Physics 110

Exam #1

September 30, 2016

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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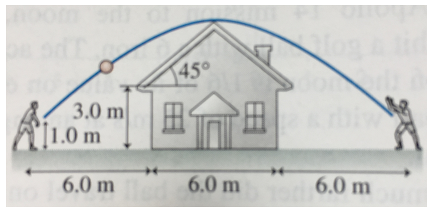
1. Suppose that you and your younger sibling decide to play a game of catch. To make the game more interesting you decide to use your house as an obstacle as shown below. You stand on the left side of your house and your sibling stands on the right. Each of you stands $6m$ from on your respective sides and the house is $6m$ wide. You throw the ball from a height of $1m$ above the ground and your sibling catches it at the same height above the ground.



   a. With what velocity (magnitude and direction) did you throw the ball so that it just barely makes it over top of the house?

   We can find the initial y-component of the velocity by examining the vertical motion to the top of the house from the point of release. We have,

$$v_{fy}^2 = 0 = v_{iy}^2 + 2a_y\Delta y = v_{iy}^2 - 2g\Delta y$$

$$v_{iy} = \sqrt{2g\Delta y} = \sqrt{2\times 9.8\tfrac{m}{s^2}\times(6m-1m)} = 9.9\tfrac{m}{s}$$

   where the height of the house is determined from

$$\tan 45 = \frac{O}{3m} \rightarrow O = 3m\tan 45 = 3m \quad .$$

$$h = 3m + O = 3m + 3m = 6m$$

   To figure out the x-component of the velocity, we need to know the time of flight of the projectile. To do this we use the vertical trajectory equation.

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 = v_{iy}t - \tfrac{1}{2}gt^2$$

$$\rightarrow 1m = 1m + 9.9\tfrac{m}{s}t - \frac{9.8\frac{m}{s^2}}{2}t^2 \rightarrow 0 = \left(9.9\tfrac{m}{s} - 4.9\tfrac{m}{s^2}t\right)t$$

$$t = \begin{cases} 0s \\ 2.02s \end{cases}$$

   Then the horizontal component of the velocity is:

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 = v_{ix}t$$

$$v_{ix} = \frac{x_f}{t} = \frac{18m}{2.02s} = 8.9\tfrac{m}{s}$$

   The velocity is $v_i = \sqrt{v_{ix}^2 + v_{ix}^2} = \sqrt{\left(8.9\tfrac{m}{s}\right)^2 + \left(9.9\tfrac{m}{s}\right)^2} = 13.3\tfrac{m}{s}$ oriented at

$$\phi = \tan^{-1}\left(\frac{v_{iy}}{v_{ix}}\right) = \tan^{-1}\left(\frac{9.9\frac{m}{s}}{8.9\frac{m}{s}}\right) = 48^0 .$$

b. Suppose that your sibling, trying to outdo you, throws the ball at twice the speed as you threw the ball. From where you were initially standing (at a location $6m$ on the left side of the house), where and by what amount would you have to move in order to catch the ball? Assume that your sibling throws the ball at the same angle as you did. If you could not get an answer to part a, assume that you threw the ball with an initial velocity of $v_i = 20\frac{m}{s}$ at $\theta = 45^0$.

The time of flight of the projectile with this new speed ($v_i = 2 \times v_{i,part\,a} = 26.6\frac{m}{s}$) is obtained from the vertical trajectory equation

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 = v_{iy}t - \tfrac{1}{2}gt^2 = (v_i \sin\theta)t - \tfrac{1}{2}gt^2$$
$$\rightarrow t = \frac{2v_i \sin\theta}{g} = \frac{2 \times 26.6\frac{m}{s} \times \sin 48}{9.8\frac{m}{s^2}} = 4.03s$$

The horizontal distance is

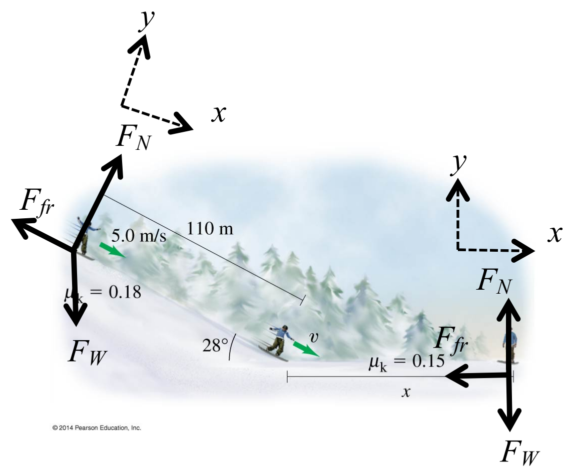
$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 = v_{ix}t = (v_i \cos\theta)t$$
$$x_f = -26.6\tfrac{m}{s} \times \cos 48 \times 4.03s = -71.8m$$

