Physics 110

Exam #1

April 15, 2013

Name_________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. To save fuel, some long-haul truck drivers try to maintain a constant speed when possible. A long-haul semi truck is traveling at $91\frac{km}{hr} \approx 25\frac{m}{s}$ approaches a car from behind that is stopped at a red light. When the truck is $116m$ from the car the light turns green. The car begins to accelerate at $3\frac{m}{s^2}$ to a final speed of $106\frac{km}{hr} \approx 29\frac{m}{s}$.

   a. Will the truck, traveling at this constant velocity, hit the accelerating car? If they do collide, at what time does the collision occur? If they do not collide provide a justification as to why they do not.

      Whether the truck and car collide depends on whether or not they are at the same location at the same time. To determine this we write the trajectory for the truck and for the car and equate them to see if there is a time they are at the same location. For the truck we have $x_{f,T} = x_{i,T} + v_{ix,T}t + \frac{1}{2}a_{x,T}t^2 = -116 + 25t$ while for the car we have $x_{f,c} = x_{i,c} + v_{ix,c}t + \frac{1}{2}a_{x,c}t^2 = \frac{3}{2}t^2$. Equating these two expressions we have a quadratic equation in time that needs to be solved. Using the quadratic equation we find:

      $1.5t^2 - 25t + 116 = 0 \rightarrow t = \dfrac{25 \pm \sqrt{(-25)^2 - 4\times 1.5\times 116}}{2\times 1.5} = \dfrac{25 \pm \sqrt{625-696}}{3}$. Since the part under the radical is negative there will never be a time at which the car and the truck are at the same location. So they never collide

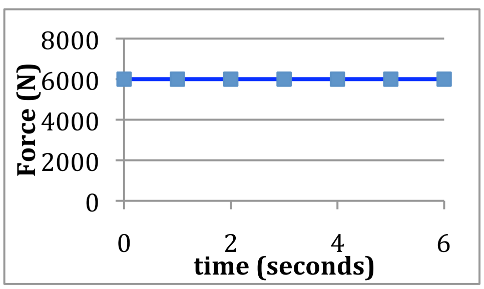
   b. If the truck does collide with the car, with respect to the car's initial position, where do they collide? If they do not collide, how closer does the truck get to the car?

      To determine how far apart they are, we need to determine at what time does the car reach the same speed as the truck. Then we can use the trajectory equations to locate the car and truck in space as this will be the distance of closest approach. Subtracting these two results at this time will give the separation distance. To determine the time at which the car reaches the truck's speed we use the velocity equation of motion: $v_{f,c} = 25\frac{m}{s} = v_{i,c} + a_c t = 3\frac{m}{s^2}t \rightarrow t = 8.3s$At this time the truck's position is $x_{f,T} = -116 + 25t = -116m + 25\frac{m}{s}\times 8.3s = 91.5m$ and the car's position is $x_{f,c} = \frac{3}{2}t^2 = \frac{3\frac{m}{s^2}}{2}(8.3s)^2 = 103.3m$. Thus the car and truck are separated by $\Delta x = 103.3m - 91.5m = 11.8m$.

c. Suppose for this particular question assume that the car and truck do not collide, but that the truck comes within a distance Δx of the car. Calling this time of closest approach t_{CA}, for any time $t > t_{CA}$ what happens?

1. The car pulls away from the truck because the car is still accelerating.
2. The truck has to slow down to avoid a collision.
3. The car and truck will continue to move forward maintaining the same separation Δx.
4. The car and trucks motions cannot be determined so it is uncertain about what will happen.

d. Suppose that ahead of the truck and car is another traffic light. Suppose that the light is changing from green to red and that the car will easily make it through the traffic light, but that the truck will not. The truck therefore needs to stop at the red light. Which of the graphs below shows the change in motion of the truck as it approaches the red light. Assume that the direction of the truck's initial velocity is taken to be the $+x$-axis and the truck comes to rest at a constant rate.

1.


