Physics 110

Exam #1

April 21, 2023

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $\left|\vec{p}\right| \approx m\left|\vec{v}\right| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or reasonable value for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

____

1. One-eighth of a mile ($\frac{1}{8}\,mile\ = 200m$) ahead of you, a large truck is driving down the road at a constant speed of $v_{i,truck} = 15\frac{m}{s}$.

   a. You decide that you want to pass the truck in $5s$ and to do this you accelerate your car, initially traveling with a constant velocity $v_{i,car} = 25\frac{m}{s}$. What constant acceleration would your car need to achieve for you to pass the truck.

   $$x_{f,car} = x_{i,car} + v_{i,car}t + \frac{1}{2}a_{car}t^2 \rightarrow x_{f,car} = v_{i,car}t + \frac{1}{2}a_{car}t^2$$
   $$x_{f,truck} = x_{i,truck} + v_{i,truck}t + \frac{1}{2}a_{truck}t^2 \rightarrow x_{f,truck} = x_{i,truck} + v_{i,car}t$$

   $$x_{f,car} = x_{f,truck} \rightarrow v_{i,car}t + \frac{1}{2}a_{car}t^2 = x_{i,truck} + v_{i,car}t$$
   $$25\frac{m}{s}(5s) + \frac{1}{2}a_{car}(5s)^2 = 200m + 15\frac{m}{s}(5s) \rightarrow a_{car} = 12\frac{m}{s^2}$$

   b. Suppose that your car cannot produce the acceleration needed in part a but can only accelerate at a maximum rate of $6\frac{m}{s^2}$. At this acceleration, how long would it take you to pass the truck in front of you?

   $$x_{f,car} = x_{f,truck} \rightarrow v_{i,car}t + \frac{1}{2}a_{car}t^2 = x_{i,truck} + v_{i,car}t$$

   $$25t + \frac{1}{2}(6)t^2 = 200 + 15t \rightarrow 3t^2 + 10t - 200 = 0 \rightarrow t = \begin{cases} -10s \\ 6.7s \end{cases}$$

   $$t = 6.7s$$

c. From your initial position behind the truck, where will you pass the truck and what will your speed be when you pass the truck?

$$x_{f,car} = v_{i,car}t + \frac{1}{2}a_{car}t^2 = 25\frac{m}{s}(6.7s) + \frac{1}{2}\left(6\frac{m}{s^2}\right)(6.7s)^2 = 302.2m$$

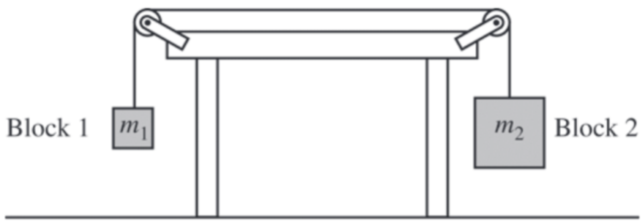
$$v_{f,car} = v_{i,car} + a_{car}t = 25\frac{m}{s} + 6\frac{m}{s^2} \times 6.7s = 65.2\frac{m}{s}$$

d. As soon as you pass the truck you notice a curve in the road ahead of you. The curve has a $30m$ radius of curvature. What coefficient of friction would there need to be between your tires and the road so that you can negotiate the curve at the constant speed you are traveling at in part c? (Note: Coefficients of friction are in general much less than one. If you tried to do this in real life, you would absolutely crash as you couldn't maintain the speed you have in part c. Do not worry about reality for this problem.)

$$F_{fr} = \mu F_n = \mu mg = ma_c = m\frac{v^2}{R} \rightarrow \mu = \frac{v^2}{Rg} = \frac{\left(65.2\frac{m}{s}\right)^2}{30m \times 9.8\frac{m}{s^2}} = 14.5$$

2. Consider the system of blocks shown below where $m_1 = 1kg$ and $m_2 = 2kg$. The system is released from rest and the block of mass m_2 falls.

a. Derive an expression (and then evaluate it) for the magnitude of the acceleration of the system. Assume the pulleys and rope are both massless and there is no friction in the pulleys.



Note: Figure not drawn to scale.

