Physics 110

Exam #2

October 28, 2016

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\,\frac{m}{s}) = 10\,\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. A karate expert strikes downward with her fist of mass $m_{fist} = 0.7kg$ breaking a $m_{brick} = 3.2kg$ brick. The spring constant for the brick is $k = 2.6 \times 10^6 \frac{N}{m}$ and the brick breaks at a deflection $d = 1.5mm$. Here we are going to model the collision between your fist and the brick as an inelastic collision and immediately after the collision your fist and the brick will be moving with the same speed, call it $v_{fist+brick}$. The brick will do work on your fist bringing it to rest after the brick has been deflected by the distance d.

   a. How much work did the brick do bringing your hand to rest, just as the brick breaks?

   $$W = \Delta K = -\Delta U_s - \Delta U_g$$
   $$W = -\left(\tfrac{1}{2}ky_f^2 - \tfrac{1}{2}ky_i^2\right) - \left(m_{fist}gy_f - m_{fist}gy_i\right) = -\tfrac{1}{2}ky_f^2 - m_{fist}gy_f$$
   $$W = -\tfrac{1}{2}\left(2.6\times10^6 \tfrac{N}{m}\right)\left(1.5\times10^{-3}m\right)^2 - \left(0.7kg \times 9.8\tfrac{m}{s^2} \times 1.5\times10^{-3}m\right) = -2.94J$$

   b. Apply conservation of energy to the situation of your hand striking the brick and determine how fast are *the brick and your fist* ($v_{fist+brick}$) are moving just after you strike the brick.

   $$W = \Delta K + \Delta U_g + \Delta U_s = 0$$
   $$0 = -\tfrac{1}{2}\left(m_{fist} + m_{brick}\right)v_{fist+brick}^2 - \left(m_{fist} + m_{brick}\right)gd + \tfrac{1}{2}kd^2$$
   $$v_{fist+brick} = \sqrt{\frac{kd^2}{m_{fist} + m_{brick}} - 2gd}$$
   $$v_{fist+brick} = \sqrt{\left(\frac{2.6\times10^6 \tfrac{N}{m} \times \left(1.5\times10^{-3}m\right)^2}{0.7kg + 3.2kg}\right) - 2\times9.8\tfrac{m}{s^2}\times1.5\times10^{-3}m} = 1.2\tfrac{m}{s}$$

c. What is the minimum speed that your hand (v_{fist}) must be moving before it collides with the brick so that you can break this karate brick? (Hint: Apply conservation of momentum from just before you strike the brick to just after you strike the brick where your fist and brick are moving at the same speed.)

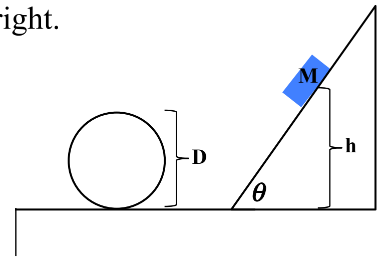
$$p_{iy} = p_{fy}$$

$$m_{fist} v_{fist} = \left(m_{fist} + m_{brick}\right) v_{fist+brick}$$

$$v_{fist} = \left(\frac{m_{fist} + m_{brick}}{m_{fist}}\right) v_{fist+brick} = \left(\frac{0.7kg + 3.2kg}{0.7kg}\right) \times 1.2\,\tfrac{m}{s}$$

$$v_{fist} = 6.7\,\tfrac{m}{s}$$

2. Suppose that you have a box of mass $M = 2kg$ released from rest at an unknown height h. The coefficient of friction between the block and the ramp oriented at an angle of $\theta = 60^0$ is $\mu = 0.5$ and the horizontal and circular potions of the track are assumed to be frictionless. The setup is shown on the right.



a. At what height h would you have to release the block so that it just barely makes it around the loop in the track? Assume that the loop has a diameter $D = 0.5m$.

To barely make it around the loop we have for the forces on the mass at the top:

$$\sum F_y : -F_N - Mg = -\frac{Mv^2}{R}$$

$$v_{top} = \sqrt{Rg} = \sqrt{0.25m \times 9.8 \tfrac{m}{s^2}} = 1.57 \tfrac{m}{s}$$

where to barely hang on is when the normal force vanishes.