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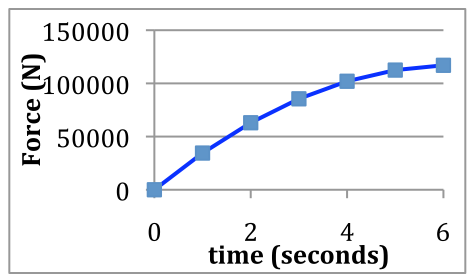

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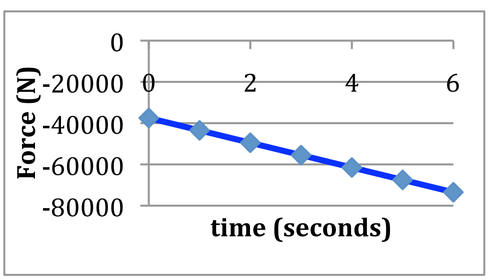

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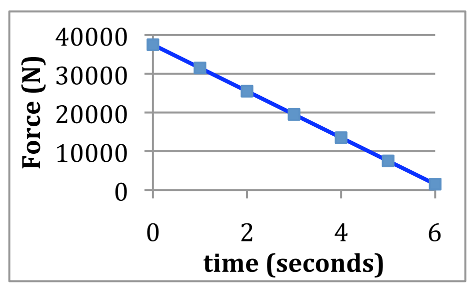

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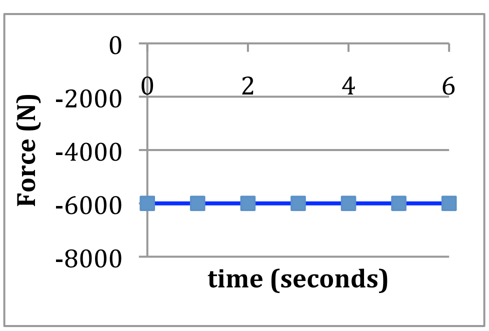

2. A model rocket is launched vertically from rest from the ground and its engines produce a constant acceleration of $13\frac{m}{s^2}$ for $8s$ after which time the fuel is used up. Assume that the rocket travels in the vertical direction only throughout it's entire motion and that air resistance is negligible.

   a. What maximum height above the ground will the rocket reach?

   There are two parts to the motion, one in which the rocket is accelerating upwards using its engines and one in which the engines shut off and gravity slows the rocket down. We have the maximum height given as the sum of these two motions: $x_{\max} = x_{acc} + x_{no\ acc} = \left(x_i + v_{iy}t_{acc} + \frac{1}{2}a_y t_{acc}^2\right) + \left(v'_{iy}t_{no\ acc} - \frac{1}{2}gt_{no\ acc}^2\right)$. At the end of the acceleration phase due to the engines providing the lift, the rocket is moving with a velocity of $v_{fy} = v_{iy} + a_y t = 13\frac{m}{s^2} \times 8s = 104\frac{m}{s}$ vertically upward. The time interval over which the rocket is not accelerating due to its engines (but is slowing down due to gravity) is given as

   $v_{fy}' = v_{iy'} - gt_{no\ acc} \rightarrow t_{no\ acc} = \dfrac{v_{fy'} = v_{iy'}}{-g} = \dfrac{0\frac{m}{s} - 104\frac{m}{s}}{-9.8\frac{m}{s^2}} = 10.6s$. Using these two numbers we find the maximum height to be:

   $$x_{\max} = x_{acc} + x_{no\ acc} = \left(+\tfrac{1}{2}\left(13\tfrac{m}{s^2}\right)(8s)^2\right) + \left(104\tfrac{m}{s} \times 10.6s - \frac{9.8\frac{m}{s^2}}{2}(10.6s)^2\right) = 967.8m$$

   b. What is the total flight time of the rocket from lift-off to landing?

   To determine the total flight time, we need to add together the time the rocket accelerates upward, the time the rocket stops using its engines and cruises to maximum height, and the time it takes the rocket to fall vertically from this maximum height back to the ground. The time it takes the rocket to fall vertically to the ground from maximum height is given from

   $y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow 0 = 967.8m - \dfrac{9.8\frac{m}{s^2}}{2}t^2 \rightarrow t = 14.1s$. Thus the total time of flight is $t_{tof} = 8s + 10.6s + 14.1s = 32.7s$.

c. Suppose that you could do this same rocket launch experiment on Titan, a moon of Saturn. Titan has a mass $M_T = 0.023 M_E$ and a radius $R_T = 0.4 R_E$, where M_E and R_E are the mass and radius of the Earth respectively. The acceleration due to gravity on Titan g_T, compared to the acceleration due to gravity on the Earth, g_E is

1. smaller and is given by $g_T = 0.14 g_E$.
2. smaller and is given by $g_T = 0.06 g_E$.
3. equal to that on Earth.
4. larger and given by $g_T = 6.96 g_E$.
5. larger and given by $g_T = 16.7 g_E$.

d. Assuming that the acceleration produced by the rocket's engines is the same, if you were to launch this rocket on the surface of Titan, the maximum height reached above the surface of Titan, compared to the height reached on Earth, would be

1. lower because the acceleration due to gravity on Titan is smaller.
2. lower because the acceleration due to gravity on Titan is larger.
3. the same since the acceleration due to gravity on Titan is the same as on Earth.
4. greater because the acceleration due to gravity on Titan is smaller.
5. greater because the acceleration due to gravity on Titan is larger.

