Physics 110

Exam #1

April 22, 2022

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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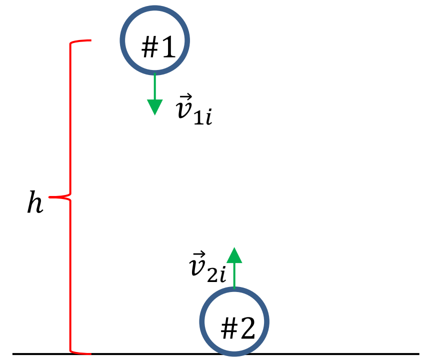
1. Suppose that you have two balls, each $22cm$ in diameter with mass $430g$. Ball #1 on the left is thrown downward with an initial speed $v_{1i} = 2\frac{m}{s}$ from a height $h = 10m$ above the ground while ball #2 on the right is simultaneously thrown up with an initial speed of $v_{2i} = 12\frac{m}{s}$ from the ground.



a. At what time do the two balls bass each other?

$$y_{1f} = y_{1i} + v_{i1}t + \frac{1}{2}a_y t^2 = h - v_{1i}t - \frac{1}{2}gt^2$$
$$y_{2f} = y_{2i} + v_{2i}t + \frac{1}{2}a_y t^2 = v_{2i}t - \frac{1}{2}gt^2$$

$$y_{2f} = y_{1f} \rightarrow h - v_{1i}t - \frac{1}{2}gt^2 = v_{2i}t - \frac{1}{2}gt^2$$

$$t = \frac{h}{v_{1i}+v_{21}} = \frac{10m}{2\frac{m}{s}+12\frac{m}{s}} = \frac{10m}{14\frac{m}{2}} = 0.71s$$

b. At what height above the ground is ball #2 when the balls pass each other?

$$y_{2f} = v_{2i}t - \frac{1}{2}gt^2 = 12\frac{m}{s} \times 0.71s - \frac{1}{2} \times 9.8\frac{m}{s^2}(0.71s)^2 = 6.1m$$

c. What is the velocity of balls #1 and #2 when they pass each other?

$$v_{if} = v_{1i} + a_y t = v_{i1} - gt = -2\frac{m}{s} - 9.8\frac{m}{s^2} \times 0.71s = -9\frac{m}{s}$$

$$v_{2f} = v_{2i} + a_y t = v_{21} - gt = 12\frac{m}{s} - 9.8\frac{m}{s^2} \times 0.71s = +5\frac{m}{s}$$

d. Ball #1 will eventually hit the ground and come to rest. Suppose that ball #1 comes to rest over a distance of $\frac{1}{4}$ of the diameter of the ball, what force (magnitude and direction) does the ground exert on the ball?

The velocity of the ball just before impact:

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y = \left(-2\frac{m}{s}\right)^2 - 2 \times 9.8\frac{m}{s^2} \times (0m - 10m) \rightarrow v_{fy} = -14.1\frac{m}{s}$$

$$F_{Ng} - F_w = ma_g \rightarrow v_{fy}^2 = 0 = v_{iy}^2 + 2a_g\Delta y$$

$$a_g = \frac{v_{iy}^2}{2\Delta y} = -\frac{\left(-14.1\frac{m}{s}\right)^2}{2\times(-0.25\times0.22m-0m)} = +1807.4\frac{m}{s^2}$$

$$F_{Ng} - F_w = ma_g \rightarrow F_{Ng} = mg + ma_g = 0.430kg \times \left(9.8\frac{m}{s^2} + 1807.4\frac{m}{s^2}\right)$$

$F_{Ng} = 781.4N$ or $781.4N$ in the positive y-direction.

2. The discus (or disk) throw in track and field has been around since the time of the ancient Greeks. In a disk throw, an athlete hurls a disk-shaped object of mass $2kg$ by spinning the disk in their outstretched arm and at some speed they release the disk in an attempt to throw the disk as far as they can. Suppose that the disk is held in the athlete's hand parallel to the ground and is spun in a horizontal circle of radius $r = 1.3m$.

a. Suppose that during the final spin, it takes the thrower $\frac{1}{3}s$ to move the discus around a horizontal circle one time. What force (magnitude and direction) would your hands need to apply to hold on to the discus?

$$|\vec{v}| = v = \frac{2\pi r}{t} = \frac{2\pi \times 1.3m}{1/3 s} = 24.5\frac{m}{s}$$

$$F = ma_c = m\frac{v^2}{r} = 2kg \times \frac{\left(24.5\frac{m}{s}\right)^2}{1.3m} = 923.8N \text{ toward the thrower.}$$

b. Suppose that the discus leaves the throwers hand horizontally parallel to the ground at a height of $h = 1.5m$. What is the maximum horizontal distance traveled by the discus?

