Physics 110

Exam #1

April 25, 2020

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times \left(2\,\tfrac{m}{s}\right) = 10\,\tfrac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. A model rocket ($m_R = 1.0kg$) is launched from the ground, starting from rest, and accelerates vertically due to an upward thrust of $F_{thrust} = 25N$ being produced by the rocket's engine. The rocket accelerates vertically for a time of $t_{acc} = 10s$ at which point the rocket's engine runs out of fuel. Assume that the rocket's engine runs out of fuel instantaneously and stops operating and that the fuel contributes a negligible amount to the rocket's mass. That is, you may assume that the rocket's mass remains $m_R = 1.0kg$ throughout the entire problem.

   a. At the point at which the rocket's engine runs out of fuel, how high above the ground is the rocket and what is its vertical speed?

   To calculate the height of the rocket and its vertical speed we need to know the acceleration of the rocket. To calculate the acceleration, we look at the forces that act on the rocket while its rising. Assume a standard Cartesian coordinate system we have:

   $$F_{thrust} - F_W = m_R a \rightarrow a = \frac{F_{thrust} - m_R g}{m_R} = \frac{25N - 1kg \times 9.8\frac{m}{s^2}}{1kg} = 15.2\frac{m}{s^2}$$

   Having the acceleration, the height of the rocket (assuming the ground is $y_i = 0m$ and $v_{iy} = 0\frac{m}{s}$) we have

   $$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 \rightarrow y_f = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}\left(15.2\tfrac{m}{s^2}\right)(10s)^2 = 760m$$

   The speed of the rocket can be calculated from either the time-independent equation of motion or from the time-dependent velocity equation of motion.

   $$v_{fy} = v_{iy} + a_y t \rightarrow v_{fy} = a_y t = 15.2\frac{m}{s^2} \times 10s = 152\frac{m}{s}$$

   Or

   $$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y \rightarrow v_{fy} = \sqrt{2a_y\Delta y} = \sqrt{2 \times 15.2\tfrac{m}{s^2} \times 760m} = 152\tfrac{m}{s}$$

   b. Once the rocket's engine runs out of fuel, the rocket continues to climb against gravity until it comes to rest? To what height *above the point at which the rocket runs out of fuel* does the rocket reach?

   When the engine stops the rocket coasts to maximum height under gravity. Taking the height of the rocket at this point to be $y_i = 0m$ and $v_{iy} = 152\frac{m}{s}$) we have the time to come to rest ($v_{fy} = 0\frac{m}{s}$)

   $$v_{fy} = v_{iy} + a_y t \rightarrow t = \frac{v_{iy}}{a_y} = \frac{152\frac{m}{s}}{9.8\frac{m}{s^2}} = 15.5s$$

   The height is:

   $$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2$$
   $$\rightarrow y_f = v_{iy}t + \frac{1}{2}a_y t^2 = 152\frac{m}{s} \times 15.5s - \frac{1}{2}\left(9.8\frac{m}{s^2}\right)(15.5s)^2 = 1178.8$$

c. When the rocket reaches its highest point above the ground, a parachute deploys instantaneously and the parachute produces an upward force of $F_{chute} = 5N$ to slow the rate of decent of the rocket back to the ground. What is the total time of flight of the projectile from its launch to the time it lands back on the ground again? Assume that the mass of the parachute is negligible.

To calculate the time to the ground we need the acceleration of the system. From the forces that act we can calculate the acceleration.

$$F_{chute} - F_W = m_R a_y \rightarrow a_y = \frac{F_{chute} - m_R g}{m_R} = \frac{5N - 1kg \times 9.8\frac{m}{s^2}}{1kg} = -4.8\frac{m}{s^2}$$

The time to the ground assuming the rocket starts from rest at its maximum height and taking the ground to be $y_f = 0m$:

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2$$
$$\rightarrow 0 = y_i + \frac{1}{2}a_y t^2 = (760m + 1178.8m) - \frac{1}{2}\left(4.8\frac{m}{s^2}\right)t^2 \rightarrow t = 28.4s$$

The time-of-flight of the rocket from launch to landing is the sum of the times in the problem.

