

Physics 110

Exam #1

September 28, 2026

Name _____

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. Erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example,
 $|\vec{p}| \approx m|\vec{v}| = (5\text{kg}) \times (2\frac{\text{m}}{\text{s}}) = 10\frac{\text{kg}\cdot\text{m}}{\text{s}}$
- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Formulas or constants not explicitly given on the formula sheet need to be derived before they can be used or full credit will not be given even if the formula or answer is correct.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 4 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Two trains A and B are on parallel straight tracks. Train A starts from rest at station A and accelerates east toward station B $4800m$ away. Train A accelerates at a rate $0.5\frac{m}{s^2}$ until it reaches its top speed of $20\frac{m}{s}$ and then continues at this constant speed.

- a. How long does it take train A to get to $20\frac{m}{s}$ from rest and how far from station A is train A when it gets to $20\frac{m}{s}$?

$$v_{fx} = v_{ix} + a_x t = a_x t \rightarrow t = \frac{v_{fx}}{a_x} = \frac{20\frac{m}{s}}{0.5\frac{m}{s^2}} = 40s$$

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2}a_x t^2 = \frac{1}{2} \cdot 0.5\frac{m}{s^2} \cdot (40s)^2 = 400m$$

- b. Using the information from the problem and from part a, what is the equation of the trajectory for train A once it reaches its top speed measured with respect to station A? That is, what is the position x_A of train A as a function of time t measured from station A after train A reaches $20\frac{m}{s}$?

$$x_{fA} = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = x_i + v_{ix}t' = 400m + 20\frac{m}{s} \cdot (t - 40s)$$

$$x_{fA} = 20\frac{m}{s} \cdot t - 400m$$

- c. At the instant train A starts moving, train B passes station B heading west at $15\frac{m}{s}$ and accelerates toward train A at $2\frac{m}{s^2}$. What is the trajectory of train B x_B as a function of time t measured with respect to station A?

$$x_{fB} = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = 4800m - 15\frac{m}{s} \cdot t - \frac{1}{2} \cdot 2\frac{m}{s^2} \cdot t^2$$

$$x_{fB} = 4800m - 15\frac{m}{s} \cdot t - 1\frac{m}{s^2} \cdot t^2$$

- d. At time t do the two trains pass each other? Note: to earn full credit you must show all of the steps that go into the calculation. It is not sufficient enough to say my calculator says...and give numbers.

$$x_{fA} = x_{fB} \rightarrow 20\frac{m}{s} \cdot t - 400m = 4800m - 15\frac{m}{s} \cdot t - 1\frac{m}{s^2} \cdot t^2$$

$$t^2 + 35t - 4400 = 0$$

$$t = \frac{-35 \pm \sqrt{(35)^2 - 4 \cdot 1 \cdot (-5200)}}{2 \cdot 1} = \begin{cases} 56.7s \\ -91.7s \end{cases} \rightarrow t = 56.7s$$

- e. How far from station A are trains A and B when they pass each other? Do your answers make sense? Explain in complete sentences.

$$x_{fA} = 400m + 20\frac{m}{s} \cdot t = 20\frac{m}{s} \cdot 56.7s - 400m = 734m$$

$$x_{fB} = 4800m - 15\frac{m}{s} \cdot t - 1\frac{m}{s^2} \cdot t^2 = 4800m - 15\frac{m}{s} \cdot 56.7s - 1\frac{m}{s^2} \cdot (56.7s)^2 = 734.6m$$

Yes, the answers make sense since the two trains have to be in the same final location at the same final time. If these distances were not the same, then the trains could not have passed each other at this time. The difference is due to rounding.

- f. At the instant train B passes train A, train B puts on its brakes and decelerates at $3\frac{m}{s^2}$ coming to rest at station A. What is the total time it takes train B to get from station B to station A?

