Physics 110

Exam #1

October 4, 2019

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. You are driving at $30\frac{m}{s}$ $(\sim 67\frac{mi}{hr})$ when you enter a one-lane tunnel. As soon as you enter the tunnel you notice a slow moving van $155m$ $(\sim 510\,ft)$ ahead traveling at a constant rate of $5\frac{m}{s}$ $(\sim 11\frac{mi}{hr})$.

   a. As soon as you enter the tunnel, you apply the brakes, but can only decelerate at a constant rate of $2\frac{m}{s^2}$ due to the road being wet. Unfortunately you will collide with the van in front of you. At what time, measured from entering the tunnel, and how far from the tunnel's entrance will the collision occur?

$$x_{fc} = x_{ic} + v_{ic}t + \tfrac{1}{2}a_c t^2 = 30t - t^2$$

$$x_{fv} = x_{iv} + v_{iv}t + \tfrac{1}{2}a_v t^2 = 155 + 5t$$

$$x_{fc} = x_{fv} \rightarrow 30t - t^2 = 155 + 5t \rightarrow 0 = t^2 - 25t + 155$$

$$t = \begin{cases} 11.4s \\ 13.6s \end{cases}$$

   We select the earlier time $11.4s$ as the collision time. From the tunnel's entrance the car collides with the van at a distance of

$$x_{fc} = x_{ic} + v_{ic}t + \tfrac{1}{2}a_c t^2 = 30\tfrac{m}{s}(11.4s) - \tfrac{1}{2}\left(2\tfrac{m}{s^2}\right)(11.4s)^2 = 212m\ .$$

   b. How fast will you be traveling when you collide with the back of the van?

$$v_{fc} = v_{ic} + a_c t = 30\tfrac{m}{s} - 2\tfrac{m}{s^2}(11.4s) = 7.2\tfrac{m}{s}$$

c. As your car collides with the van, the van exerts a force on you. Call this force $\vec{F}_{v,c}$. In addition, your car exerts a force on the van during the collision. Call this force $\vec{F}_{c,v}$. If no other external forces act on the car and van during the collision and if $m_v >> m_c$, which of the following statements is true?

1. $\vec{F}_{c,v} \geq \vec{F}_{v,c}$.
2. $\vec{F}_{c,v} = -\vec{F}_{v,c}$.
3. $\vec{F}_{c,v} \leq \vec{F}_{v,c}$.
4. All of the above are true statements.
5. None of the above are true statements.

d. During the time between when you enter the tunnel and collide with the van, friction is helping to slow your car down. Which of the following gives a possible expression for the coefficient of friction between your tires and the road?

1. $\mu = mg$.
2. $\mu = \frac{F_{fr}}{m}$.
3. $\mu = g$.
4. $\mu = \frac{a}{g}$.
5. $\mu = \frac{g}{a}$.

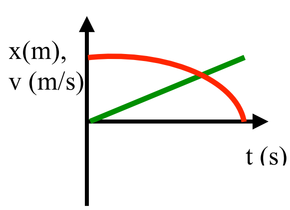
2. A Boeing 737 with mass $75,000kg$ $(\sim 164,000lbs)$ taxis onto a runway. Awaiting the control tower's clearance for takeoff the plane sits on the end of the runway, with overall length of $2600m$ $(\sim 1.6mi)$, at rest. When the control tower gives the plane clearance for takeoff, the pilots increase engine power and when the desired engine power is reached, the brakes are released and the plane accelerates from rest down the runway.

   a. If the pilots want the plane to takeoff after it has traveled a distance of $1600m$ $(\sim 1mi)$, what minimum acceleration would the airplane need and how long would it take the airplane to takeoff and become airborne? Assume that the plane needs to have a velocity of $71\frac{m}{s}$ $(\sim 150\frac{mi}{hr})$ before it can take off from the runway.

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow a = \frac{v_{fx}^2}{2\Delta x} = \frac{\left(71\frac{m}{s}\right)^2}{2\left(1600m\right)} = 1.58\frac{m}{s^2}$$

$$v_{fx} = v_{ix} + a_x t \rightarrow t = \frac{v_{fx}}{a_x} = \frac{71\frac{m}{s}}{1.58\frac{m}{s^2}} = 44.9s$$

   b. Taking the origin to be located where the airplane starts at the beginning of the runway, which of the following would give correct position-versus time (shown in red) and velocity versus time plots (shown in green)?

1.