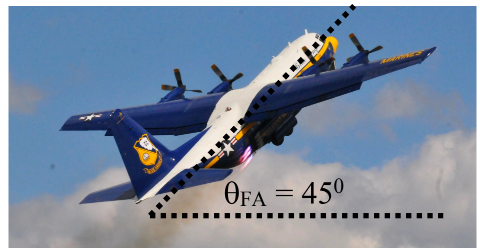
$$t_{tof} = t_{engine} + t_{no\ engine} + t_{fall} = 10s + 15.5s + 28.4s = 53.9s$$

d. When the rocket makes contact with the ground upon landing, the ground exerts a force (F_{ground}) on the rocket to bring it to rest. During the time the force from the ground is bringing the rocket to rest, which of the following statements is true concerning the force exerted on the rocket by the ground?

1. $F_{ground} < F_W$.
2. $F_{ground} = F_W$.
3. $F_{ground} > F_W$.
4. All of the above are true statements.
5. None of the above are true statements.

2. The Blue Angels flight demonstration squadron transports their personnel and associated equipment to and from various airshow locations using a Lockheed Martin C-130 cargo plane affectionately named “Fat Albert.” For takeoff, Fat Albert accelerates down the runway and near the end of the runway, a set of rockets on each side fire (as shown in picture 1). The rockets allow Fat Albert to takeoff at much larger angles ($\theta_{FA} = 45^0$) measured with respect to the horizontal than your normal passenger plane ($\theta_{pp} = 15^0$) would. Suppose that Fat Albert has a mass $m_{FA} = 40{,}000kg$ and takes off from the runway with a speed of $75\frac{m}{s}$ ($\sim 165\frac{mi}{hr}$), and climbs away from the ground at a constant at a constant angle of $\theta_{FA} = 45^0$(as shown in picture 2) with an acceleration of $a_{FA} = 0.75g = 7.35\frac{m}{s^2}$.

Picture 1: Fat Albert rocket take-off



Picture 2: Fat Albert climbing at $\theta_{FA} = 45^0$.

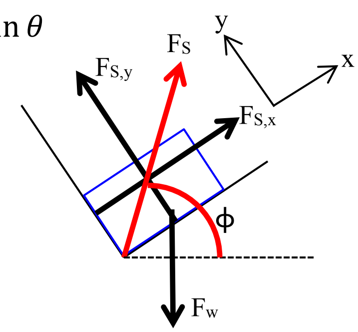
a. What is the net force on a pilot (in both magnitude and direction measured with respect to the ground) due to the airplane seat that a pilot of Fat Albert is sitting in as the plane climbs away from the runway at $\theta_{FA} = 45^0$measured with respect to the ground? Assume that a pilot’s mass is $60kg$ and hint, the seat back and bottom both exert forces on you.

From the diagram on the right, we can sum the horizontal and vertical forces to determine the net horizontal and vertical force that acts on the pilot.

Horizontal forces

$$F_{Sx} - F_{Wx} = F_{Sx} - mg\sin\theta = ma_x \rightarrow F_{Sx} = ma_x + mg\sin\theta$$

$$\rightarrow F_{Sx} = 60kg\left(7.35\tfrac{m}{s^2}\sin 45 + 9.8\tfrac{m}{s^2}\sin 45\right) = 856.8N$$



Vertical forces

$$F_{Sy} - F_{Wy} = F_{Sy} - mg\cos\theta = ma_y = 0$$

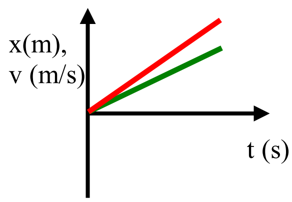
$$\rightarrow F_{Sy} = mg\cos\theta = 60kg \times 9.8\frac{m}{s^2}\cos 45 = 415.8N$$

$$F_S = \sqrt{F_{Sx}^2 + F_{Sy}^2} = \sqrt{(856.8N)^2 + (415.8N)^2} = 952.4N$$

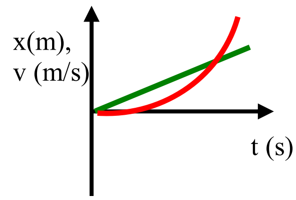
$$\phi = 45^0 + \tan^{-1}\left(\frac{F_{Sy}}{F_{Sx}}\right) = 45^0 + \tan^{-1}\left(\frac{415.8N}{856.8N}\right) = 70.8^0$$

b. Taking the origin to be located where the airplane starts at the beginning of the runway, which of the following would give correct position-versus time (shown in red) and velocity versus time plots (shown in green) from the time the plane starts moving until it takes-off?