When train B passes train A, its speed is:

$$v_{fxB} = v_{ixB} + a_{xB}t = -15\frac{m}{s} - 2\frac{m}{s^2}(56.7s) = -128.4\frac{m}{s}$$

To stop the train from this speed it takes a time t_{stop} :

$$v_{fxB} = 0\frac{m}{s} = v_{ixB} + a_{xB}t = -128.4\frac{m}{s} + 3\frac{m}{s^2} \cdot t_{stop} \rightarrow t_{stop} = \frac{128.4\frac{m}{s}}{3\frac{m}{s^2}} = 42.8s$$

The total travel time:

$$t_{total} = t_{to\ meet\ A} + t_{stop} = 56.7s + 42.8s = 99.5s$$

2. A small rocket is accelerated upwards at a rate of $15\frac{m}{s^2}$ from rest on the ground. The upward acceleration is provided by a small rocket engine that consumes fuel. The rocket engine burns its fuel for a time $t_{burn} = 10s$ at which point the rocket's fuel is all used up. After the rocket's fuel is consumed the rocket continues upward under gravity until it momentarily comes to rest at a height h above the ground. For all parts below, assume the motion takes place only in the vertical direction, air resistance is negligible unless otherwise stated, and you can ignore the small change in mass of the rocket as the fuel is consumed.

- a. What is the velocity of the rocket when it runs out of fuel?

$$v_{fy} = v_{iy} + a_y t = a_y t_{burn} = 15\frac{m}{s^2} \cdot 10s = 150\frac{m}{s}$$

- b. How long after the rocket runs out of fuel does it take the rocket to reach its maximum height h above the ground?

$$v_{fy} = v_{iy} + a_y t \rightarrow 0 = v_{iy} - g t_{no\ fuel} \rightarrow t_{no\ fuel} = \frac{v_{iy}}{g} = \frac{150\frac{m}{s}}{9.8\frac{m}{s^2}} = 15.3s$$

- c. What is the maximum height that the rocket reaches above the ground?

$$y_f = y_i + v_{iy} t + \frac{1}{2} a_y t^2 = \frac{1}{2} a_{burn} t_{burn}^2 + v_{iy} t_{no\ fuel} - \frac{1}{2} g t_{no\ fuel}^2$$

$$y_f = \left(\frac{1}{2} \cdot 15\frac{m}{s^2} \cdot (10s)^2 \right) + 150\frac{m}{s} \cdot 15.3s - \frac{1}{2} \cdot 9.8\frac{m}{s^2} \cdot (15.3s)^2$$

$$y_f = 750m + 2295m - 1174m = 1898m$$

- d. From the moment that the rocket reaches its maximum height gravity will accelerate the rocket back down toward the ground until it ultimately impacts the ground. What will be the impact speed of the rocket with the ground if it falls from rest from its maximum height h ?

$$v_{fy} = v_{iy} + a_y t = -gt_{ground} = -9.8 \frac{m}{s^2} \cdot 19.7s = -192.9 \frac{m}{s}$$

Impact speed: $192.9 \frac{m}{s}$

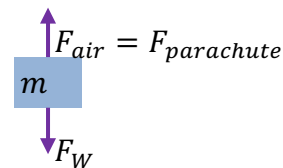
Where we calculated the time to fall $1898m$ from rest under gravity:

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 = -\frac{1}{2}gt_{ground}^2$$

$$\rightarrow t_{ground} = \sqrt{-\frac{2y_f}{g}} = \sqrt{-\frac{2 \cdot (-1898m)}{9.8 \frac{m}{s^2}}} = 19.7s$$

- e. At the speed you calculated in part d, the rocket will probably be destroyed upon impact with the ground. Ideally, we'd like the rocket to impact the ground with a much smaller speed. To help achieve this, the rocket deploys a parachute to help it glide gently to the ground. Explain in complete sentences and with as much physics as possible why the rocket would hit the ground at a much lower speed if the parachute was deployed. Hints: It might be helpful to draw a picture to help with the explanations and air resistance is not negligible. In fact, the air resistance force goes as the square of the velocity of the object.

Here, air resistance is not negligible. The upward air experts a force on the parachute opposite to the weight of the rocket.



Since forces are vectors, the sum of the forces vertically are:

With no parachute:

$F_{net,y} = -F_W = -mg = ma_y \rightarrow a_{y,no parachute} = -g$ and the rocket falls due to gravity.

