Physics 110

Exam #1

September 30, 2013

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. A car starts from rest on a long straight road and accelerates at a constant rate of $3\,\frac{m}{s^2}$ until its speed is $50\,\frac{m}{s}$ $(\sim 115mph)$. Once it reaches $50\,\frac{m}{s}$, it continues to travel at a constant velocity for $667m$ $(\sim 0.4mi)$, at which time the car's driver applies the brakes and the car decelerates to rest at a rate of $5\,\frac{m}{s^2}$. From the time the car starts from rest and returns to rest again, the car covers a total distance of $1335m$ $(\sim 0.83mi)$.

   a. For how much time does the car travel before the brakes are applied?

   There are two phases to the car's motion. One, the car accelerates from rest to $50\,\frac{m}{s}$ and then travels at a constant velocity for the second part. We'll break the motion into two pieces and calculate the time for each. The total travel time will be the sum of these two times.

   Acceleration: $v_{fx} = v_{ix} + a_x t_{acc} \rightarrow t_{acc} = \dfrac{v_{fx} - v_{ix}}{a_x} = \dfrac{50\,\frac{m}{s} - 0\,\frac{m}{s}}{3\,\frac{m}{s^2}} = 16.7s$

   Constant velocity: The time to travel the $667m$ traveling at $50\,\frac{m}{s}$ is given by

   $$v_x = \frac{\Delta x}{\Delta t} \rightarrow \Delta t = t_{const} = \frac{\Delta x}{v_x} = \frac{667m}{50\,\frac{m}{s}} = 13.3s$$

   Thus the total time the car travels before applying the brakes is

   $$t_{total} = t_{acc} + t_{const} = 16.7s + 13.3s = 30s$$

   b. Suppose that an airplane flying overhead spots the car sitting at rest and decides to race the car. At the instant that the car starts to move, the airplane, traveling horizontally at a constant speed of $55\,\frac{m}{s}$ $(\sim 124mph)$ passes directly above the car. Will the car ever overtake the airplane before the car needs to slow down or will the airplane beat the car in the race?

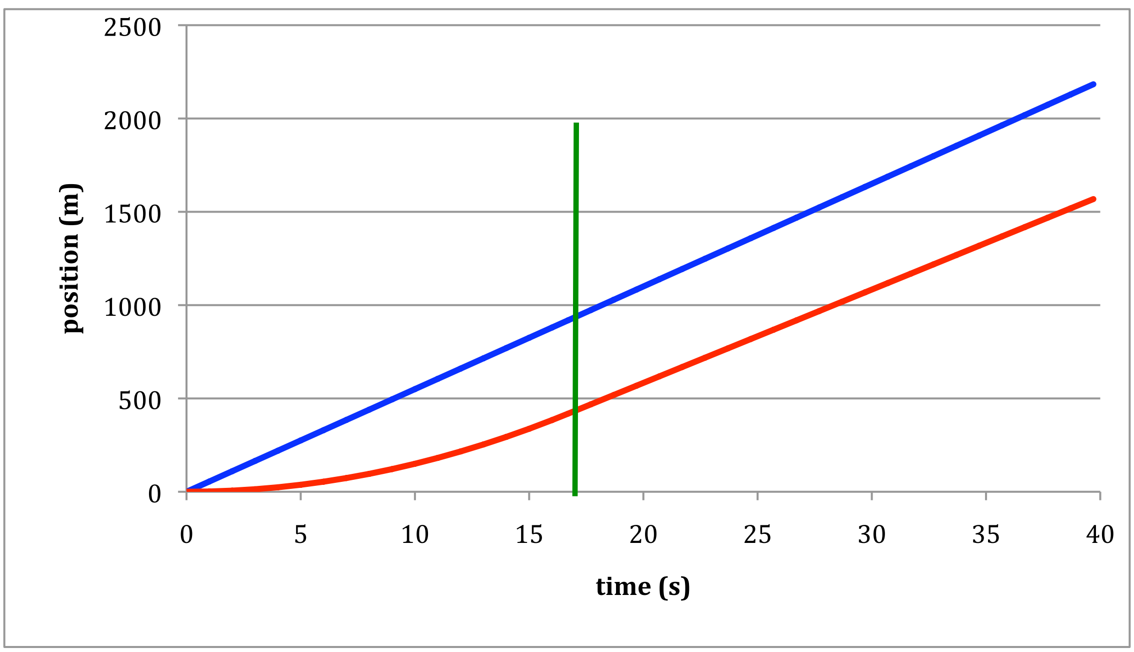
   While the car accelerates it travels some distance given by $x_{f,acc} = x_{i,acc} + v_{ix}t_{acc} + \frac{1}{2}a_x t_{acc}^2 = \frac{1}{2}\left(3\,\frac{m}{s^2}\right)(16.7s)^2 = 418.3m$ at which point it travels at a constant velocity. Thus the trajectory for the car is given by $x_{f,c} = x_{i,c} + v_{ix}t_{catch} + \frac{1}{2}a_x t_{catch}^2 = 418.3 + 50t_{catch}$. While the car is accelerating the plane is also moving overhead at a constant velocity. When the car stops accelerating the plane is located at $x_{f,p} = x_{i,p} + v_{ix}t_{acc} + \frac{1}{2}a_x t_{acc}^2 = 55\,\frac{m}{s} \times 16.7s = 918.5m$. Thus the plane's trajectory is given as $x_{f,p} = x_{i,p} + v_{ix}t_{catch} + \frac{1}{2}a_x t_{catch}^2 = 918.5 + 55t_{catch}$. The car catches the plane if there is a time at which they have the same location. Thus we equate the two trajectories and determine the time.