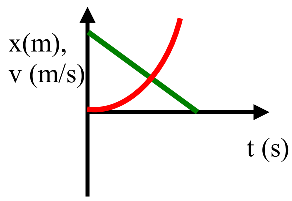
1.



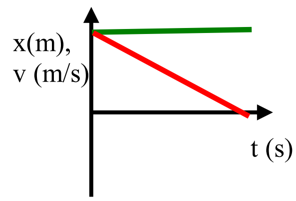
2.



3.

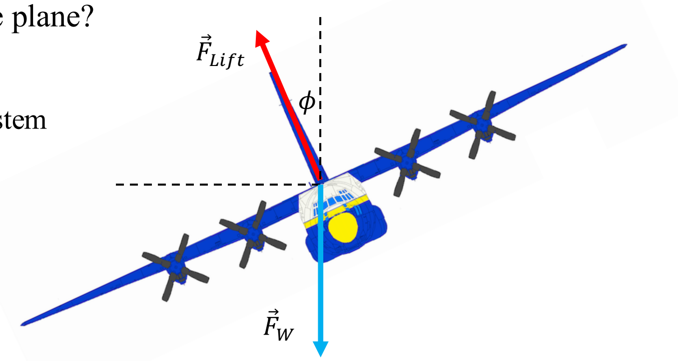


4.



5. None of the above accurately describes the position and velocity of the airplane as a function of time.

c. Suppose that the pilots of Fat Albert perform a maneuver for the airshow attendees where they fly the plane in a horizontal circle of radius $R = 1600m$ ($\sim 1\ mile$) at a constant speed of $75\frac{m}{s}$ ($\sim 165\frac{mi}{hr}$), by tilting the wings of the airplane by an angle ϕ measured with respect to the vertical. When the plane flies, airflow over the wings of the plane generates a lifting force $\vec{F}_{lift}$ that is always perpendicular to the wings of the plane. At what angle ϕmeasured with respect to the vertical did the pilots tilt the plane?



Assuming a standard Cartesian coordinate system

Horizontal forces:

$F_{lift,x} = F_{lift} \sin\phi = m_{FA} a_\perp = m_{FA} \frac{v^2}{R}$

Vertical forces:

$F_{lift,y} - F_W = F_{lift} \cos\phi - m_{FA} g = m_{FA} a_y = 0$

$\rightarrow F_{lift} = \frac{m_{FA} g}{\cos\phi}$

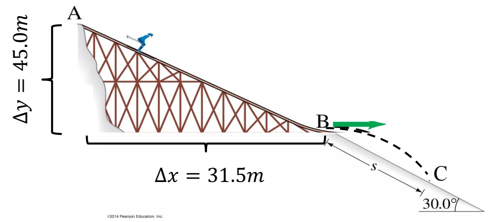
$\rightarrow F_{lift} \sin\phi = m_{FA} g \tan\phi = m_{FA} \frac{v^2}{R} \rightarrow \tan\phi = \frac{v^2}{Rg} = \frac{(75\frac{m}{s})^2}{1600m \times 9.8\frac{m}{s^2}} = 0.3587$

$\rightarrow \phi = 19.7^0$

d. In terms of the weight of the plane, what is the magnitude of the lift force F_{lift} generated by the airflow over the wings?

$F_{lift} = \frac{m_{FA} g}{\cos\phi} = \frac{F_W}{\cos\phi} = \frac{F_W}{\cos 19.7} = 1.06 F_W$

3. A $m_{skier} = 55kg$ skier starts from rest at point A at the top of a ski jump ramp. The skier skis down the ramp and leaves the jump at point B with her velocity horizontal.



a. If friction between the skier and the ski slope is negligible, what will be the skier's horizontal launch speed at point B?

To calculate the launch speed, we need to know the acceleration of the skier down the incline. Assuming a tilted coordinate system the forces that act on the skier:

Horizontal forces:
$F_{Wx} = m_{skier} g \sin\theta = m_{skier} a_x$
$\rightarrow a_x = g \sin\theta$

From the geometry we can determine the angle.
$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{45m}{31.5m} = 1.492 \rightarrow \theta = 55^0$

Vertical forces:
$F_N - F_{Wy} = F_N - m_{skier} g \cos\theta = m_{skier} a_y = 0$
$\rightarrow F_N = m_{skier} g \cos\theta$

$v_{fxB}^2 = v_{ixA}^2 + 2a_x \Delta p_{BA}$
$\rightarrow v_{fxB} = \sqrt{2a_x \Delta p_{BA}} = \sqrt{2g \sin\theta \Delta p_{BA}} = \sqrt{2 \times \left(9.8\frac{m}{s^2} \sin 55\right) \times 54.9m} = 29.7\frac{m}{s}$
Where, $\Delta p_{BA} = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(31.5m)^2 + (45m)^2} = 54.9m.$

b. What is the distance s measured along the ramp to where the skier strikes the ground at point C?