With the parachute:

$$F_{net,y} = F_{air} - F_W = F_{air} - mg = ma_{y,parachute}$$

Since the air force goes as the square of the velocity, the air force is initially zero. As the object speeds up, the air force gets larger and at some speed the air force equals the weight and the object falls at a constant rate and does not fall faster and faster as it would with no air resistance. Thus, the downward speed is lower and the impact with the ground happens at a lower speed.

- f. When the rocket ultimately hits the ground, the ground brings the rocket to rest. Assuming that the parachute did not deploy and that the rocket hit the ground with the speed calculated in part d, what force was exerted by the ground on the rocket bringing it to rest? Assume that the mass of the rocket is $2500g$ and that when the rocket hits the ground it makes a hole $10cm$ deep as it comes to rest. Does your answer make sense? Explain.

$$v_{fy}^2 = v_{iy}^2 + 2a_{ground}\Delta y_{ground} \rightarrow 0 = v_{iy}^2 + 2a_{ground}\Delta y_{ground}$$

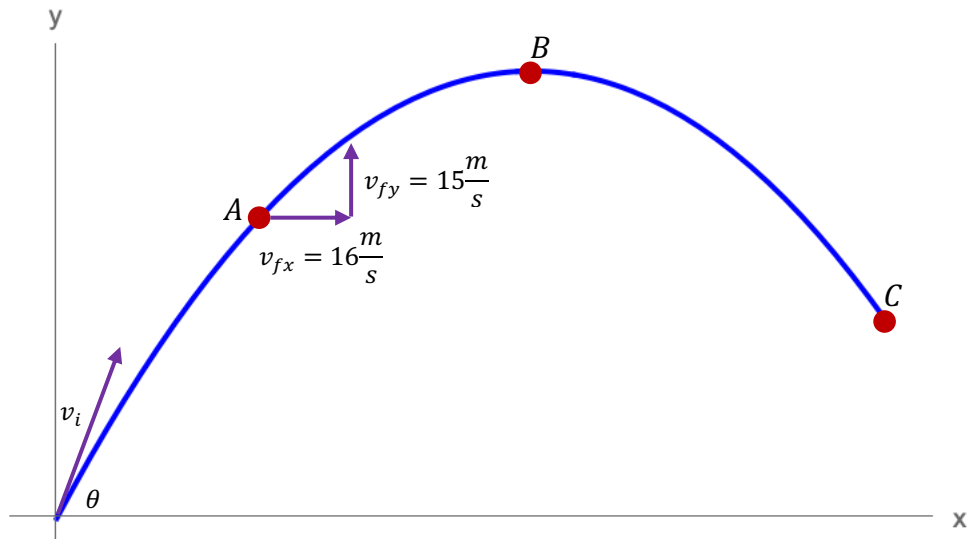
$$\rightarrow a_{ground} = \frac{v_{iy}^2}{2\Delta y_{ground}} = -\frac{\left(-192.9\frac{m}{s}\right)^2}{2(-0.1m - 0m)} = 1.9 \times 10^5 \frac{m}{s^2}$$

The direction of the acceleration is vertically up as I would expect. To slow the rocket the acceleration has to point opposite the rocket's downward velocity.

$$F_{ground} = ma_{ground} = 2.5kg \cdot 1.9 \times 10^5 \frac{m}{s^2} = 4.7 \times 10^5 N$$

To slow the rocket down when it impacts the ground, the force must be opposite to the motion of the rocket. Thus, the force must be vertically up, which this is.

3. Suppose that it's a sunny day on a distant planet in a galaxy far far away and that you are doing some experiments involving projectile motion. On this planet the acceleration due to gravity is g_{planet} and is constant. Suppose that you launch a ball of mass m from the ground on this planet at an angle θ measured with respect to the horizontal with a speed v_i . The motion of the projectile is parabolic and is shown below. At a time $t = 1s$ after launch the ball is at point A where the x- and y-components of the velocity of the ball are $16\frac{m}{s}$ and $15\frac{m}{s}$ respectively. At a time $t = 2s$ after launch the ball is at point B and this is the maximum height h_{max} that the ball reaches above the ground.