   $x_{f,c} = x_{f,p} \rightarrow 418.3 + 50t_{catch} = 918.5 + 55t_{catch} \Rightarrow t_{catch} = -100.4s$. Since the time is negative, there is no positive time, given the parameters above that the car catches the plane.

c. If the car overtakes the airplane, how far from the cars starting position will this happen? If the car does not ever catch the airplane, how close does the car get to the airplane?

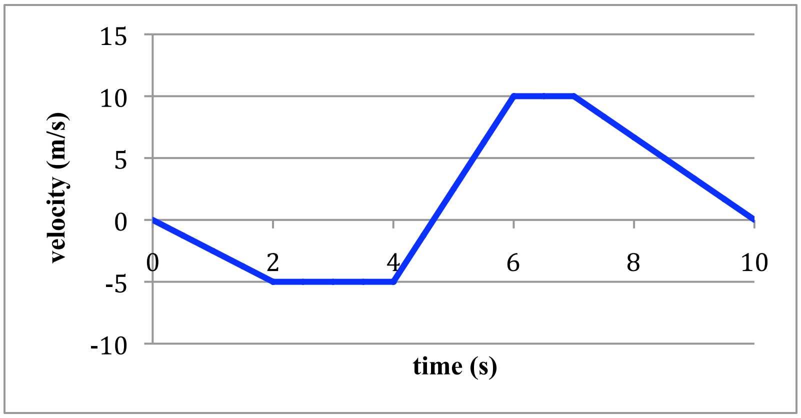
Since the car doesn't catch the plane, we have to find the distance of closest approach that the car get's to the airplane before it has to begin to slow down. From part a, I know that the car has to travel for a total time of $30s$ before it begins to brake. In this total time the car travels for $16.7s$ accelerating to a constant velocity. After the car reaches $50\frac{m}{s}$ the plane continues to pull away from the car. Thus after the car stops accelerating is when the car and plane are as close as they can get. Thus the distance of closest approach for the car is given by
$\Delta x = x_{f,p} - x_{f,c} = 918.5m - 418.3m = 500.2m$

This can be seen graphically below, plotting the two trajectories, where the red curve is the car and the blue curve is the plane. The green vertical line is when the car stops accelerating.