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \rightarrow x_f = v_i t \rightarrow t = \frac{x_f}{v_i}$$

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow 0 = h - \frac{1}{2}gt^2 = h - \frac{1}{2}g\left(\frac{x_f}{v_i}\right)^2 \rightarrow x_f = \sqrt{\frac{2h}{g}}v_i$$

$$x_f = \sqrt{\frac{2h}{g}}v_i = \sqrt{\frac{2 \times 1.5m}{9.8\frac{m}{s^2}}} \times 24.5\frac{m}{s} = 13.6m$$

c. Suppose that the thrower was unhappy with their first throw and decides for their second attempt to throw the disk as far as they could, they would launch the discus at the same speed as in part a, but at an angle of 38.4^0 measured with respect to the horizontal. Does the horizontal distance reached by the discus increase or decrease and by what factor does the distance reached increase or decrease? To earn full credit, you must show whether the distance increases or decreases.

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 \rightarrow x_f = v_{ix}t \rightarrow t = \frac{x_f}{v_i \cos\theta}$$
$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow 0 = h + (v_i \sin\theta)t - \frac{1}{2}gt^2$$

$$0 = h + (v_i \sin\theta) \times \frac{x_f}{v_i \cos\theta} - \tfrac{1}{2}g\left(\frac{x_f}{v_i \cos\theta}\right)^2 = h + x_f \tan\theta - \frac{g}{2v_i^2 \cos^2\theta}x_f^2$$

$$x_f = \begin{cases} -1.84m \\ +62.8m \end{cases}$$

Thus, the discus goes farther by a factor of $f = \frac{62.8m}{13.6m} = 4.7$ times.

d. At what time(s) will the discus be at a height $h = 6.7m$ above the ground and where horizontally will the discus be, measured from the where the discus was released. Assume that the discus is launched with the initial velocity given in part c.

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 \rightarrow 6.7m = 1.5m + \left(24.5\tfrac{m}{s}\sin 38.4\right)t - \left(\tfrac{1}{2} \times 9.8\tfrac{m}{s^2}\right)t^2$$

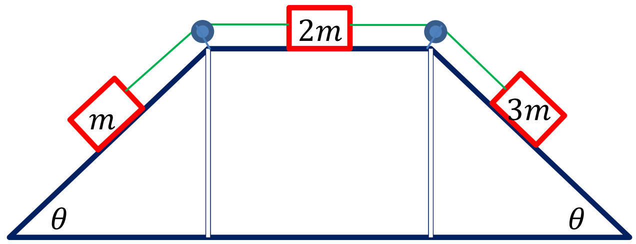
$$t = \begin{cases} 0.39s \\ 2.7s \end{cases}$$

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 \rightarrow x_f = v_{ix}t = (v_i \cos\theta)t$$

$$x_{f1} = \left(24.5\tfrac{m}{s}\cos 38.4\right) \times 0.39s = 7.5m$$

$$x_{f2} = \left(24.5\tfrac{m}{s}\cos 38.4\right) \times 2.7s = 51.8m$$

3. Blocks m, $2m$, and $3m$ are connected by very light ropes. The blocks are released from rest and the $3m$ block slides down the incline. Both inclines are oriented at an angle of $\theta = 40^0$, measured with respect to the ground.



a. Applying Newton's laws of motion and assuming all surfaces are frictionless, what are the expressions for the for forces for each mass parallel to the surface that each mass is on?

For m taking up the ramp as the positive x-direction

$$F_{TL} - F_{wx} = F_{TL} - mg\sin\theta = ma$$

For $2m$ taking to the right as the positive x-direction

$$F_{TR} - F_{TL} = 2ma$$

For $3m$ taking up the ramp for the positive x-direction

$$F_{TR} - F_{wx} = F_{TR} - 3mg\sin\theta = -3ma$$

b. What is the acceleration of the $2m$ block?

$$F_{TR} = 3mg\sin\theta - 3ma$$
$$F_{TL} = mg\sin\theta + ma$$

$$F_{TR} - F_{TL} = 3mg\sin\theta - 3ma - (mg\sin\theta + ma) = 2ma$$
$$\rightarrow 6ma = 2mg\sin\theta \rightarrow a = \frac{1}{3}g\sin\theta = \frac{1}{3} \times 9.8\frac{m}{s^2} \times \sin 40 = 2.1\frac{m}{s^2}$$

c. What is the magnitude and direction of the difference in tension forces in the ropes connected to the $2m$ block, if $m = 1kg$?

$$F_{TR} - F_{TL} = 2ma = 2 \times 1kg \times 2.1\frac{m}{s^2} = 4.2N$$

d. Starting from rest, how long would it take the $3m$ block to slide a distance $\Delta x = 0.75m$ along the incline and what would be its speed after this distance?

