Physics 110

Fall 2011

Exam #1

September 26, 2011

Name____________________

Multiple Choice	/ 10
Problem #1	/ 36
Problem #2	/ 27
Problem #3	/ 27
Total	/ 100

In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.

Part I: Free Response Problems

The three problems below are worth 90 points total and each subpart is worth 9 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.

1. A top fuel drag race involves two dragsters (one is shown below left) that accelerate from rest and try to beat each other a distance of ¼ mile (*400m*) on a straight racetrack. Suppose that you watch one of these cars start from rest, accelerate at a constant rate and at the end of the ¼ mile stretch of track deploy a parachute (shown on the right), which causes the car to decelerate back to rest over a further distance of ½ mile (*800m*).

a. If you wanted the car to cover the ¼ mile track in *4.5s*, what acceleration would be required?

$$x_f = \frac{1}{2}a_x t^2 \rightarrow a_x = \frac{2x_f}{t^2} = \frac{2\times 400m}{(4.5s)^2} = 39.5\,\tfrac{m}{s^2}$$

b. How fast would your dragster be traveling at the end of the ¼ mile track just before you deploy the parachute?

$$v_{fx} = v_{ix} + a_x t = 39.5\,\tfrac{m}{s^2}\times 4.5s = 177.8\,\tfrac{m}{s}$$

c. Suppose that you deploy your parachute as soon as you cross the ¼ mile mark on the track and decelerate to rest. What is the assumed deceleration over the remaining ½ mile of track that the dragster covers?

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow a_x = -\frac{v_{ix}^2}{2\Delta x} = -\frac{\left(177.8\,\tfrac{m}{s}\right)^2}{2\times 800m} = -19.8\,\tfrac{m}{s^2}$$

d. If your dragster had a mass *m= 816kg*, what force would need to be produced to stop your dragster?

$$F_x = ma_x = 816kg\times\left(-19.8\,\tfrac{m}{s^2}\right) = -16123N$$

2. Suppose that two identical balls are tossed off of a building that is 20m tall. Each ball is thrown with an initial speed of $v_i = 12m/s$ but ball #1 is thrown at an angle of 30° above the horizontal, while ball #2 is thrown at 30° below the horizontal.

   a. What are the flight times for *each* ball?

   For Ball #1

$$y_{f,1} = y_{i,1} + v_{iy,1}t_1 + \tfrac{1}{2}a_y t_1^2 \rightarrow y_{f,1} = +\left(v_i \sin\theta\right)t_1 - \tfrac{1}{2}gt_1^2 \rightarrow 0 = 20 + \left(12\sin 30\right)t_1 - 4.9t_1^2$$

$$t_1 = \begin{cases} -1.5s \\ +2.7s \end{cases}$$

   For Ball #2

$$y_{f,2} = y_{i,2} + v_{iy,1}t_2 + \tfrac{1}{2}a_y t_2^2 \rightarrow y_{f,2} = -\left(v_i \sin\theta\right)t_2 - \tfrac{1}{2}gt_2^2 \rightarrow 0 = 20 - \left(12\sin 30\right)t_2 - 4.9t_2^2$$

$$t_1 = \begin{cases} +1.5s \\ -2.7s \end{cases}$$

   b. What are the velocities of *each* ball just before impact with the ground?

   For Ball #1

$$v_{f,1} = \sqrt{v_{fx,1}^2 + v_{fy,1}^2}\ @\ \phi_1 = \tan^{-1}\left(\frac{v_{fy,1}}{v_{fx,1}}\right) = 23.2\tfrac{m}{s}\ @\ \phi_1 = -63^0$$

$$v_{fx,1} = v_{ix,1} = v_i\cos\theta = 12\tfrac{m}{s}\cos 30 = 10.4\tfrac{m}{s}$$

$$v_{fy,1} = v_{iy,1} - gt_1 = v_i\sin\theta - gt_1 = 12\tfrac{m}{s}\sin 30 - 9.8\tfrac{m}{s^2} \times 2.7s = -20.5\tfrac{m}{s}$$

   For Ball #2

$$v_{f,2} = \sqrt{v_{fx,2}^2 + v_{fy,2}^2}\ @\ \phi_2 = \tan^{-1}\left(\frac{v_{fy,2}}{v_{fx,2}}\right) = 23.2\tfrac{m}{s}\ @\ \phi_1 = -63^0$$

$$v_{fx,2} = v_{ix,2} = v_i\cos\theta = 12\tfrac{m}{s}\cos 30 = 10.4\tfrac{m}{s}$$

$$v_{fy,2} = -v_{iy,2} - gt_2 = -v_i\sin\theta - gt_2 = -12\tfrac{m}{s}\sin 30 - 9.8\tfrac{m}{s^2} \times 1.5s = -20.5\tfrac{m}{s}$$

   c. What is the horizontal distance between the points where each ball strikes the ground?