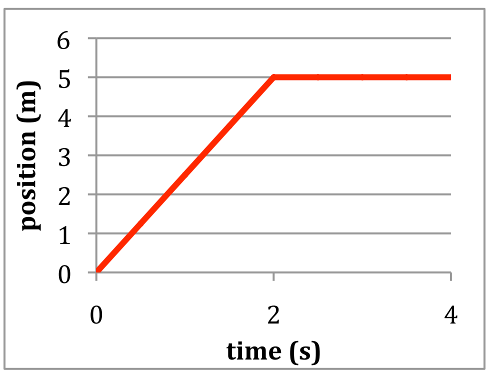
Technically speaking of course, the distance between them keeps increasing for any time greater than the start. So probably the actual distance of closest approach is at time zero, but I was looking for the time when the car stopped accelerating although this was not explicitly stated in the problem.

2. Below is a velocity versus time graph for an object moving in one-dimension.

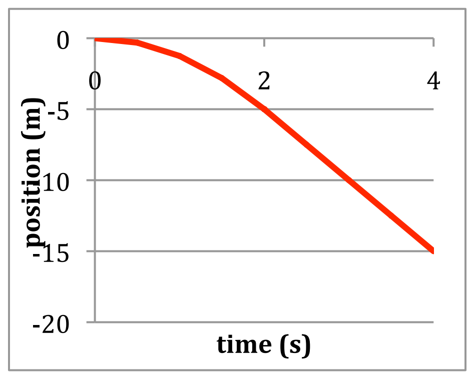


a. Over the time interval $0s \le t \le 4s$, a possible position versus time graph might look like

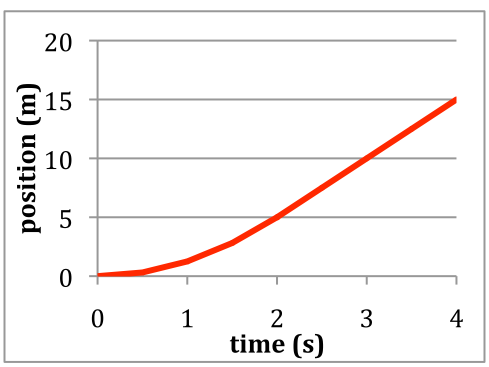
1.



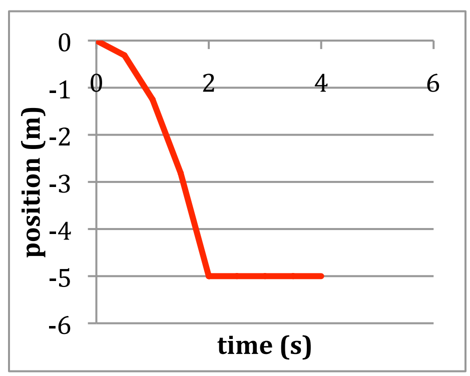
2.



3.



4.



The object accelerates from rest to a negative velocity at a constant (slope of the velocity versus time graph is negative) and then it continues at a constant velocity (slope of the velocity versus time graph is zero.) The acceleration produces a position versus time graph that is parabolic and since it is negative the parabola opens down (the object gets farther away from the origin as time progresses. When it's velocity becomes constant the position versus time graph takes on a linear shape and since the velocity is constant and negative, the object continues to get farther away from the origin in the negative x-direction. Thus the only graph that shows these two features is graph 2.

b. Describe in words the motion of the object during the time interval $4s \le t \le 6s$, and determine at what time is the object at rest?

During this time interval, the object is traveling at an initial velocity of $5\frac{m}{s}$ in the negative x-direction. The object experiences an acceleration directed opposite its velocity (a positive acceleration) and the object begins to slow down. Eventually the object comes to rest and this time is given by

$v_{fx} = v_{ix} + a_x t \rightarrow 0\frac{m}{s} = -5\frac{m}{s} + \left(\frac{10\frac{m}{s} - \left(-5\frac{m}{s}\right)}{6s - 4s}\right)t \rightarrow t = 0.67s$, which has to be added to the start of the time interval, so the object comes to rest at a time $4.67s$. The acceleration persists and the object experiences a change it its direction and its speed. Now it is traveling in the positive x-direction and its velocity now reaches a maximum of $10\frac{m}{s}$ due to the positive acceleration of $7.5\frac{m}{s^2}$.

c. What is the total distance traveled by the object between $0s \le t \le 10s$?