3. A daredevil is trying a death-defying stunt in which she will be launched across a $500\,ft \approx 167m$ wide chasm. She will be launched $10m$ from the left edge of the chasm by a cannon at a speed v at an angle of $\theta = 40^o$ above the horizontal.

a. At what speed would the daredevil need to be launched so that she lands on a safety mat located $10m$ from the right edge of the chasm? (Hint: Assume that she is launched $2m$ above the ground and lands in the safety mat that has a height of $1m$ above the ground.)

The time of flight is given from the combination of the vertical and horizontal trajectories. For the horizontal and vertical trajectories we have:
$x_f = v_{ix}t = (v_i \cos\theta)t$ and $y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 = y_i + (v_i \sin\theta)t - \frac{1}{2}gt^2$.
Eliminating time from the horizontal trajectory and substituting into the vertical trajectory, we have:

$$y_f = y_i + x_f \tan\theta - \tfrac{1}{2}g\left(\frac{x_f^2}{v_i^2\cos^2\theta}\right) \rightarrow 1m = 2m + 187m \times \tan 40 - \frac{9.8\frac{m}{s^2}}{2}\left(\frac{(187m)^2}{v_i^2\cos^2 40}\right)$$

$$\rightarrow v_i = 43\tfrac{m}{s}$$

b. How long will the daredevil be in the air enjoying the view?

Using the horizontal trajectory, say, we have the time given by

$$x_f = v_{ix}t = (v_i\cos\theta)t \rightarrow t = \frac{x_f}{v_i\cos\theta} = \frac{187m}{43\frac{m}{s}\times\cos 40} = 5.7s$$

c. Suppose that the stunt safety committee decides that the mat needs to be moved farther from the right edge of the chasm to keep the daredevil safe. Every time she performs the stunt, she uses the same cannon and is launched at the same speed as in part a. If she were to land farther from the right edge of the chasm the launch angle would have to

1. be increased from the initial angle of $\theta = 40^o$.
2. be decreased from the initial angle of $\theta = 40^o$.
3. be kept the same at $\theta = 40^o$.
4. be changed, but in a way that cannot be determined from the information given in the problem.

d. Suppose that you decided that you wanted, in light of the concerns over safety, to include air resistance into your calculations. Air resistance is a force that opposes the motion of the object and this force produces an additional acceleration on the system that opposes the velocity of the object. Compared to the case where there is no air resistance the maximum horizontal and vertical distances reached by the daredevil in the presence of air of resistance will be (assuming that the launch velocity is the same as in part a)

1. both greater in both directions than those reached with no air resistance.
2. both smaller in both directions than those reached with no air resistance.
3. greater and smaller respectively since air resistance only affects the horizontal component of motion.
4. smaller and greater respectively since air resistance only affects the vertical component of motion.

Useful formulas:

Motion in the r = x, y or z-directions

$r_f = r_0 + v_{0r}t + \frac{1}{2}a_r t^2$

$v_{fr} = v_{0r} + a_r t$

$v_{fr}^2 = v_{0r}^2 + 2a_r \Delta r$

Uniform Circular Motion

$a_r = \frac{v^2}{r}$

$F_r = ma_r = m\frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles: $C = 2\pi r$, $A = \pi r^2$

Triangles: $A = \frac{1}{2}bh$

Spheres: $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$

Quadratic equation: $ax^2 + bx + c = 0$,

whose solutions are given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\text{magnitude of a vector} = \sqrt{v_x^2 + v_y^2}$$

$$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\, m/s^2$ $\quad G = 6.67\times10^{-11}\, Nm^2/kg^2$

$N_A = 6.02\times10^{23}\, atoms/mole$ $\quad k_B = 1.38\times10^{-23}\, J/K$

$\sigma = 5.67\times10^{-8}\, W/m^2K^4$ $\quad v_{sound} = 343\, m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T):\ \beta = 3\alpha$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\,\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times10^{-12}\,\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L};\ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$