substance is bone.

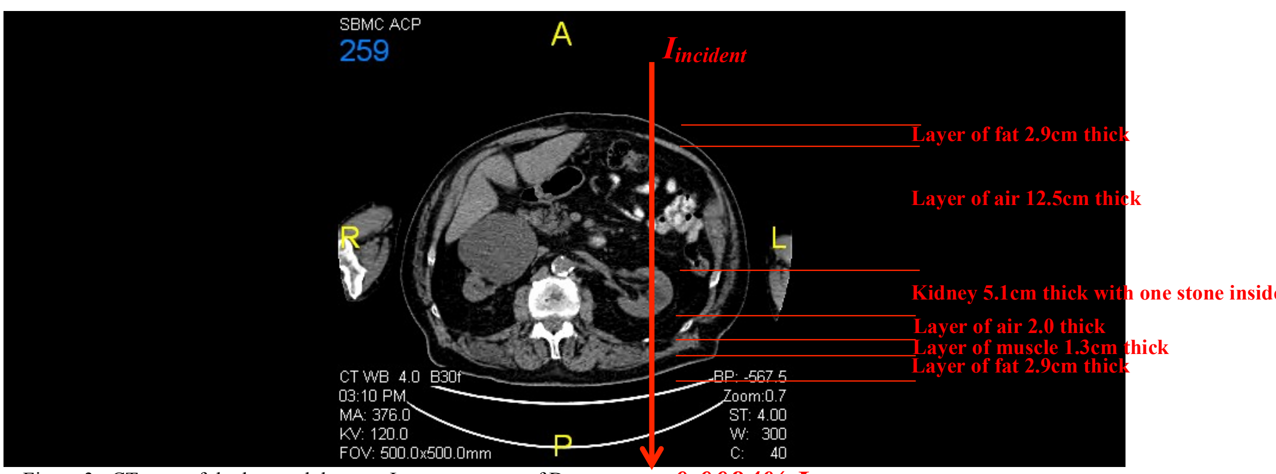

Figure 3: CT scan of the lower abdomen. Image courtesy of Dr. William Moore, MD.

b. Suppose that you have an anode made out of tungsten and this anode was used to create a beam of tungsten x-rays. The principle K_α line of tungsten is approximately $58keV = 9.3\times10^{-15}J$. How many photons are produced every second from an anode (of diameter $1cm$) if approximately 1% of the incident electrons actually produce x-ray photons? The current from the CT scan is $I = 376mA = 376\frac{mC}{s}$ and the current is given as $I = \frac{\Delta Q}{\Delta t}$, where each electron has $1e^- = 1.6\times10^{-19}C$ of charge. Suppose further that this entire beam of x-ray photons were incident on the patient over spot in $1cm$ diameter, how much energy (in Joules) would be incident on the patient every second?

Here, 1% of the incident electrons actually produces photons, we have: $\frac{\#x-rays}{s} = 0.01\left(\frac{\#e^{-1}}{s}\right) = 0.01\left(\frac{I}{e}\right) = 0.01\left(376\times10^{-3}\frac{C}{s}\times\frac{1e^-}{1.6\times10^{-19}C}\right) = 2.35\times10^{16}$. The energy per unit time is therefore, $\frac{E}{t} = 2.35\times10^{16}\frac{xray}{s}\times\frac{9.3\times10^{-15}J}{xray} = 219\frac{J}{s}$.

c. We've said in class that you'd really like x-ray detectors that are small in size and have a fast response time. Assuming that you can manufacture x-ray detectors with increasingly smaller detector size, does this mean that you can continue to increase the spatial resolutions indefinitely? Is there a limit to the spatial resolutions that you can achieve? Explain your answer fully.

The feature size that you can image is on the order of the wavelength of the light used. Thus the smaller the detectors are size the finer the features you can image in theory. However, in practice you are limited by the wavelength of the x-ray no matter how small you make the detector. So yes there is a limit to the spatial resolution you can make (it's the wavelength of the x-rays you're using) no matter how small you can make your detector.

3. Endoscopy is a procedure where an endoscope is used to look inside of the body. Suppose that you have the two pictures shown below in figure 4. The photograph on the left is a normal view of the folds of the stomach while the view on the right shows a bleeding gastric ulcer (the dark mass on the left side of the right image in figure 4.) These two images were obtained by endoscopy using a scope placed down a patient's esophagus. Suppose that you want to use an optical scope to view the lining of the stomach. The core and cladding of the optical fiber are made out of thin borosilicate glass with the index of refraction of the core $n_{core} = 1.62$ and cladding $n_{cladding} = 1.52$. Light is incident from air onto the front surface of the scope at an angle of $\theta = 25^0$ with respect to the normal to the surface as shown in figure 5.