The total distance traveled will be the sum of the distances traveled over each time interval. Over the first interval the object starts from rest and accelerates. It covers a distance given by $\Delta x_{f1} = v_{i1}t_1 + \frac{1}{2}a_1 t_1^2 = \frac{1}{2}\left(\frac{-5\frac{m}{s} - 0\frac{m}{s}}{2s}\right)(2s)^2 = -5m$, or $5m$ total distance traveled. [This can also be obtained from

$\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1} \rightarrow \Delta x_1 = \bar{v}_1 \Delta t_1 = \left(\frac{0 + 5\frac{m}{s}}{2}\right)2s = 5m$.] Over the second interval the object travels at a constant velocity and covers a distance given by

$\Delta x_{f2} = v_{i2}t_2 + \frac{1}{2}a_2 t_2^2 = -5\frac{m}{s} \times 2s = -10m$ or $10m$ more for a total of $15m$ total distance traveled. [This can also be obtained from

$\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2} \rightarrow \Delta x_2 = \bar{v}_2 \Delta t_2 = \left(\frac{5\frac{m}{s} + 5\frac{m}{s}}{2}\right)2s = 10m$.] Over the third interval we have to be more careful since using the trajectory equation gives the displacement, not the distance. So we must use the average velocity definition. Breaking this 4th interval up into two pieces (one from $4 \le t \le 4\frac{2}{3}$ and one from $4\frac{2}{3} \le t \le 6$) we have

$\bar{v}_3 = \frac{\Delta x_3}{\Delta t_3} \rightarrow \Delta x_3 = \bar{v}_3 \Delta t_3 = \left(\frac{5\frac{m}{s} + 0\frac{m}{s}}{2}\right)\frac{2}{3}s = \frac{5}{3}m$ and

$\bar{v}_3 = \frac{\Delta x_3}{\Delta t_3} \rightarrow \Delta x_3 = \bar{v}_3 \Delta t_3 = \left(\frac{0\frac{m}{s} + 10\frac{m}{s}}{2}\right)\frac{4}{3}s = \frac{40}{6}m$. Thus I have traveled a distance of

$\frac{5}{3}m + \frac{40}{6}m = \frac{50}{6}m = 8.3m$. Now I've covered $23.3m$ total. Over the fourth interval the object is again traveling with a constant velocity and it covers a distance given by

$\Delta x_{f4} = v_{i4}t_4 + \frac{1}{2}a_4 t_4^2 = 10\frac{m}{s} \times 1s = 10m$ bringing my distance traveled to $33.3m$. [This can also be obtained from $\bar{v}_4 = \frac{\Delta x_4}{\Delta t_4} \rightarrow \Delta x_4 = \bar{v}_4 \Delta t_4 = \left(\frac{10\frac{m}{s} + 10\frac{m}{s}}{2}\right)1s = 10m$.] And over the last interval the object covers a distance given by

$\Delta x_{f5} = v_{i5}t_5 + \tfrac{1}{2}a_5 t_5^2 = \left(10\tfrac{m}{s} \times 3s\right) + \tfrac{1}{2}\left(\dfrac{0\tfrac{m}{s} - 10\tfrac{m}{s}}{3s}\right)(3s)^2 = 15m$, adding another $15m$ to my total and now I've covered $48.3m$ in total distance. [This can also be obtained from $\bar{v}_5 = \dfrac{\Delta x_5}{\Delta t_5} \rightarrow \Delta x_5 = \bar{v}_5 \Delta t_5 = \left(\dfrac{0\tfrac{m}{s} + 10\tfrac{m}{s}}{2}\right)3s = 15m$.]

Note, by the way, this is different than the total displacement. The displacement is a vector and is given by the vector sum of the displacements. The total displacement is $\Delta x_{total} = -5m - 10m + 5m + 10m + 15m = +15m$ or the object ends up $15m$ to the right of where it started.

d. Over the time interval $7s \le t \le 10s$, the acceleration of the object is

1. positive and increasing with time.
2. positive and decreasing with time
3. positive and constant in time.
4. negative and constant in time.
5. negative and increasing with time.
6. negative and decreasing with time.

