Physics 110

Exam #1

October 2, 2020

Name_______________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. A train ($m_T = 10000kg$) travels between two stations, A and B. The train starts from rest at station A and accelerates at a constant rate of $a_1 = 0.1\frac{m}{s^2}$ for a time t_1 at which point the train operator applies the brakes and the train decelerates to rest at a rate of $a_2 = -0.5\frac{m}{s^2}$ for a time t_2 coming to rest at station B. The distance between stations A and B is $5km$.

   a. How long does it take to bring the train to rest from the point the brakes are applied until the train arrives at station B? That is, what is the time t_2? Hints: Start with the expression for the trajectory for the train and express time t_1 in terms of t_2.

   The train accelerates from rest to speed in a time t_1 at a_1.
   Thus, $v_f = v_i + a_x t \rightarrow v = a_1 t_1$.
   The train decelerates from speed to rest in a time t_2 at a_2.
   Thus, $v_f = v_i + a_x t \rightarrow 0 = v - a_2 t_2$.

   Therefore, $v = a_1 t_1 = a_2 t_2$ and $t_1 = \frac{a_2}{a_1} t_2$.

   The trajectory of the train:

   $$x_{total} = x_A + x_B = \left(\tfrac{1}{2} a_1 t_1^2\right) + \left(v t_2 - \tfrac{1}{2} a_2 t_2^2\right) = \tfrac{1}{2} a_1 \left(\frac{a_2}{a_1} t_2\right)^2 + a_2 t_2^2 - \tfrac{1}{2} a_2 t_2^2$$

   $$x_{total} = \frac{a_2^2}{2a_1} t_2^2 + \tfrac{1}{2} a_2 t_2^2 = \frac{1}{2}\left(\frac{a_2^2}{a_1} + a_2\right) t_2^2$$

   $$t_2 = \sqrt{\frac{2x_{total}}{\frac{a_2^2}{a_1} + a_2}} = \sqrt{\frac{2 \times 5000m}{\frac{\left(0.5\frac{m}{s^2}\right)^2}{0.1\frac{m}{s^2}} + 0.5\frac{m}{s^2}}} = 57.7s$$

   b. What was the speed of the train, v, just before the brakes were applied and how long (time t_1) did the train accelerate from station A before applying the brakes?

   $$v = a_2 t_2 = 0.5\frac{m}{s^2} \times 57.7s = 28.9\frac{m}{s}$$

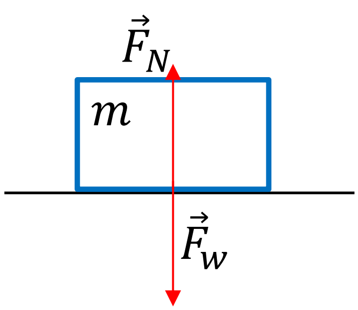
   $$t_1 = \frac{a_2}{a_1} t_2 = \frac{0.5\frac{m}{s^2}}{0.1\frac{m}{s^2}} \times 57.7s = 288.5s$$

c. How far from station A was the train when the brakes were applied?

$$x_A = x_{iA} + v_{ix,A}t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}a_1 t_1^2 = \tfrac{1}{2}\left(0.1\tfrac{m}{s^2}\right)(288.5s)^2 = 4161.6m$$

d. Which of the following force diagrams would give the correct forces that act on the train as it's decelerating to rest?

1.



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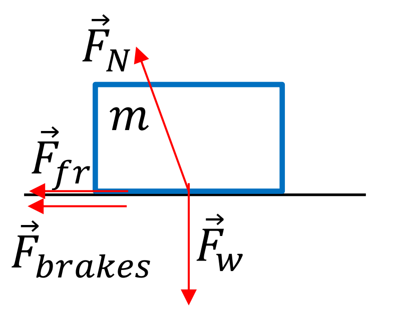

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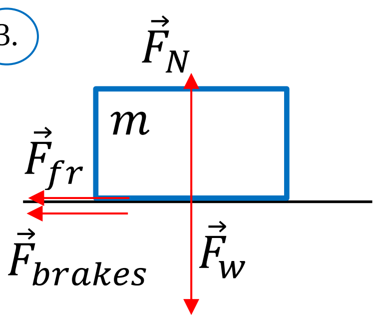

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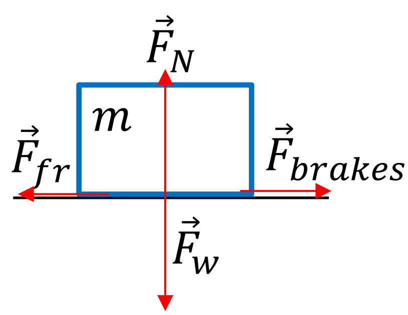
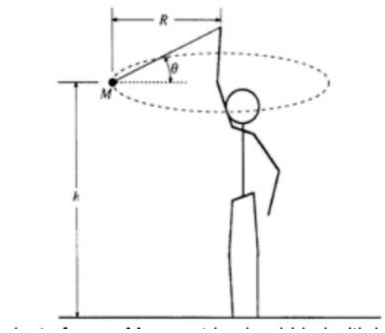


2. A ball of mass $M = 0.5kg$ on the end of a string of length $L = 1m$ is whirled with an increasing speed in a horizontal circle of radius $R = 0.75m$. At a particular speed, v_i, the string breaks and the ball is launched tangent to the circular path at a height $h = 1.5m$ above the ground, as shown in the diagram below.



a. What is the speed of the ball, v_i, when the string breaks?