Thus the ball will land $71.8m$ to the left of where it was thrown, so you'll have to move to the left by an amount $71.8m - 18m = 53.8m$.

c. In order to the ball to be launched at any initial velocity, you have to apply a force to the ball and accelerate it from rest to the initial velocity that you want. The magnitude of the force needed to accelerate the ball to any initial velocity is given by

1. $F = \frac{mv}{t}$, where t is the time of flight of the ball from the front to the back of the house.
2. $F = \frac{mv^2}{2\Delta x}$, where Δx is the distance over which the ball is accelerated.
3. $F = \frac{mv^2}{2\Delta x}$, where Δx is the horizontal distance the ball covers from the front to the back of the house.
4. $F = mvt$, where t is the time the ball is in contact with your hand.

2. A $75kg$ snowboarder has an initial velocity of $5\,\frac{m}{s}$ at the top of a 28^0 incline as shown below. After sliding down the $110m$ long incline (on which there is friction between the snowboard and the snow) the snowboarder attains a speed v .



   a. What is the speed of the snowboarder at the bottom of the hill?

   Assuming a tilted coordinate system on the incline, we break up the forces in to their components perpendicular to and parallel to the incline. We have:

$$\sum F_x : F_{wx} - F_{fr} = F_w \sin\theta - \mu_k F_N = ma_x = ma$$

$$\sum F_y : F_N - F_{wy} = F_N - F_w \cos\theta = ma_y = 0 \rightarrow F_N = mg\cos\theta$$

$$\therefore a = \frac{F_w \sin\theta - \mu_k F_N}{m} = g\left(\sin\theta - \mu_k \cos\theta\right)$$

$$a = 9.8\,\tfrac{m}{s^2}\left(\sin 28 - 0.18\cos 28\right) = 3.04\,\tfrac{m}{s^2}$$

   The speed of the snowboarder at the bottom of the hill is given by

$$v_{fx}^2 = v_{fx}^2 + 2a_x\Delta x$$

$$\rightarrow v_{fx} = \sqrt{\left(5\,\tfrac{m}{s}\right)^2 + 2\times 3.04\,\tfrac{m}{s^2}\times 110m} = 26.4\,\tfrac{m}{s}.$$

   b. At the bottom of the hill, the snowboarder then slides across a flat horizontal surface and comes to rest after a distance $\Delta x = x$. What is the distance $\Delta x = x$ that the snowboarder covers and how long does it take the snowboarder to come to rest?

   For the snowboarder on the horizontal surface, we take a traditional Cartesian coordinate system. Breaking the forces up in to components perpendicular to and parallel to the ground we have:

$$\sum F_x : -F_{fr} = -\mu_k F_N = ma_x = ma$$

$$\sum F_y : F_N - F_w = F_N - F_w = ma_y = 0 \rightarrow F_N = mg$$

$$\therefore a = \frac{-\mu_k F_N}{m} = -\mu_k g = -0.15\times 9.8\,\tfrac{m}{s^2} = -1.47\,\tfrac{m}{s^2}$$

   Thus the time for the snowboarder to come to rest is given by

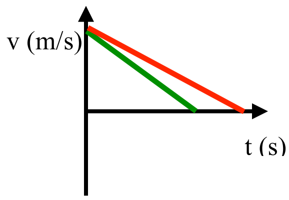
$$v_{fx} = v_{ix} + a_x t \rightarrow t = -\frac{v_{ix}}{a} = -\left(\frac{26.4\,\frac{m}{s}}{-1.47\,\frac{m}{s^2}}\right) = 18s.$$

   The distance traveled by the snowboarder in coming to rest is

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 = \left(26.4\,\tfrac{m}{s}\times 18s\right) - \tfrac{1}{2}\left(1.47\,\tfrac{m}{s}\right)\left(18s\right)^2 = 237.1m.$$

   Another way: $v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow \Delta x = -\dfrac{v_{ix}^2}{2a_x} = -\left(\dfrac{\left(26.4\,\frac{m}{s}\right)^2}{2\times 1.47\,\frac{m}{s^2}}\right) = 237.1m.$

c. The velocity, as a function of time, for the snowboarder sliding across the horizontal surface (with $\mu_k = 0.15$) is plotted on the graph below in green? If the horizontal surface were instead frictionless, the velocity versus time graph in the frictionless case (plotted in red) compared to the case of friction (plotted in green) would most closely resemble

1.