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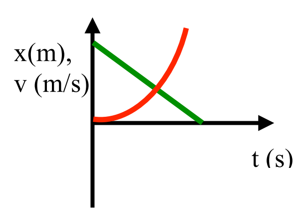

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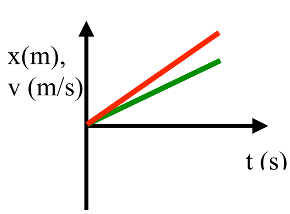

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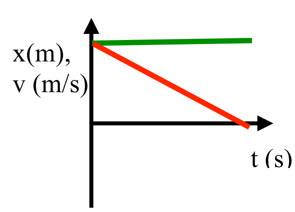


5. None of the above accurately describes the position and velocity of the airplane as a function of time.

c. In part a, you calculated the minimum acceleration the aircraft would need to become airborne. Ideally of course, for safety concerns, you would like an acceleration greater than what you found in part a, so that you become airborne sooner. Suppose that you want the acceleration of the airplane to be *twice the value that you found in part a* and that the airplane takes off and climbs into the air at a constant angle of 15^0, measured with respect to the ground. What is the net force on you (in both magnitude and direction measured with respect to the ground) due to the airplane seat that your are sitting in as a passenger? Assume that your mass is $60kg$ and hint, the seat back and bottom both exert forces on you.

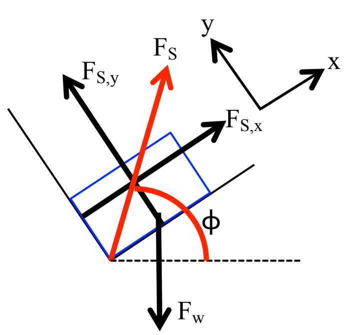
$$a = 2a_{part\,a} = 2\times 1.58\tfrac{m}{s^2} = 3.16\tfrac{m}{s^2}$$

$$\sum F_x: \; F_{S,x} - mg\sin\theta = ma_x \to F_{S,x} = ma + mg\sin\theta = 60kg\left(3.16\tfrac{m}{s^2} + 9.8\tfrac{m}{s^2}\sin 15\right) = 342N$$

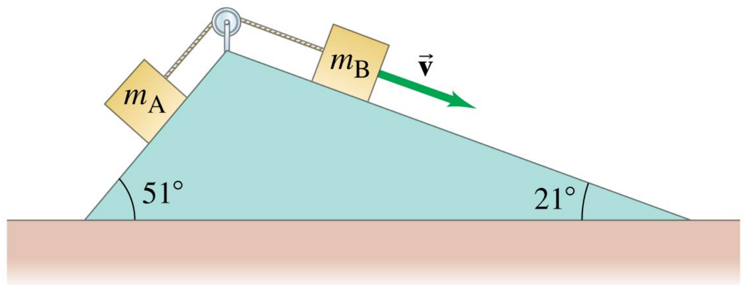
$$\sum F_y: \; F_{S,y} - mg\cos\theta = ma_y \to F_{S,y} = mg\cos\theta = 60kg\times 9.8\tfrac{m}{s^2}\cos 15 = 568N$$

$$F_{net,S} = \sqrt{F_{S,x}^2 + F_{S,y}^2} = \sqrt{\left(342N\right)^2 + \left(568N\right)^2} = 663N$$

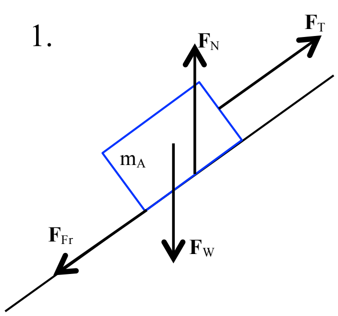
$$\phi = \tan^{-1}\left(\frac{F_{S,y}}{F_{S,x}}\right) + 15^0 = \tan^{-1}\left(\frac{568N}{342N}\right) + 15^0 = 74^0$$



d. Suppose instead of the airplane taking off, you have the following situation. A block of mass m_A is connected to a block of mass m_B by a light rope that passes over a "massless" pulley, as shown below. Assume that there is friction between each block and the ramp with coefficient of friction μ. If block with mass m_A slides up the ramp while the block with mass m_B slides down the ramp, which of the following could be a possible free-body (force) diagram for block m_A?



1.



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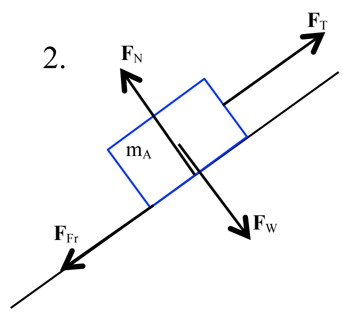

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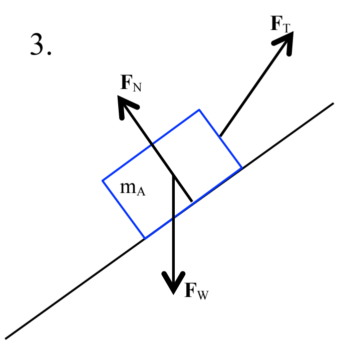