Assuming a standard cartesian coordinate system we have in the:

x-direction: $F_{Tx} = Ma_c = M\frac{v_i^2}{R} \to F_T \cos\theta = M\frac{v_i^2}{R}$

y-direction: $F_{Ty} - F_w = Ma_y = 0 \to F_{Ty} = F_w \to F_T \sin\theta = Mg \to F_T = \frac{Mg}{\sin\theta}$

Thus, $F_T \cos\theta = \frac{Mg}{\sin\theta}\cos\theta = M\frac{v_i^2}{R} \to v_i = \sqrt{\frac{gR\cos\theta}{\sin\theta}} = \sqrt{\frac{gR}{\tan\theta}}$

From the diagram, $\cos\theta = \frac{R}{L} \to \theta = \cos^{-1}\frac{R}{L} = \cos^{-1}\frac{0.75m}{1m} = 41.4^0$.

$$v_i = \sqrt{\frac{gR}{\tan\theta}} = \sqrt{\frac{9.8\frac{m}{s^2}\times 0.75m}{\tan 41.4}} = 2.89\frac{m}{s}$$

b. When the string breaks the ball flies off tangent to the circular path. Which of the following best explains why this happens?

1. Newton's zeroth law of motion.
2. Newton's first law of motion.
3. Newton's second law of motion.
4. Newton's third law of motion
5. None of Newton's laws adequately explain the motion of the ball when the string breaks.

c. As soon as the string breaks, the ball becomes a projectile in flight. What is the time of flight of the ball to the ground and how far horizontally does the ball travel before striking the ground?

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 \rightarrow y_f = 0 = h - \tfrac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1.5m}{9.8\frac{m}{s^2}}} = 0.55s$$

$$x_f = x + v_{ix}t + \tfrac{1}{2}a_x t^2 \rightarrow x_f = v_i t = 2.89\tfrac{m}{s} \times 0.55s = 1.60m$$

d. With what velocity does the ball impact the ground?

$$v_{fx} = v_{ix} + a_x t \rightarrow v_{fx} = v_{ix} = v_i = 2.89\tfrac{m}{s}$$

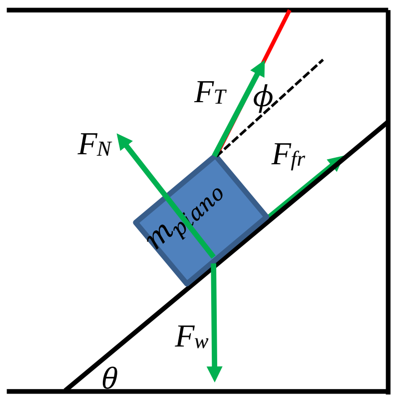
$$v_{fy} = v_{iy} + a_y t \rightarrow v_{fy} = -gt = -9.8\tfrac{m}{s^2} \times 0.55s = -5.39\tfrac{m}{s}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(2.89\tfrac{m}{s}\right)^2 + \left(-5.39\tfrac{m}{s}\right)^2} = 6.11\tfrac{m}{s}$$

$$\tan\phi = \frac{v_{fy}}{v_{fx}} \rightarrow \phi = \tan^{-1}\frac{v_{fy}}{v_{fx}} = \tan^{-1}\left(\frac{-5.39\frac{m}{s}}{2.89\frac{m}{s}}\right) = -61.8^0$$

3. A $m_{piano} = 10{,}000kg$ piano is on a ramp inclined at an angle $\theta = 30^0$ measured with respect to the horizontal. To keep the piano from sliding down the ramp, a rope is tie d to the piano and the ceiling, with the angle of the rope, $\phi = 12^0$being measured with respect to the surface of the ramp as shown in the diagram below. Friction exists between the piano and the ramp with coefficient of friction $\mu = 0.1$.

   a. What is the magnitude of the tension in the rope connecting the piano to the ceiling?



Assuming a tilted cartesian coordinate system with y perpendicular to the ramp pointing vertically up and x-parallel to the ramp pointing down the ramp, we have:

x-direction:

$F_{wx} - F_{fr} - F_{Tx} = m_{piano}a_x = 0$

$F_w \sin\theta - \mu F_N - F_T\cos\phi = 0$

y-direction:

$F_N - F_{wy} + F_{Ty} = m_{piano}a_y = 0$

$F_N - F_w\cos\theta + F_T\sin\phi = 0 \rightarrow F_N = m_{piano}g\cos\theta - F_T\sin\phi$

Thus,

$F_w\sin\theta - \mu F_N - F_T\cos\phi = mg\sin\theta - \mu\left(m_{piano}g\cos\theta - F_T\sin\phi\right) - F_T\cos\phi = 0$