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \rightarrow \Delta x = \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{\frac{2 \times 0.7m}{2.1\frac{m}{s^2}}} = 0.85s$$

$$v_{fx} = v_{ix} + a_x t = at = 2.1\frac{m}{s^2} \times 0.85s = 1.8\frac{m}{s}$$
$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow v_{fx} = \sqrt{2a\Delta x} = \sqrt{2 \times 2.1\frac{m}{s^2} \times 0.75m} = 1.8\frac{m}{s}$$

Physics 110 Formula sheet

Vectors

$v = \sqrt{v_x^2 + v_y^2}$

$\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Momentum & Force

$\vec{p} = m\vec{v} \rightarrow p_x = mv_x;\ p_y = mv_y$

$\Delta\vec{p} = \vec{F}\Delta t \rightarrow \vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a} \rightarrow F_x = ma_x; F_y = ma_y$

$F_{fr} = \mu F_N$

$F_w = mg$

$F_s = -kx$

$F_G = G\frac{M_1 M_2}{r^2}$

$F_c = ma_c = m\frac{v^2}{R}$

Motion Definitions

Displacement: $\Delta x = x_f - x_i$

Average velocity: $v_{avg} = \frac{\Delta x}{\Delta t}$

Average acceleration: $a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y \end{cases}$

Work & Energy

$\begin{cases} W_T = \int \vec{F}\cdot d\vec{r} = Fdr\cos\theta = \Delta K_T \\ W_R = \int \vec{\tau}\cdot d\vec{\theta} = \tau d\theta = \Delta K_R \end{cases}$

$W_{net} = W_T + W_R = \Delta K_T + \Delta K_R = -\Delta U$

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgy$

$U_s = \frac{1}{2}kx^2$

$\Delta E = \Delta E_R + \Delta E_T$

$\Delta E = \Delta K_R + \Delta K_T + \Delta U_g + \Delta U_s = \begin{cases} 0 \\ W_{fr} \end{cases}$

Rotational Motion Definitions

Angular displacement: $\Delta s = R\Delta\theta$

Angular velocity: $\omega = \frac{\Delta\theta}{\Delta t} \rightarrow v = R\omega$

Angular acceleration: $\alpha = \frac{\Delta\omega}{\Delta t} \rightarrow \begin{cases} a_t = r\alpha \\ a_c = r\omega^2 \end{cases}$

Rotational Equations of Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Rotational Momentum & Force

$\vec{\tau} = \vec{r}\times\vec{F};\ \tau = r_\perp F = rF_\perp = rF\sin\theta$

$\tau = \frac{\Delta L}{\Delta t} = I\alpha$

$L = I\omega$

$\Delta\vec{L} = \vec{\tau}\Delta t \rightarrow \vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$

Fluids

$\rho = \frac{m}{V}$

$P = \frac{F}{A}$

$P_y = P_{air} + \rho g y$

1,2,3, … open pipes

$F_B = \rho g V$

1,3,5, … closed pipes

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$; compressible

$A_1 v_1 = A_2 v_2$; incompressible

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

Simple Harmonic Motion

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$; $\omega = \sqrt{\frac{k}{m}}$

$T_p = 2\pi\sqrt{\frac{l}{g}}$; $\omega = \sqrt{\frac{g}{l}}$

Geometry/Algebra

Circles: $A = \pi r^2$ $C = 2\pi r = \pi D$

Spheres: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2} b h$

Quadratics: $a x^2 + b x + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sound

$v_s = f\lambda = (331 + 0.6T)\frac{m}{s}$

$\beta = 10 \log \frac{I}{I_o}$

$f_n = n f_1 = n\frac{v}{2L}; n =$

$f_n = n f_1 = n\frac{v}{4L}; n =$

Waves

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = n f_1 = n\frac{v}{2L};\ n = 1,2,3,\ldots$

$I = 2\pi^2 f^2 \rho v A^2$

Equations of Motion for SHM

$$x(t) = \begin{cases} x_{max}\sin\left(\frac{2\pi}{T}t\right) \\ x_{max}\cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

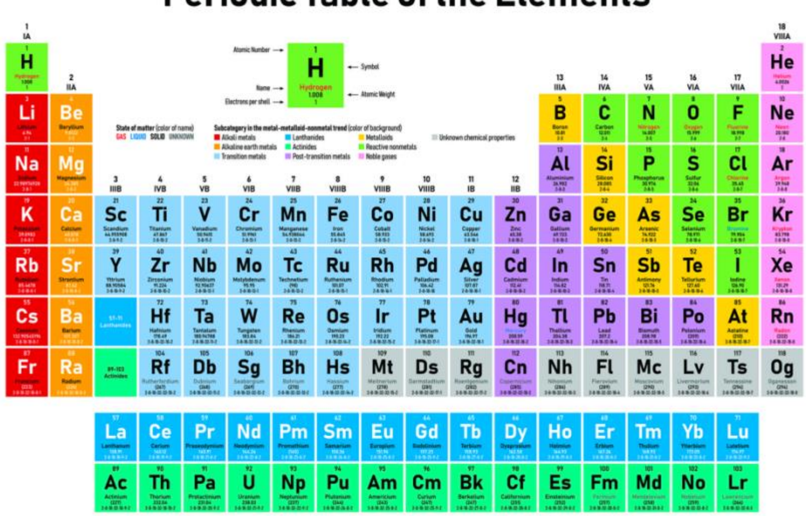
$$v(t) = \begin{cases} v_{max}\cos\left(\frac{2\pi}{T}t\right) \\ -v_{max}\sin\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$a(t) = \begin{cases} -a_{max}\sin\left(\frac{2\pi}{T}t\right) \\ -a_{max}\cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

$v = \pm v_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$

$v = \pm \omega x_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$

Periodic Table of the Elements



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