- a. What is the launch speed v_i of the ball?

$$v_{fy} = v_{iy} + a_y t = v_{iy} - g_{planet} t$$

$$\text{At point A: } v_{fyA} = 15\frac{m}{s} = v_{iy} - g_{planet} \cdot 1s$$

$$\text{At point B: } v_{fyB} = 0\frac{m}{s} = v_{iy} - g_{planet} \cdot 2s \rightarrow g_{planet} = \frac{v_{iy}}{2s}$$

$$\rightarrow v_{fyA} = 15\frac{m}{s} = v_{iy} - g_{planet} \cdot 1 = v_{iy} - \frac{v_{iy}}{2} = \frac{v_{iy}}{2} \rightarrow v_{iy} = 2 \cdot 15\frac{m}{s} = 30\frac{m}{s}$$

$$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{\left(16\frac{m}{s}\right)^2 + \left(30\frac{m}{s}\right)^2} = 34\frac{m}{s}$$

- b. At what angle θ was the ball launched?

$$\tan \theta = \frac{v_{iy}}{v_{ix}} \rightarrow \theta = \tan^{-1} \left(\frac{v_{iy}}{v_{ix}} \right) = \tan^{-1} \left(\frac{30 \frac{m}{s}}{16 \frac{m}{s}} \right) = 61.9^\circ$$

- c. What is the value of g_{planet} ?

$$g_{planet} = \frac{v_{iy}}{2s} = \frac{30 \frac{m}{s}}{2s} = 15 \frac{m}{s^2}$$

- d. What is the maximum height h_{max} that the ball reaches above the ground?

The ball reaches maximum height at point B at a time $t = 2s$.

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 = v_{iy}t - \frac{1}{2}g_{planet}t^2 = 30 \frac{m}{s} \cdot 2s - \frac{1}{2} \cdot 15 \frac{m}{s^2} \cdot (2s)^2 = 30m$$

Or,

$$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \rightarrow 0 = v_{iy}^2 - 2g_{planet} \Delta y \rightarrow \Delta y = \frac{v_{iy}^2}{2g_{planet}} = \frac{\left(30 \frac{m}{s}\right)^2}{2 \cdot 15 \frac{m}{s^2}} = 30m$$

- e. At point C there is a cliff that the ball will land on. What is the impact speed of the ball with the cliff at point C if the ball lands on the cliff $t = 3.5s$ after launch? Assume there is no air resistance on the ball.

$$v_{fx} = v_{ix} + a_x t = v_{ix} = 16 \frac{m}{s}$$

$$v_{fy} = v_{iy} + a_y t = v_{iy} - g_{planet} t = 30 \frac{m}{s} - 15 \frac{m}{s^2} \cdot 3.5s = -22.5 \frac{m}{s}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(16 \frac{m}{s}\right)^2 + \left(-22.5 \frac{m}{s}\right)^2} = 27.6 \frac{m}{s}$$

- f. To bring the ball to rest on the cliff, the cliff will apply a force to the ball to slow it down and make it stop. At what angle ϕ measured with respect to the horizontal will the force from the ground need to be applied to bring the ball to rest?

To bring the ball to rest, the acceleration (and the force) from the ground has to be opposite to the velocity. The impact angle of the ball with the ground:

$$\tan \theta = \frac{v_{fy}}{v_{fx}} \rightarrow \theta = \tan^{-1} \left(\frac{v_{fy}}{v_{fx}} \right) = \tan^{-1} \left(\frac{-27.6 \frac{m}{s}}{16 \frac{m}{s}} \right) = -54.6^\circ \text{ or } 54.6^\circ \text{ below the positive x-axis.}$$

The force from the ground has to be exactly opposite this angle, or $\phi = 54.6^\circ$ above the negative x-axis.

Or with respect to the positive x-axis $\phi = 180^\circ - 54.6^\circ = 125.4^\circ$ counterclockwise.