Physics 110

Exam #1

April 23, 2021

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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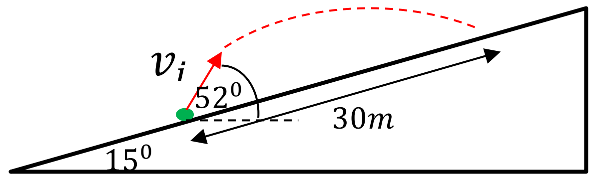
1. Suppose that you are standing on a hill inclined at an angle of 15^0measured with respect to the horizontal. A ball of mass $m = 0.1kg$ is launched with an initial speed v_i, at an angle of 52^0 (also measured with respect to the horizontal) as shown below. The ball lands $30m$ away along the hill from where it was launched.

   a. What was the launch speed of the ball?



   To use the equations of motion we need to know the horizontal and vertical displacements of the ball, which is the only reason the hill is here.

   The horizontal displacement: $\cos 15 = \frac{\Delta x}{30m} \rightarrow \Delta x = 30m \cos 15 = 29m$

   The vertical displacement: $\sin 15 = \frac{\Delta y}{30m} \rightarrow \Delta y = 30m \sin 15 = 7.8m$

   After the displacements are found, we don't need the hill. This is a projectile launched in the air that travels a horizontal distance x across the ground and a vertical distance y up in the air.

   $\Delta x = v_{ix}t + \frac{1}{2}a_x t^2 = (v_i \cos\theta)t \rightarrow t = \frac{\Delta x}{v_i \cos\theta}$

   $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 = (v_i \sin\theta)t - \frac{1}{2}gt^2 = \Delta\text{x} \tan\theta - \frac{g\Delta x^2}{v_i^2 \cos^2\theta}$

   $7.8m = 29m \tan 52 - \frac{9.8\frac{m}{s^2}(29m)^2}{v_i^2 \cos^2 52} \rightarrow v_i = 19.3\frac{m}{s}$

   b. What is the time of flight of the ball from the time it was launched until it impacts the hill?

   $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow 0 = -\Delta y + v_{iy}t - \frac{1}{2}gt^2 = -7.8 + (19.3 \sin 52)t - 4.9t^2$

   $t = \frac{-15.2 \pm \sqrt{(15.2)^2 - 4(-4.9)(-7.8)}}{-9.8} = \begin{cases} 0.64s \\ 2.45s \end{cases}$

   The time of flight is $2.45s$.

   You can also obtain the time of fight from the horizontal motion:
   $t = \frac{\Delta x}{v_i \cos\theta} = \frac{29m}{19.3\frac{m}{s}\cos 52} = 2.44s$

c. What was the impact velocity of the ball with the hill?

$$v_{fx} = v_{ix} + a_x t = v_{ix} = v_i \cos\theta = 19.3\tfrac{m}{s}\cos 52 = 11.9\tfrac{m}{s}$$

$$v_{fx} = v_{ix} + a_x t = (19.3\sin 52) - \left(9.8\tfrac{m}{s2}\right) \times 2.45s = -8.9\tfrac{m}{s}$$

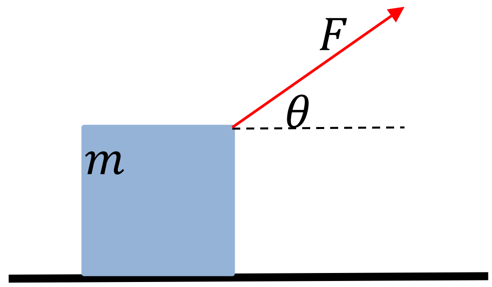
$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(11.9\tfrac{m}{s}\right)^2 + \left(-8.9\tfrac{m}{s}\right)^2} = 14.9\tfrac{m}{s}$$

$$\phi = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) = \tan^{-1}\left(\frac{-8.9\frac{m}{s}}{11.9\frac{m}{s}}\right) = -36.8^0$$

d. Suppose that the ball was launched with the same initial velocity as in part a, except this time the ball was launched down the hill rather than up. Which of the following gives the time of flight for the ball launched down the hill to the time of flight of the ball launched up the hill?

1. The time of flight of the ball down the hill is greater than the time of flight of the ball up the hill.
2. The time of flight of the ball down the hill is less than the time of flight of the ball up the hill.
3. The time of flight of the ball down the hill is the same as the time of flight of the ball up the hill.
4. None of the above answers give the correct relationship between the time of flights for the ball.