Here we don't know the horizontal or vertical distance the skier travels. But we can relate the two unknowns using the geometry in the system.

$$\tan\phi = \frac{y}{x} \rightarrow y = x\tan\phi$$

Then we locate the point by using the horizontal and vertical trajectories.

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 \rightarrow x_f = x = v_B t \rightarrow t = \frac{x}{v_B}$$

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 \rightarrow -x\tan\phi = -\tfrac{1}{2}gt^2 = -\tfrac{1}{2}g\left(\frac{x}{v_B}\right)^2 = -\frac{gx^2}{2v_B^2}$$

$$\rightarrow 0 = x\tan\phi - \frac{gx^2}{2v_B^2}$$

The solutions are:

$x = 0m$

$$x = \frac{2v_B^2}{g}\tan\phi = \frac{2\left(29.7\frac{m}{s}\right)^2\tan 30}{9.8\frac{m}{s^2}} = 103.9m$$

Then the distance s is given by:

$$\cos\phi = \frac{x}{s} \rightarrow s = \frac{x}{\cos\phi} = \frac{103.9m}{\cos 30} = 120m$$

c. Suppose instead that there is friction between the skier and the ski slope with coefficient of friction $\mu = 0.5$. Will the skier land before or after point C and by how much before or after? No credit is given for simply saying before or after. You need to properly justify before or after with a calculation and a number.

To calculate the new launch speed, we need to know the acceleration of the skier down the incline. Assuming a tilted coordinate system the forces that act on the skier:

Horizontal forces:

$$F_{Wx} - F_{fr} = m_{skier}g\sin\theta - \mu F_N = m_{skier}g\sin\theta - \mu m_{skier}g\cos\theta = m_{skier}a_x$$

$$\rightarrow a_x = g\sin\theta - \mu g\cos\theta$$

$$v_{fxB}^2 = v_{ixA}^2 + 2a_x\Delta p_{BA} \rightarrow v_{fxB} = \sqrt{2a_x\Delta p_{BA}} = \sqrt{2g\sin\theta\Delta p_{BA}}$$

$$v_{fxB} = \sqrt{2\times 9.8\tfrac{m}{s^2}(\sin 55 - 0.5\cos 55)\times 54.9m} = 23.8\frac{m}{s}$$

$$s = \frac{x}{\cos\phi} = \frac{\frac{2v_B^2}{g}\tan\phi}{\cos\phi} = \frac{\frac{2\left(23.8\frac{m}{s}\right)^2\tan 30}{9.8\frac{m}{s^2}}}{\cos 30} = 77.1m$$

So, the skier lands before point C by an amount $120m - 77.1m = 42.9m$.

d. Suppose instead of the skier leaving the ramp horizontally at point B, the skier leaves the ski jump an angle θ measured counter-clockwise from the horizontal with $0^0 < \theta \leq 45^0$. Let s' be the distance along the slope the skier now lands. Compared to the distance s you calculated in part b with no friction, which of the following gives the correct relation between s' and s?

1. $s' < s$.
2. $s' = s$.
3. $s' > s$.
4. $s' = 0$. The skier only rises and falls vertically but never goes forward in space.
5. There is not enough information to answer this question.

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r};\quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles — $C = 2\pi r$, $A = \pi r^2$

Triangles — $A = \frac{1}{2}bh$

Spheres — $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$

Quadratic equation: $ax^2 + bx + c = 0$,

whose solutions are given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\,{}^{m}\!/\!_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/\!_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}\!/\!_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}\!/\!_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}\!/\!_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}\!/\!_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\cdot dt$

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_s = -k\vec{x}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

Work/Energy

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = Fd\,\mathrm{Cos}\,\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T):\ \beta = 3\alpha$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\,\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times10^{-12}\,\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L};\ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho vA^2$