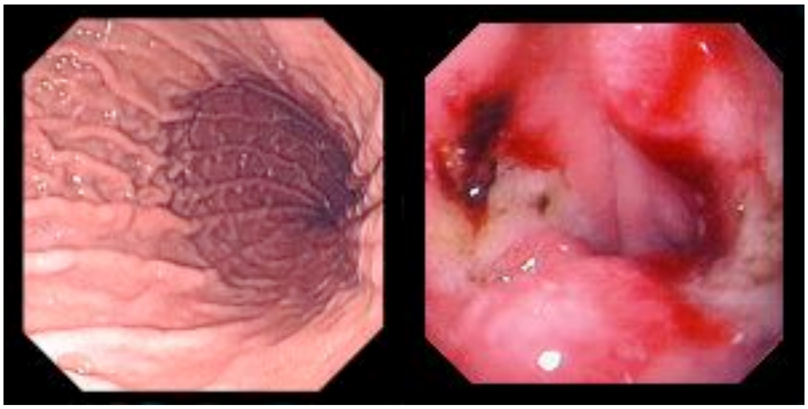

Figure 4: Left: normal view of the interior lining of the stomach. Right: A 77 year-old man presented with acute hematemesis (vomiting blood). Exam revealed this ulcer (at a 9:00 o'clock position) with a white, fibrinous base, and a dark, protruding visible vessel, signifying the site of recent bleeding. Images from: http://www.endoatlas.com/st_ul_01.html

a. What is the angle of refraction for light in the core and will the light be totally internally reflected in the scope? If the light were not totally internally reflected in the scope, would you have to increase or decrease the angle of incidence ($\theta = 25^0$) on the front surface of the scope?

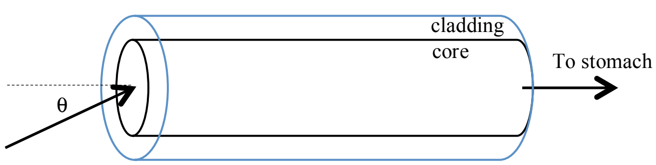

Figure 5: Schematic of the endoscope used to view the stomach.

The angle of refraction is:

$$n_{air}\sin\theta = n_{core}\sin\theta_{core} \rightarrow \theta_{core} = \sin^{-1}\left(\frac{n_{air}}{n_{core}}\sin\theta\right) = \sin^{-1}\left(\frac{1}{1.62}\sin 25\right) = 15.1^0.$$

On the upper surface the light will be incident at an angle of:

$\theta_{upper} = 90^0 - \theta_{core} = 90^0 - 15.1^0 = 74.9^0$.

Whether the light will be totally internally reflected or not depends on the critical angle. The critical angle is: $n_{core}\sin\theta_{crit} = n_{cladding}\sin\theta_{cladding} \rightarrow \theta_{crit} = \sin^{-1}\left(\frac{n_{cladding}}{n_{core}}\sin 90\right) = 69.7^0$ and since the light is incident at an angle larger than the critical angle the light will be totally internally reflected.

b. Suppose that you wanted to use a laser to cauterize the ulcer and stop the bleeding for the patient who presented in the right image of figure 4. What type of laser(s) would you use to stop the bleeding and why? Explain in as much detail as you can your choice(s).

Ideally since the ulcer is full of blood and the major component of blood is hemoglobin, which preferentially absorbs light in the ultraviolet and blue-green portion of the electromagnetic spectrum (see figure 3.24 on page 91 in Kane) and reflects in the red portion of the electromagnetic spectrum. Ideally you would want a laser to lase in the UV or blue-green portion of the spectrum. Some choices for the laser surgery would be: $Argon\ ion$ laser with a wavelength of $514nm$ or the $Nd:YAG$ which lases at $532nm$.

c. Suppose that you were to use a laser that has a power output of $25\frac{W}{cm^2}$. If this laser were incident on a portion of the ulcer for $10s$, would you be able to cauterize (burn) a $1cm^2$ piece of tissue? If not, what change(s) could you make so that you could cauterize the ulcer? (Hint: To cauterize the tissue, you must get the tissue's temperature to rise to 48^0C (from http://www.nist.gov/fire/fire_behavior.cfm) from its initial temperature taken to be 37^0C (normal body temperature). The energy in the tissue shows up as heat and the amount of heat is given as $heat = mc\Delta T = \left(4.19\frac{J}{^0C}\right)\Delta T$, for a $1g$ mass of tissue that say, you want to heat.)

The energy given to the tissue is: $I = \dfrac{E}{tA} \rightarrow E = ItA = \left(25\frac{W}{cm^2}\right)\times 10s \times 1cm^2 = 250J$. This energy shows up as heat in the tissue and the change in the tissue's temperature would be $heat = 250J = \left(4.19\frac{J}{^0C}\right)\Delta T \rightarrow \Delta T = 59.7^0C = T_f - T_i \rightarrow T_f = 96.7^0C$. This is almost the temperature that water boils. So, the answer is you would cause a photocoagulation burn (cauterize) but more importantly you'd actually start to vaporize the tissue at this point. So, this is probably not a great idea. What you could do is try to limit the power output of the laser and make it lower – which may or may not be so easy to physically do. One way to lower the energy given to the ulcer as heat (and lower the resulting temperature of the ulcer) is to reduce the treatment time.

Material	Speed (m/s)	Z ($x10^6 kg/m^2 s$)	μ (cm^{-1})	US intensity loss (dB/cm)
Air	343	0.0004	$3.7x10^{-5}$	12
Blood	1570	1.6	0.212	0.15
Bone	3500	7.8	0.50	14.2
Kidney Stone	4300	4.7	Unknown	20
Fat	1460	1.4	0.179	0.6
Water	1480	1.5	0.206	0.0022
Muscle	1580	2.2	0.217	1.4

Table 1: Speeds of sound, acoustic impedance, x-ray attenuation coefficients (at 60keV), and US intensity loss in various materials. Values are taken from http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html, http://www.nist.gov/pml/data/xraycoef/, and Physics of Radiology, 2nd Ed., Anthony Wolbarst, p120.