2. A block of mass m is pulled across a horizontal surface by a constant, externally applied force F applied at an angle θ measured with respect to the horizontal as shown on the right. The block accelerates to the right with a magnitude a and friction exists between the block and the horizontal surface with coefficient of friction μ.



a. Derive an expression for the coefficient of friction μbetween the block and the horizontal surface.

Vertical forces: $\sum F_y$: $F_N + F_y - F_w = F_N + F\sin\theta - mg = ma_y = 0$

$$\rightarrow F_N = mg - F\sin\theta$$

Horizontal forces: $\sum F_x$: $F_x - F_{fr} = F\cos\theta - \mu F_N = ma_x = ma$

$$\rightarrow \mu = \frac{F\cos\theta - ma}{F_N} = \frac{F\cos\theta - ma}{mg - F\sin\theta}$$

b. If the applied force is large enough (and call this largest force F_{max}), the block will lose contact with the horizontal surface. What is the expression for the maximum acceleration a_{max}that the block can attain just before the block loses contact with the horizontal surface?

When the block loses contact with the surface, $F_N = 0$.

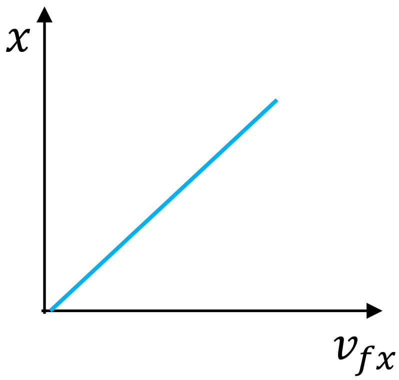
Vertical forces: $\sum F_y$: $F_N + F_y - F_w = F\sin\theta - mg = ma_y = 0$

$$\rightarrow F = F_{max} = \frac{F_W}{\sin\theta} = \frac{mg}{\sin\theta}$$

Horizontal forces: $\sum F_x$: $F_x - F_{fr} = F\cos\theta - \mu F_N = F\cos\theta = ma_x = ma$

$$\rightarrow a_{max} = \frac{F_{max}\cos\theta}{m} = \frac{mg\cos\theta}{m\sin\theta} = \frac{g}{\tan\theta}$$

c. Which of the following position versus velocity graphs gives the motion of the block as the block moves across the horizontal surface?

1.


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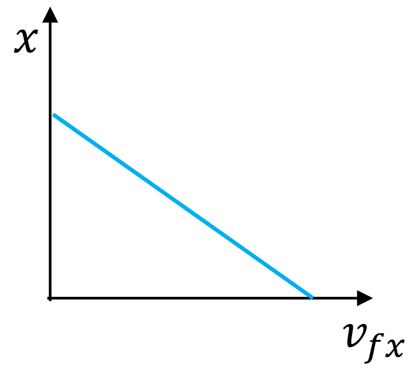

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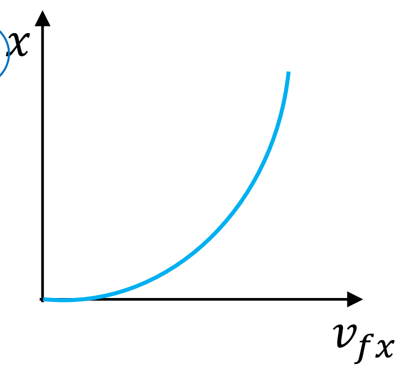

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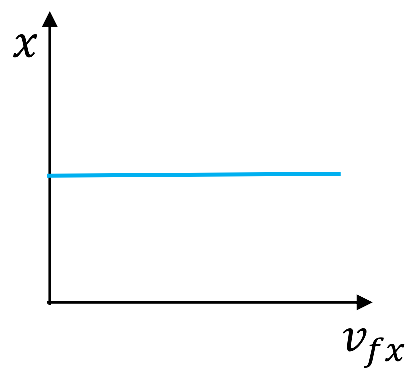
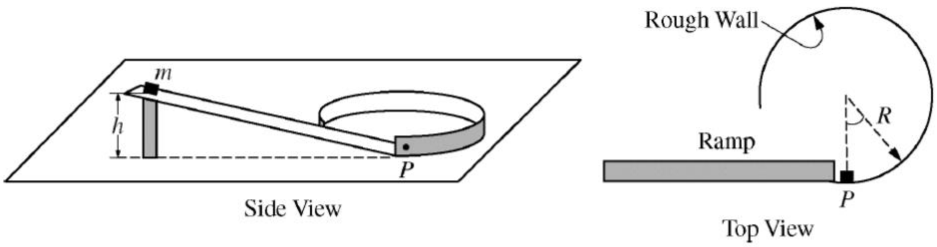