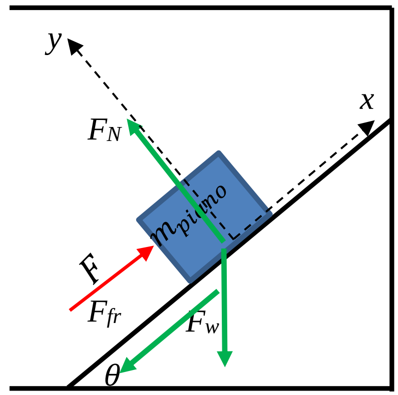
$F_T = \frac{mg(\sin\theta - \mu\cos\theta)}{\cos\phi - \mu\sin\phi} = \frac{10000kg\times 9.8\frac{m}{s2}(\sin 30 - 0.1\cos 30)}{\cos 12 - 0.1\sin 12} = 42316N$

   b. What is the magnitude of the reaction force from the ramp?

$F_N = m_{piano}g\cos\theta - F_T\sin\phi$

$F_N = 10000kg \times 9.8\frac{m}{s^2}\cos 30 - 42316N\sin 12 = 76073N$

c. Suppose instead of the rope holding the piano motionless on the ramp, you had the following situation. A person of mass $m_{person} = 70kg$ applies a force $F = 5000N$ to the piano parallel to the surface of the ramp which is inclined at an angle $\theta = 30^0$ measured with respect to the horizontal. There is still friction between the piano and the ramp with coefficient of friction $\mu = 0.1$. Using the coordinate system shown what will be the magnitude and direction of the acceleration of the piano along the ramp?



I'm going to assume that I push the piano up the ramp, thus the acceleration should point up the ramp in the positive x-direction. If I get a negative sign in my answer, the piano does not accelerate up the ramp, but down.

x-direction:
$$-F_{wx} - F_{fr} + F = m_{piano} a_x$$

y-direction:
$$F_N - F_{wy} = m_{piano} a_y = 0$$
$$F_N = F_{wy} = F_w \cos\theta = m_{piano} g \cos\theta$$

Thus,
$$-F_{wx} - F_{fr} + F = -F_w \sin\theta - \mu F_N + F = m_{piano} a$$
$$a = \frac{F - m_{piano} g(\sin\theta + \mu\cos\theta)}{m_{piano}}$$
$$a = \frac{5000N - 10000kg \times 9.8\frac{m}{s^2}(\sin 30 - 0.1\cos 30)}{10000kg} = -3.55\frac{m}{s^2}$$

The magnitude of the acceleration is $3.55\frac{m}{s^2}$ and since this is opposite the direction is opposite to what I assumed. Thus, the piano accelerates down the ramp.

d. Suppose that you are standing on the edge of a cliff and you drop a rock of mass m. When the rock has fallen a distance of y, you drop a second rock of mass m. As the rocks continue to fall, which of the following is a correct statement about their velocities?
1. Both of their velocities increase at the same rate.
2. The velocity of the first frock increases faster than the velocity of the second rock.
3. The velocity of the second rock increases faster than the velocity of the first rock.
4. Both of their velocities remain constant as the rocks fall.
5. There is not enough information to answer this question.

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \mathrm{D}x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \mathrm{D}y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2pr}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2pr$	$A = \frac{1}{2}bh$	$A = 4pr^2$
$A = pr^2$		$V = \frac{4}{3}pr^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\ {}^{m}\!/\!_{s^2}$ $\quad G = 6.67´\ 10^{-11}\ {}^{Nm^2}\!/\!_{kg^2}$

$N_A = 6.02´\ 10^{23}\ {}^{atoms}\!/\!_{mole}$ $\quad k_B = 1.38´\ 10^{-23}\ {}^{J}\!/\!_{K}$

$s = 5.67´\ 10^{-8}\ {}^{W}\!/\!_{m^2K^4}$ $\quad v_{sound} = 343\ {}^{m}\!/\!_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\cdot dt$

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_s = -k\vec{x}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

Work/Energy

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = Fd\,\mathrm{Cos}\,\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + a\mathrm{D}T)$

$A_{new} = A_{old}(1 + 2a\mathrm{D}T)$

$V_{new} = V_{old}(1 + b\mathrm{D}T): \ b = 3a$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\mathrm{D}Q = mc\mathrm{D}T$

$P_C = \frac{\mathrm{D}Q}{\mathrm{D}t} = \frac{kA}{L}\mathrm{D}T$

$P_R = \frac{\mathrm{D}Q}{\mathrm{D}T} = esA\mathrm{D}T^4$

$\mathrm{D}U = \mathrm{D}Q - \mathrm{D}W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = fl = (331 + 0.6T)\,\frac{m}{s}$

$b = 10\log\frac{I}{I_0}; \quad I_o = 1´\ 10^{-12}\ \frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$w = 2pf = \frac{2p}{T}$

$T_S = 2p\sqrt{\frac{m}{k}}$

$T_P = 2p\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2pt}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2pt}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2pt}{T}\right)$

$v = fl = \sqrt{\frac{F_T}{m}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2p^2f^2rvA^2$