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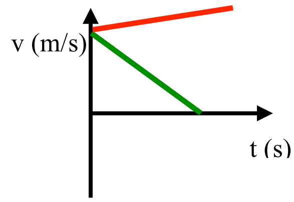

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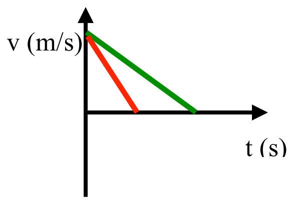

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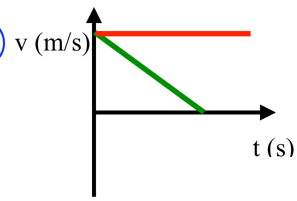


3. A carnival "rotor" ride is shown below. The riders stand in a vertical cylindrically walled "room" which has a radius of $5.5m$. The ride starts up and when it's rotating at full speed of v, the floor suddenly drops away and the riders are magically stuck to the walls. The ride is shown below.

a. From a carefully labeled free body diagram of the forces that act on the rider, what are the coefficient of static friction, μ_s, between a rider and the wall and the reaction force of the wall on a rider? Assume that typical a rider has mass of $65kg$ and that the ride rotates at a rate of 30 revolutions per minute. Hint: the speed of an object is the ratio of the distance traveled by the object to the time it takes to travel that distance.

From the free body diagram and applying a traditional Cartesian coordinate system we have breaking the forces up in the horizontal and vertical directions

$$\sum F_x : F_N = ma_x = m\frac{v^2}{r}$$

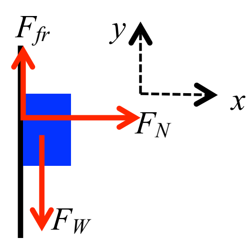
$$\sum F_y : F_{fr} - F_w = ma_y = 0 \to F_{fr} = \mu_s F_N = mg \to F_N = \frac{mg}{\mu_s}$$

$$\therefore \frac{mg}{\mu_s} = m\frac{v^2}{r} \to \mu_s = \frac{gr}{v^2} = \frac{9.8\frac{m}{s^2} \times 5.5m}{\left(17.3\frac{m}{s}\right)^2} = 0.18$$

The velocity of the rider is given by $v = \frac{\Delta x}{\Delta t} = \frac{2\pi r}{T} = \frac{2\pi \times 5.5m}{2s} = 17.3\frac{m}{s}$ and the time for one revolution is given by $T = \frac{1\min}{30rev} \times \frac{60s}{1\min} = \frac{2s}{rev}$.

The reaction force of the wall on the rider is given by

$$F_N = \frac{mg}{\mu_s} = \frac{65kg \times 9.8\frac{m}{s^2}}{0.18} = 3539N.$$



b. In part a, we assumed that a typical rider has a mass of $65kg$. In fact, anyone with any mass can ride the ride and remain attached to the wall. The fact that anyone can ride this ride is because

1. the coefficient of static friction is independent of the mass of the rider.
2. the normal force from the wall on the rider is independent of the mass of the rider.
3. the normal force from the wall on the rider and the frictional force always oppose the weight of the rider.
4. the frictional force opposes the weight of the rider and the normal force from the floor.

c. At some point the ride begins to slow down and eventually will come to a stop when the ride is over. During this period of slowing down, the speed of the riders on the ride will be one half of the maximum speed v. At this point, what will the acceleration of the riders be down the wall? Assume that the coefficient of kinetic friction between the riders and the wall is $\mu_k = 0.12$.

Assuming the same coordinate system as in part a, we have in for the forces in the horizontal and vertical directions:

$$\sum F_x : F_N = ma_x = m\frac{v^2}{r}$$

$$\sum F_y : F_{fr} - F_w = ma_y \rightarrow a_y = \frac{F_{fr} - F_w}{m} = \frac{\frac{\mu_k mv^2}{r} - mg}{m}$$

$$a_y = \frac{\mu_k v^2}{r} - g = \frac{0.12\left(\frac{17.3\frac{m}{s}}{2}\right)^2}{5.5m} - 9.8\tfrac{m}{s^2} = -8.2\tfrac{m}{s^2}$$

Useful formulas:

Motion in the r = x, y or z-directions

$r_f = r_0 + v_{0r}t + \frac{1}{2}a_r t^2$

$v_{fr} = v_{0r} + a_r t$

$v_{fr}^2 = v_{0r}^2 + 2a_r \Delta r$

Uniform Circular Motion

$a_r = \frac{v^2}{r}$

$F_r = ma_r = m\frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles *Triangles* *Spheres*

$C = 2\pi r$ $A = \frac{1}{2}bh$ $A = 4\pi r^2$

$A = \pi r^2$ $V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$magnitude\ of\ a\ vector = \sqrt{v_x^2 + v_y^2}$

$direction\ of\ a\ vector \to \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\ m/s^2$ $G = 6.67\times10^{-11}\ Nm^2/kg^2$

$N_A = 6.02\times10^{23}\ atoms/mole$ $k_B = 1.38\times10^{-23}\ J/K$

$\sigma = 5.67\times10^{-8}\ W/m^2K^4$ $v_{sound} = 343\ m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T) :\ \beta = 3\alpha$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta :\ v = r\omega :\ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times10^{-12}\ \frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L};\ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1 v_1 = A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$