d. Suppose that, at some point in the motion of the block that the constant, externally applied force F is suddenly removed. At the point the applied force F is removed the mass has acquired a speed v_i. Derive expressions for how long will it take the block to come to rest and how far will the block have been displaced when the applied force vanishes? Assume $F < F_{max}$ for this part.

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow 0 = v_{ix}^2 - 2\left(\frac{F_{fr}}{m}\right)\Delta x \rightarrow \Delta x = \frac{mv_{ix}^2}{2F_{fr}} = \frac{mv_{ix}^2}{2\mu F_N} = \frac{mv_i^2}{2\mu mg} = \frac{v_i^2}{2\mu g}$$

$$v_{fx} = v_{ix} + a_x t \rightarrow 0 = v_i - \left(\frac{F_{fr}}{m}\right)t \rightarrow t = \frac{mv_t}{F_{fr}} = \frac{mv_i}{\mu F_N} = \frac{mv_i}{\mu mg} = \frac{v_i}{\mu g}$$

3. A block of mass $m = 0.5kg$ is released from rest from a height $h = 1m$ above a horizontal table. The block slides down the ramp, inclined at an angle $\theta = 70^0$ measured with respect to the horizontal.



a. Assuming that the acceleration of the block is constant down the ramp, what is the speed of the block at the bottom of the ramp if friction exists along the ramp with coefficient of friction $\mu = 0.2$?

Assuming a tilted coordinate system with the positive y-direction perpendicular to the ramp and up away from the ground and the positive x-axis parallel to the ramp and toward the ground.

Vertical forces: $\sum F_y$: $F_N - F_{wy} = ma_y = 0 \to F_N = F_w \cos\theta = mg\cos\theta$
Horizontal forces: $\sum F_x$: $-F_{fr} + F_{wx} = ma_x \to a = \frac{F_w \sin\theta - \mu F_N}{m}$
$a = \frac{mg\sin\theta - \mu mg\cos\theta}{m} = (\sin\theta - \mu\cos\theta)g = (\sin 70 - 0.2\cos 70) \times 9.8\frac{m}{s^2}$
$a = 8.5\frac{m}{s^2}$

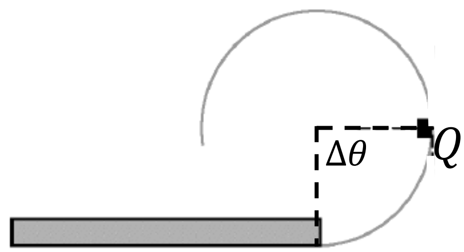
$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x = 2a_x\Delta x \to v_{fx} = \sqrt{2a_x\Delta x} = \sqrt{2 \times 8.5\frac{m}{s^2} \times 1.06m}$

$v_{fx} = 4.3\frac{m}{s}$

Where the distance down the ramp is given by:

$$\sin\theta = \frac{h}{\Delta x} \to \Delta x = \frac{h}{\sin\theta} = \frac{1m}{\sin 70} = 1.06m$$

b. When the block reaches the bottom of the incline it slides along the horizontal surface of the table guided in the circular path by a circular wall of radius $R = 0.5m$. Three ***is*** friction between the block and the wall ($\mu = 0.2$) but ***not*** between the block and the floor. Since there is friction between the block and the wall the block slows down as it moves along the table due to the block's interaction with the wall. What is the speed of the block when it is at point Q. Assume that the block has moved through an angle $\Delta\theta = \frac{\pi}{2}\ radians$. The displacement of the block along the wall is given by $\Delta x = R\Delta\theta$, for $\Delta\theta$measured in radians.