For m_1:

$$F_T - F_{W1} = F_T - m_1 g = m_1 a$$

For m_2:

$$F_T - F_{W2} = F_T - m_2 g = -m_2 a$$

$$\rightarrow m_1 g + m_1 a - m_2 g = -m_2 a \rightarrow a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

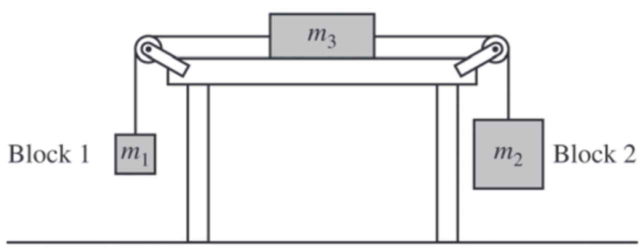
$$a = \left(\frac{2kg - 1kg}{2kg + 1kg}\right) \times 9.8\frac{m}{s^2}$$

$$a = 3.3\frac{m}{s^2}$$

b. What is the magnitude of the tension force in the rope connecting the block of mass m_1 to the pulley?

$$F_T - m_1 g = m_1 a \rightarrow F_T = m_1 g + m_1 a = 1kg\left(9.8\frac{m}{s^2} + 3.3\frac{m}{s^2}\right) = 13.1N$$

c. Suppose that a 3rd block of unknown mass m_3 is added to the system. The block of mass m_3 is on the horizontal table and the table is considered frictionless. What is the mass of m_3 such that the magnitude of the acceleration of the system is $\frac{g}{8} = 1.23\frac{m}{s^2}$. Hints: The tension forces on either side of block 3 are not the same and the tension forces in the system are not the same as what you found in part b.



Note: Figure not drawn to scale.

For m_1:

$F_{TL} - F_{W1} = F_{TL} - m_1 g = m_1 a \rightarrow F_{TL} = m_1 g + m_1 a$

For m_3:

$-F_{TL} + F_{TR} = m_3 a$

For m_2:

$F_{TR} - F_{W2} = F_{TR} - m_2 g = -m_2 a \rightarrow F_{TR} = m_2 g - m_2 a$

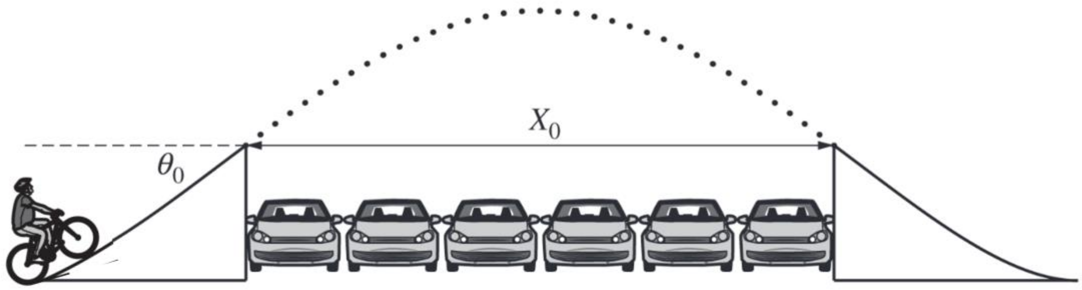
$$-F_{TL} + F_{TR} = m_3 a \rightarrow -m_1 g - m_1 a + m_2 g - m_2 a = m_3 a \rightarrow a = \left(\frac{m_2 - m_1}{m_3 + m_2 + m_1}\right) g$$

$$a = \frac{g}{8} = \left(\frac{2kg - 1kg}{m_3 + 2kg + 1kg}\right) g \rightarrow m_3 = 5kg$$

d. In part a, there was only one tension force in the rope connecting blocks m_1 and m_2. In part c, there are two different tension forces in the system of ropes. Explain why there are two tension forces in part c and which tension force must be larger, the tension force in the rope that connects blocks m_1 and m_3, or the tension force in the rope that connects blocks m_2 and m_3.

We must accelerate mass m_3 to the right. To do this the tension force on the right-hand side of m_3 has to be greater than the tension force on the left-hand side of m_3. Thus, the two tension forces cannot be equal and the difference in the tension forces (which points to the right) causes m_3 to accelerate to the right. If the two tension forces were the same on mass m_3, then the mass m_3 would not accelerate left or right.

3. A stunt cyclist builds a ramp to jump over some parked cars. The cyclist will accelerate themselves up the ramp and launch themselves over the parked cars. To test the ramp, the cyclist starts from rest at the bottom of the ramp inclined at $\theta_0 = 30^0$ measured with respect to the ground. The cyclist jumps over six parked cars and lands on the second ramp on the right.