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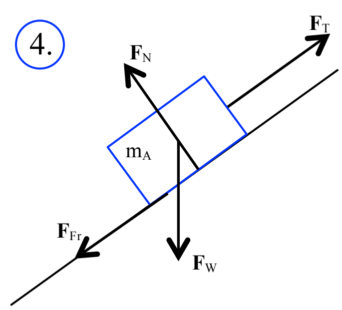
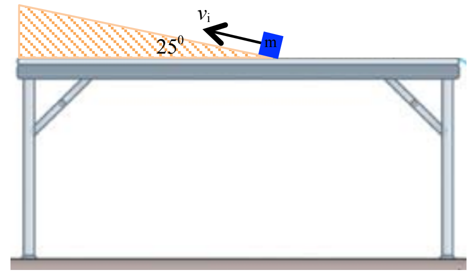


5. None of the above would give the correct free-body (force) diagram for the block of mass m_A.

3. Consider a small ramp located on the edge of a table, where the top of the table is located $1.5m$ above the ground. A $0.5kg$ block is given an initial speed of $|\vec{v}_i| = 10\frac{m}{s}$ directed along up along the ramp inclined at an angle of 25^0 measured with respect to the top of the table as shown on the right. The block slides along the ramp, which is $1.0m$ long, and is launched from the top of the ramp. Friction between the block and the ramp exists and let the coefficient of friction be $\mu = 0.8$.



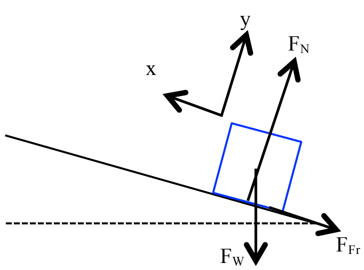
a. With what speed does the block leave the top of the ramp?

$$\sum F_{y:}\ F_N - mg\cos\theta = ma_y = 0 \rightarrow F_N = mg\cos\theta$$

$$\sum F_{x:}\ -F_{Fr} - mg\sin\theta = -ma_x \rightarrow \mu F_N + mg\sin\theta = \mu mg\cos\theta + mg\sin\theta = ma_x$$

$$\therefore a_x = g\sin\theta + \mu g\cos\theta = 9.8\tfrac{m}{s^2}\left(\sin 25 + 0.8\cos 25\right) = 11.3\tfrac{m}{s^2}$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x = \left(10\tfrac{m}{s}\right)^2 - 2\left(11.3\tfrac{m}{s^2}\right)\left(1m\right) \rightarrow v_{fx} = 8.8\tfrac{m}{s}$$



b. How far horizontally from the end of the ramp does the block land?

$$v_{ix} = v_i \cos\theta = 8.8\tfrac{m}{s}\cos 25 = 7.98\tfrac{m}{s}$$
$$v_{iy} = v_i \sin\theta = 8.8\tfrac{m}{s}\sin 25 = 3.72\tfrac{m}{s}$$

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2$$
$$-(1.5m + 1m\sin 25) = \left(3.72\tfrac{m}{s}\right)t - \tfrac{1}{2}\left(9.8\tfrac{m}{s^2}\right)t^2$$
$$4.9t^2 - 3.72t - 1.92 = 0$$
$$t = \begin{cases} 1.11s \\ -0.35s \end{cases}$$

Choosing the positive value for the time we have the time of flight of the projectile as $1.11s$. The horizontal displacement of the projectile from the end of the table is given by $x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 = v_{ix}t = 7.98\tfrac{m}{s}(1.11s) = 8.86m$.

c. Which of the following would allow the block to travel a greater horizontal distance measured from the end of the ramp?

1. Decreasing the launch angle of the ramp.
2. Decreasing the coefficient of friction between the block and the ramp.
3. Increasing the initial speed of the block at the bottom of the ramp.
4. All of the above would allow the block to travel a greater horizontal distance.
5. None of the above would allow the block to travel a greater horizontal distance.

d. Imagine that the magnitude of the acceleration due to gravity were not $g = 9.8\tfrac{m}{s^2}$, but rather say g', where $g' < g$. In this case, which of the following quantities would change?

1. The time of flight of the projectile through the air would be smaller if $g' < g$.
2. The impact speed of the projectile with the ground would be greater since the vertical component of the projectile's velocity is greater if $g' < g$.
3. The horizontal displacement of the projectile would be greater if $g' < g$.
4. The vertical displacement of the projectile would be greater if $g' < g$.
5. There is not enough information to answer this question.

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\,{}^{m}\!/\!{}_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/\!{}_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}\!/\!{}_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}\!/\!{}_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}\!/\!{}_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}\!/\!{}_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\cdot dt$

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_s = -k\vec{x}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

Work/Energy

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = Fd\,\mathrm{Cos}\,\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\,\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$