Name PIXE Homework #2 - Physics 100 Union College Fall 2016

## 1. Describe the PIXE process.

PIXE stands for proton induced x-ray emission spectroscopy. This is a technique whereby a high-energy proton interacts with a target nucleus ejecting an electron from an inner atomic orbital of the target atom. An electron from a higher atomic orbital de-excites to the vacancy in the lower atomic orbital with an emission of an x-ray photon. The energy of the photon is characteristic of the atomic nucleus.

2. What are the  $K_{\alpha}$  and  $K_{\beta}$  transition energies for the following elements:  ${}^{19}_{9}F$ ,  ${}^{59}_{27}Co$ , and  ${}^{197}_{79}Au$ ? (Please use the equations given in the lecture and not the table of x-ray energies to do the calculation.)

The x-ray energies are given by 
$$\Delta E = -Z^2 \frac{me^4}{2(4\pi\epsilon_0)^2 \hbar^2} \left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$
, where for the  $\alpha$ -

transition we have  $n_u = 2 \rightarrow n_l = 1$  and for a  $\beta$ -transition  $n_u = 3 \rightarrow n_l = 1$ . Thus for a  $K_{\alpha}$  transition (evaluating the constants) we have

$$\Delta E_{\alpha} = -Z^{2} \frac{me^{4}}{2(4\pi\epsilon_{0})^{2} \hbar^{2}} \left(\frac{1}{n_{l}^{2}} - \frac{1}{n_{u}^{2}}\right) = -Z^{2} \left[ \frac{9.11 \times 10^{-31} kg \times \left(1.6 \times 10^{-19} C\right)^{4}}{32\pi^{2} \left(8.85 \times 10^{-12} \frac{C^{2}}{Nm^{2}}\right)^{2} \left(\frac{6.63 \times 10^{-34} Js}{2\pi}\right)^{2}} \right] \left(\frac{1}{1} - \frac{1}{4}\right) \Delta E_{\alpha} = \left(1.626 \times 10^{-18} J\right) Z^{2} = (10.16) Z^{2}$$

, while for a  $K_{\beta}$  transition

$$\Delta E_{\beta} = -Z^{2} \frac{me^{4}}{2(4\pi\varepsilon_{0})^{2} \hbar^{2}} \left(\frac{1}{n_{l}^{2}} - \frac{1}{n_{u}^{2}}\right) = -Z^{2} \left[\frac{9.11 \times 10^{-31} kg \times (1.6 \times 10^{-19} C)^{4}}{32\pi^{2} (8.85 \times 10^{-12} \frac{C^{2}}{Nm^{2}})^{2} \left(\frac{6.63 \times 10^{-34} Js}{2\pi}\right)^{2}}\right] \left(\frac{1}{1} - \frac{1}{9}\right)$$

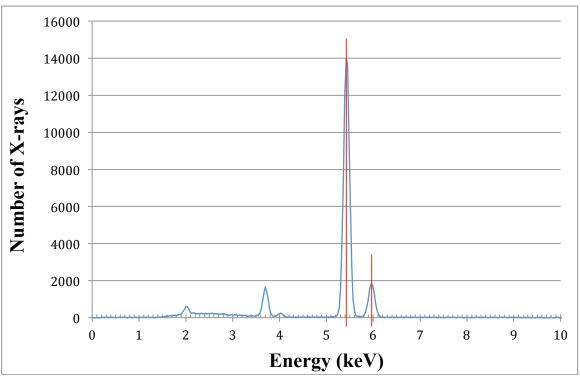
$$\Delta E_{\beta} = (1.927 \times 10^{-18} J)Z^{2} = (12.04 eV)Z^{2}$$

Element	$\Delta E_{\alpha} (keV)$	$\Delta E_{\alpha}(J)$	$\Delta E_{\beta} (keV)$	$\Delta E_{\beta}(J)$
$^{19}_{9}F$	0.822	$1.31 \times 10^{-16}$	0	0
<sup>59</sup> Co	7.407	$1.19 \times 10^{-15}$	8.78	$1.40 \times 10^{-15}$
<sup>197</sup> <sub>79</sub> Au	63.41	$1.02 \times 10^{-14}$	75.14	$1.20 \times 10^{-14}$

3. What is the ratio of the  $K_{\alpha}$  energies of  $^{238}_{92}U$  to  $^{12}_{6}C$ ?

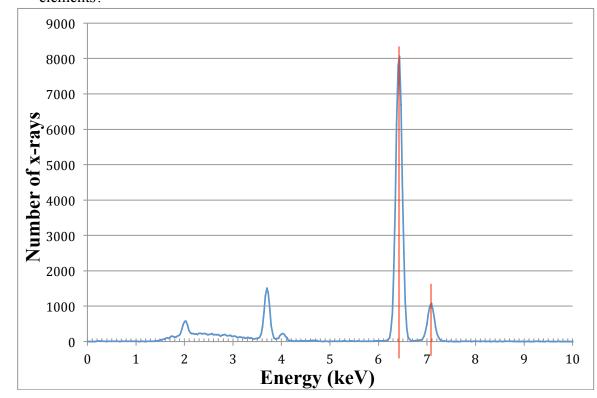
Using the formula above:  $\frac{U_{K_{\alpha}}}{C_{K_{\alpha}}} = \frac{(92)^2}{(6)^2} = 235$  where all of the constants cancel by taking the ratio.

4. Given the *PIXE* spectrum of a single element standard shown below, what are the elements?



The energy of the  $K_{\alpha}$  peak is approximately 5.4keV, while the  $K_{\alpha}$  peak is 6.0keV. Using the formula from problem #2 we can calculate the atomic number of the unknown element. Using the  $K_{\alpha}$  energy, we have  $\Delta E_{\alpha} = 5.4 \text{keV} = (10.16 \text{eV})Z^2 \rightarrow Z = 22.5$  and thus the element could be either Ti (Z = 22) or V(Z = 23). From the  $K_{\beta}$  energy, we have  $\Delta E_{\beta} = 6.0 \text{keV} = (12.04 \text{eV})Z^2 \rightarrow Z = 22.4$  and thus the element could be either Ti (Z = 22) or V(Z = 23).

5. Given the *PIXE* spectrum of a single element standard shown below, what are the elements?



The energy of the  $K_{\alpha}$  peak is approximately 6.4keV, while the  $K_{\alpha}$  peak is 7.1keV. Using the formula from problem #2 we can calculate the atomic number of the unknown element. Using the  $K_{\alpha}$  energy, we have  $\Delta E_{\alpha} = 6.4 \text{keV} = (10.16 \text{eV})Z^2 \rightarrow Z = 25$  and thus the element could be Mn (Z = 25). From the  $K_{\beta}$  energy, we have  $\Delta E_{\beta} = 7.1 \text{keV} = (12.04 \text{eV})Z^2 \rightarrow Z = 24.3$  and thus the element could be either Cr (Z = 24) or Mn (Z = 25).