Top View

The distance traveled by the block is: $\Delta x = 0.5m \times \frac{\pi}{2} = \frac{\pi}{4}m$

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$

$$v_{fx}^2 = v_{ix}^2 - 2\left(\frac{F_{fr}}{m}\right)\Delta x$$

$$v_{fx}^2 = v_{ix}^2 - 2\left(\frac{\mu F_N}{m}\right)\Delta x = v_{ix}^2 - 2\left(\frac{\mu m v_{fx}^2}{mR}\right)\Delta x = v_{ix}^2 - 2\frac{\mu v_{fx}^2}{R}\Delta x$$

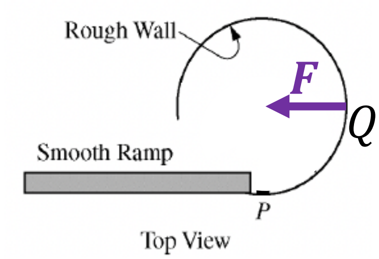
$$v_{fx} = \sqrt{\frac{v_{ix}^2}{1 + \frac{2\mu}{R}\Delta x}} = \sqrt{\frac{\left(4.3\frac{m}{s}\right)^2}{1 + \frac{2 \times 0.2 \times \frac{\pi}{4}m}{0.5m}}} = 3.4\frac{m}{s}$$

c. What is the magnitude of the normal force from the wall on the block at point Q?

$$F_N = m\frac{v_Q^2}{R} = 0.5kg \times \frac{\left(3.4\frac{m}{s}\right)^2}{0.5m} = 11.3N$$

d. Which of the following gives the net force on the block in the plane of the table at point Q?

1.



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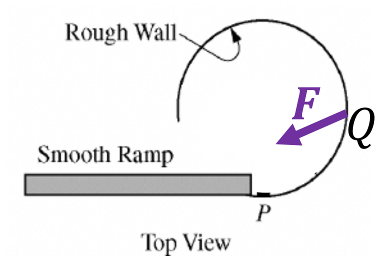

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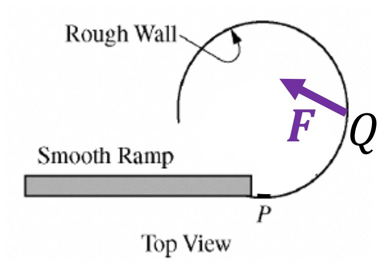

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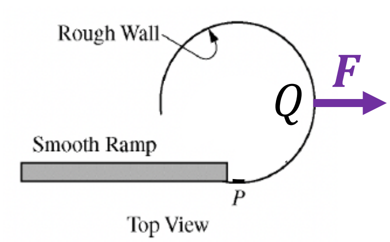

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x Dx \\ v_{fy}^2 = v_{iy}^2 + 2a_y Dy \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2pr}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2pr$	$A = \frac{1}{2}bh$	$A = 4pr^2$
$A = pr^2$		$V = \frac{4}{3}pr^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8 \,{}^{m}/_{s^2} \quad G = 6.67´\ 10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02´\ 10^{23}\,{}^{atoms}/_{mole} \quad k_B = 1.38´\ 10^{-23}\,{}^{J}/_{K}$

$s = 5.67´\ 10^{-8}\,{}^{W}/_{m^2K^4} \quad v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\cdot dt$

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_s = -k\vec{x}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

Work/Energy

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = Fd\,\mathrm{Cos}\,\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + aDT)$

$A_{new} = A_{old}(1 + 2aDT)$

$V_{new} = V_{old}(1 + bDT) : \ b = 3a$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$DQ = mcDT$

$P_C = \frac{DQ}{Dt} = \frac{kA}{L}DT$

$P_R = \frac{DQ}{DT} = esADT^4$

$DU = DQ - DW$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$

$a_r = r\omega^2$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho g d$

$F_B = \rho g V$

$A_1 v_1 = A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho g h_2$

Simple Harmonic Motion/Waves

$w = 2pf = \frac{2p}{T}$

$T_S = 2p\sqrt{\frac{m}{k}}$

$T_P = 2p\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2pt}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2pt}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2pt}{T}\right)$

$v = fl = \sqrt{\frac{F_T}{m}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2p^2 f^2 rvA^2$

Sound

$v = fl = (331 + 0.6T)\,\frac{m}{s}$

$b = 10\log\frac{I}{I_0}; \quad I_o = 1´\ 10^{-12}\,\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \quad f_n = nf_1 = n\frac{v}{4L}$