Note: Figure not drawn to scale.

a. Suppose the cyclist wants to leave the end of the ramp at a speed of $14\frac{m}{s}$. If the ramp is $5m$ long, what force would the cyclist need to generate to propel themselves up the ramp to launch themselves at this speed? The cyclist and the bike have a total mass $m_{total} = 100kg$ and there is friction on the ramp with coefficient $\mu = 0.1$.

The net acceleration of the cyclist up the ramp:

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x = 2a_x\Delta x \rightarrow a_x = \frac{v_{fx}^2}{2\Delta x} = \frac{\left(14\frac{m}{s}\right)^2}{2 \times 5m} = 19.6\frac{m}{s^2}$$

From the forces on the cyclist parallel to the ramp:

$$F_{bike} - F_{fr} - F_{Wx} = F_{bike} - \mu F_N - F_W \sin\theta = ma_x$$
$$F_{bike} = ma_x + \mu F_N + F_W \sin\theta = ma_x + \mu mg\cos\theta + mg\sin\theta$$
$$F_{bike} = 100kg \times \left(19.6\frac{m}{s^2} + 0.1 \times 9.8\frac{m}{s^2} \times \cos 30 + 9.8\frac{m}{s^2}\sin 30\right) = 2535N$$

b. What total length of cars that the cyclist covers, X_0?

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 = v_{ix}t = (v_i\cos\theta)t = 14\frac{m}{s} \times 1.43s \times \cos 30 = 17.3m$$

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 \rightarrow 0 = \left(v_i\sin\theta - \tfrac{1}{2}gt\right)t$$

$$\rightarrow t = \begin{cases} 0s \\ \dfrac{2v_i\sin\theta}{g} = \dfrac{2 \times 14\frac{m}{s} \times \sin 30}{9.8\frac{m}{s^2}} = 1.43s \end{cases}$$

c. What is the maximum height above the launch point that the cyclist reaches?

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y \rightarrow \Delta y = \frac{v_{iy}^2}{2g} = \frac{v_i^2 \sin^2\theta}{2g} = \frac{\left(14\frac{m}{s}\right)^2 \sin^2 30}{2 \times 9.8\frac{m}{s^2}} = 2.5m$$

Or

$$v_{fy} = v_{iy} + a_y t \rightarrow t = \frac{v_{iy}}{g} = \frac{14\frac{m}{s}\sin 30}{9.8\frac{m}{s^2}} = 0.42s$$
$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 = (v_i \sin\theta)t - \tfrac{1}{2}gt^2$$
$$y_f = \left(14\frac{m}{s}\sin 30\right) \times 0.72s - \frac{1}{2} \times 9.8\frac{m}{s^2} \times (0.72s)^2 = 2.5m$$

d. The cyclist eventually lands on the ramp on the right side of the cars. If the cyclist does not apply the brakes, what will be the speed of the cyclist at the bottom of the ramp. Assume the ramp on the right is frictionless, has a length of $5m$, and is inclined at 30^0 to the horizontal.

The impact speed:
$$v_{fx} = v_{ix} + a_x t = v_{ix} = v_i \cos\theta = 14\frac{m}{s}\cos 30 = 12.1\frac{m}{s}$$
$$v_{fy} = v_{iy} + a_g t = v_i \sin\theta - gt = 14\frac{m}{s}\sin 30 - 9.8\frac{m}{s^2} \times 1.43s = -7.0\frac{m}{s}$$
$$v_{fx} = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(12.1\frac{m}{s}\right)^2 + \left(7.0\frac{m}{s}\right)^2} = 14\frac{m}{s}$$
As I would expect since the motion is symmetric.

From the forces that act on the mass parallel to the ramp we have
$$F_{Wx} = mg\sin\theta = ma_x \rightarrow a_x = g\sin\theta$$

The final speed of the cyclist
$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow v_{fx} = \sqrt{v_{ix}^2 + 2g\sin\theta\,\Delta x} = \sqrt{\left(14\frac{m}{s}\right)^2 + 2g}$$
$$v_{fx} = \sqrt{\left(14\frac{m}{s}\right)^2 + 2 \times 9.8\frac{m}{s^2}\sin 30 \times 5m} = 15.7\frac{m}{s}$$