$$\Delta x = x_{f,2} - x_{f,1} = \left(v_i\cos\theta\right)\left[t_2 - t_1\right] = \left(12\tfrac{m}{s}\cos 30\right)\left[2.7s - 1.5s\right] = 12.5m$$

3. In class we've started talking about Newton's laws of motion. This problem involves an application of Newton's 2nd and 3rd laws of motion and the equations of motion. Suppose that you (of mass *70kg*) stand on a table the top of which is ½ m above the ground.

   a. If you walk off of the edge of the table, what is your time of flight to the ground? (Please keep 3 decimal places in your answer for the time.)

   $$y_f = y_i + v_{iy}t + \tfrac{1}{2}at^2 \rightarrow -\tfrac{1}{2} = -\tfrac{1}{2}\times 9.8\tfrac{m}{s^2}t^2 \rightarrow t = 0.319s$$

   b. In the time you spend accelerating towards the ground, by Newton's 3rd law the Earth is accelerating up towards you, since you both exert equal magnitude and oppositely directed gravitational forces on each other. In the time that you fall to the ground, what distance does the Earth move up toward you? (Hint: The mass of the Earth is $6x10^{24}kg$.)

   $$\left|F_{E,you}\right| = \left|F_{you,E}\right| \rightarrow m_E a_E = m_{you}a_{you}$$

   $$\rightarrow a_E = \frac{m_{you}a_{you}}{m_E} = \frac{70kg\times 9.8\frac{m}{s^2}}{6\times 10^{24}kg} = 1.14\times 10^{-22}\tfrac{m}{s^2}$$

   $$x_{f,E} = \tfrac{1}{2}a_E t^2 = \tfrac{1}{2}\times 1.14\times 10^{-22}\tfrac{m}{s^2}\times (0.319s)^2 = 5.8\times 10^{-24}m$$

   c. Of course it should be obvious that the acceleration and displacement of the Earth that you calculated are incredibly small, due to the large mass of the Earth. (You on the other hand have a small mass and hence a rather large acceleration and a rather large displacement compared to that of the Earth.) Suppose instead that you wanted the Earth to move upwards through a measurable distance, say *2cm*. To do this you take *N, 70kg* people and go to the North Pole. You all stand on tables and jump off at the same time. How many people would you need to bring to the North Pole to accomplish this feat of displacing the Earth by *2cm*? Compare your answer *N*, with the population of the Earth, which is about 6 billion people.

   $$x_{f,E} = \tfrac{1}{2}a_E t^2 \rightarrow a_{E,new} = \frac{2x_{f,E}}{t_{new}^2} = \frac{2\times 0.02m}{(0.313s)^2} = 0.408\tfrac{m}{s^2};$$

   where the time is calculated from the distance you fall, 0.48m

   $$x_{f,you} = \tfrac{1}{2}a_y t_{new}^2 \rightarrow t_{new} = \sqrt{\frac{2x_{f,you}}{a_{you}}} = \sqrt{\frac{2\times 0.48m}{9.8\frac{m}{s^2}}} = 0.313s$$

   $$\left|F_{E,you}\right| = \left|F_{you,E}\right| \rightarrow m_E a_E = Nm_{you}a_{you}$$

   $$\rightarrow N = \frac{m_E a_E}{m_{you}a_{you}} = \frac{6\times 10^{24}kg\times 0.408\frac{m}{s^2}}{70kg\times 9.8\frac{m}{s^2}} = 3.6\times 10^{21}$$

   $\sim 6x10^{11}$ times the population of the Earth.

Part II: Multiple-Choice

Circle your best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 2.5 points for a total of 10 points.