3. In a local drinking establishment a patron slides an empty glass down the bar. The bartender, momentarily distracted, does not see the sliding glass and the glass slides horizontally off of the bar and ultimately smashes on the floor. Suppose that the glass leaves the top of the bar with a speed of v_i and that the mass of the empty mug is m.

   a. What are the time of flight of the glass to the ground and the initial velocity with which the glass left the top of the bar if the top of the bar is $1.3m$ above the ground and the glass impacts the ground $2.1m$ from the edge of the bar?

   Assuming that vertically up from the bar is the positive y-direction and to the right is the positive x-direction, the vertical distance that the projectile has to fall through determines the time of flight. Thus we have: $y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow 0 = 1.3 - \frac{9.8\frac{m}{s^2}}{2}t_{tof}^2 \rightarrow t_{tof} = \sqrt{\frac{2\times 1.3m}{9.8\frac{m}{s^2}}} = 0.52s$. In this time, the projectile is traveling horizontally and covers before landing. Thus the initial velocity of the projectile is given by $x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \rightarrow v_{ix} = \frac{x_f}{t} = \frac{2.1m}{0.52s} = 4.0\frac{m}{s}$.

   b. What is the impact velocity of the glass on the ground?

   We need to determine the components of the impact velocity of the glass. The horizontal velocity is a constant of the motion, so $v_{fx} = v_{ix} = 4.0\frac{m}{s}$. The vertical component of the velocity changes with time and the final y-component of the velocity is given by $v_{fy} = v_{ig} + a_y t = -9.8\frac{m}{s^2}\times 0.52s = -5.1\frac{m}{s}$. The magnitude of the impact velocity $v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(4.0\frac{m}{s}\right)^2 + \left(-5.1\frac{m}{s}\right)^2} = 6.5\frac{m}{s}$ at an angle $\phi = \tan^{-1}\left(\frac{-5.1\frac{m}{s}}{4.0\frac{m}{s}}\right) = -51.9^o$.

c. Suppose that another customer wants to try the same thing but decides to slide their full glass down the bar. Suppose that the full glass leaves the top of the bar horizontally and that it leaves with the same initial velocity as the empty mug, v_i when it leaves the top of the bar. Now, however, this glass has a mass that is 3 times that of the empty mug. When the glass impacts the floor, its impact velocity will be

1. greater than the empty glass since the time it takes the full glass to hit the floor will be greater.
2. be the same as the empty glass since the time it takes both glasses to hit the floor is the same.
3. less than the empty glass since the time it takes the full glass to hit the floor will be less.
4. will be different, but in a way that cannot be predicted from the information given.

Since the mass is not a factor in determining the impact velocity, only the time of flight, the time to hit the floor for both is the same so the impact velocity is the same.

d. Compared to the empty glass, when the full glass hits the floor, the magnitude of the impact force (the force with which either glass hits the floor) will be

1. greater for the empty glass.
2. greater for the full glass.
3. the same for both glasses.
4. smaller for the full glass.

The impact velocity is the same for both, so the impact acceleration, (change in velocity with respect to time) is the same for both, assuming the collision time is the same. Thus what determines the magnitude of the collision force is the mass. Since the full glass has more mass, by Newton's 2nd law it has a larger impact force.

Useful formulas:

Motion in the r = x, y or z-directions

$r_f = r_0 + v_{0r}t + \frac{1}{2}a_r t^2$

$v_{fr} = v_{0r} + a_r t$

$v_{fr}^2 = v_{0r}^2 + 2a_r \Delta r$

Uniform Circular Motion

$a_r = \frac{v^2}{r}$

$F_r = ma_r = m\frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	Triangles	Spheres
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

$\text{Quadratic equation}: ax^2 + bx + c = 0,$

$\text{whose solutions are given by}: \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\text{magnitude of a vector} = \sqrt{v_x^2 + v_y^2}$$

$$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}/_{s^2} \qquad G = 6.67 \times 10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02 \times 10^{23}\,{}^{atoms}/_{mole} \qquad k_B = 1.38 \times 10^{-23}\,{}^{J}/_{K}$

$\sigma = 5.67 \times 10^{-8}\,{}^{W}/_{m^2K^4} \qquad v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1 v_1 = A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$

Sound

$v = f\lambda = (331 + 0.6T)\,\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1 \times 10^{-12}\,\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$