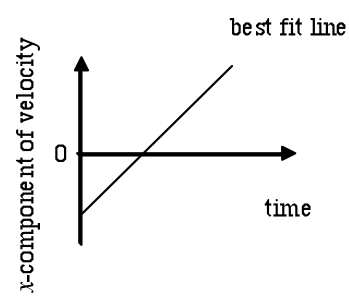
1. You observe three carts moving to the left.
    Cart A moves with constant speed.
    Cart B is speeding up.
    Cart C is slowing down.
    Which cart or carts are experiencing a net acceleration to the left?

    a. Cart A b. Cart B c. Cart C d. Carts B and C

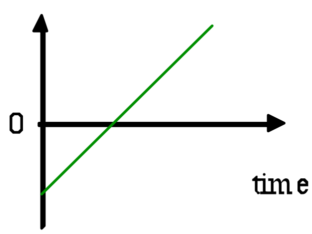
2. Suppose that instead of the Earth being created in its present size and shape that it were somehow actually created twice as massive as it really is with twice the radius. The approximate acceleration due to gravity on this "new" Earth would be equal to

    a. g b. $g/4$ c. $g/2$ d. $4g$

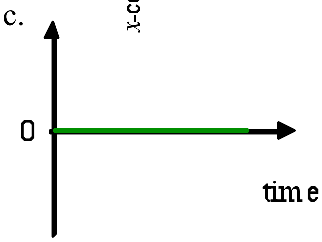
3. The x-component of a particle's velocity is sampled every 5 seconds. The data are fit with a straight line as shown in the figure to the right. Assuming the fit is a good approximation to the motion, which of the following best represents the x-component of the displacement of the particle?



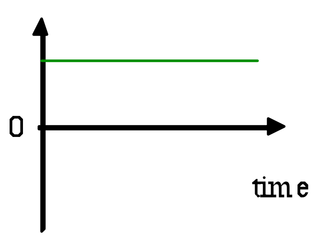
a.



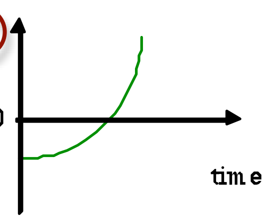
c.



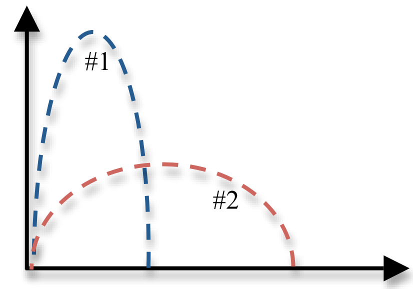
b.



d.



4. Two objects are fired into the air and the motion of the projectile is shown? Projectile #1 reaches the greater height (the blue curve), but projectile #2 (the red curve) covers twice the horizontal distance as projectile #1. Which one is in the air for the greater amount of time?



   a. Projectile #1, because it travels higher than projectile #2.
   b. Projectile #2, because it travels the greater horizontal distance.
   c. Both Projectiles spend the same amount of time in the air.
   d. Projectile #2, because it has the smaller initial speed and therefore travels more slowly than projectile #1.

Motion in the r = x, y or z-directions

$r_f = r_i + v_{ir}t + \frac{1}{2}a_r t^2$

$v_{fr} = v_{ir} + a_r t$

$v_{fr}{}^2 = v_{ir}{}^2 + 2a_r \Delta r$

Uniform Circular Motion

$a_r = \dfrac{v^2}{r}$

$F_r = ma_r = m\dfrac{v^2}{r}$

$v = \dfrac{2\pi r}{T}$

$F_G = G\dfrac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles — $C = 2\pi r$, $A = \pi r^2$

Triangles — $A = \frac{1}{2}bh$

Spheres — $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\textit{magnitude of a vector} = \sqrt{v_x{}^2 + v^2{}_y}$$

$$\textit{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8 \, m/s^2$ $\quad G = 6.67 \times 10^{-11} \, Nm^2/kg^2$

$N_A = 6.02 \times 10^{23} \, atoms/mole$ $\quad k_B = 1.38 \times 10^{-23} \, J/K$

$\sigma = 5.67 \times 10^{-8} \, W/m^2K^4$ $\quad v_{sound} = 343 \, m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}\left[T_F - 32\right]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \dfrac{\Delta Q}{\Delta t} = \dfrac{kA}{L}\Delta T$

$P_R = \dfrac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\frac{m}{s}$

$\beta = 10\log\dfrac{I}{I_0}; \quad I_o = 1 \times 10^{-12} \frac{W}{m^2}$

$f_n = nf_1 = n\dfrac{v}{2L}; \ f_n = nf_1 = n\dfrac{v}{4L}$

Fluids

$\rho = \dfrac{M}{V}$

$P = \dfrac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1 v_1 = A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \dfrac{2\pi}{T}$

$T_S = 2\pi\sqrt{\dfrac{m}{k}}$

$T_P = 2\pi\sqrt{\dfrac{l}{g}}$

$v = \pm\sqrt{\dfrac{k}{m}}A\left(1 - \dfrac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\dfrac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\dfrac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\dfrac{F_T}{\mu}}$

$f_n = nf_1 = n\dfrac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$