The speed at the bottom of the hill is found from conservation of energy between the top of the loop and the bottom:

$$\Delta E_{system} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$

$$0 = \left(\tfrac{1}{2} Mv_{bottom}^2 - \tfrac{1}{2} Mv_{top}^2\right) + \left(Mgy_{bottom} - Mgy_{top}\right)$$

$$0 = \left(\tfrac{1}{2} Mv_{bottom}^2 - \tfrac{1}{2} Mv_{top}^2\right) - Mgy_{top}$$

$$v_{bottom} = \sqrt{v_{top}^2 + 2gy_{top}} = \sqrt{\left(1.57 \tfrac{m}{s}\right)^2 + 2 \times 9.8 \tfrac{m}{s^2} \times 0.5m} = 3.5 \tfrac{m}{s}$$

To find the height of the hill, again use conservation of energy:

$$\Delta E_{system} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = -W_{fr}$$

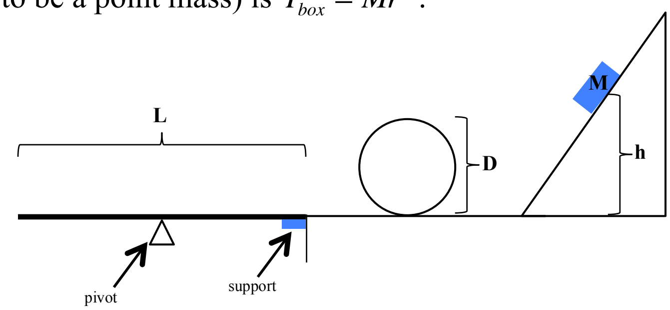
$$0 = \tfrac{1}{2} Mv_{bottom}^2 - Mgh = -\left(\mu Mg\cos\theta\right)x = -\left(\mu Mg\cos\theta\right)\left(\frac{h}{\sin\theta}\right)$$

$$h = \frac{v_{bottom}^2}{2g\left(1 - \mu/\tan\theta\right)} = \frac{\left(3.5 \tfrac{m}{s}\right)^2}{2 \times 9.8 \tfrac{m}{s^2}\left(1 - 0.5/\tan 60\right)} = 0.88m$$

b. What is the speed of the box when it leaves the horizontal surface on the left?

From part a, $v_{bottom} = 3.5 \tfrac{m}{s}$ since the surface is frictionless.

c. To the left of the loop and at the end of the horizontal section of track, the box encounters one last section of horizontal track. This last section of track has a mass of $M_{track} = 3kg$ and the section is $L = 2m$ long. This last section of track is supported on the right end by a non-moveable support while the left end of the track is free to rotate about the pivot as shown below. The pivot is located at the midpoint of the track and there is friction present on this last section with coefficient of friction $\mu = 0.5$. Does the system rotate? If it does, what is the angular acceleration of the system? If it does not rotate, what is the reaction force of the support on the right end of the last segment of track? Hints: The moment of inertia of the box (assumed to be a point mass) is $I_{box} = Mr^2$.



To determine whether or not the segment of track pivots or not, we need to know where the mass comes to rest. To do this we calculate the work done.

$$\Delta E_{system} = -W_{fr} = \Delta K_T = -\tfrac{1}{2} M v_{bottom}^2$$

$$\mu M g x = \tfrac{1}{2} M v_{bottom}^2$$

$$x = \frac{v_{bottom}^2}{2\mu g} = \frac{\left(3.5 \tfrac{m}{s}\right)^2}{2 \times 0.5 \times 9.8 \tfrac{m}{s^2}} = 1.25m$$

Since this is on the left side of the pivot, so there will be a net torque and the system will rotate.

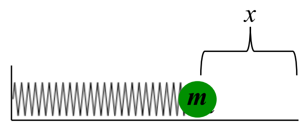
To calculate the angular acceleration we use:

$$\tau_{net} = I\alpha \rightarrow \alpha = \frac{\tau_{net}}{I} = \frac{rF\sin\phi}{I} = \frac{RMg}{I} = \frac{RMg}{MR^2} = \frac{g}{R}$$

$$\alpha = \frac{9.8 \tfrac{m}{s^2}}{0.25m} = 39.2 \tfrac{rad}{s^2}$$

3. A ball of mass $m = 0.050kg$ and radius $r = 1.75cm$ is fired from a spring ($k = 375\frac{N}{m}$) that is on a horizontal surface. Initially the spring is compressed by an amount $x = 15cm$ and the system is released from rest. When the spring is released from rest the ball rolls without slipping across the horizontal surface toward the pendulum arm as shown in the figure below.

   a. When the spring reaches its equilibrium length, the ball loses contact with the spring. What is the translational speed of the ball when it loses contact with the spring? Hint: The moment of inertia of a ball rotated about any axis is $I = \frac{2}{5}mr^2$.



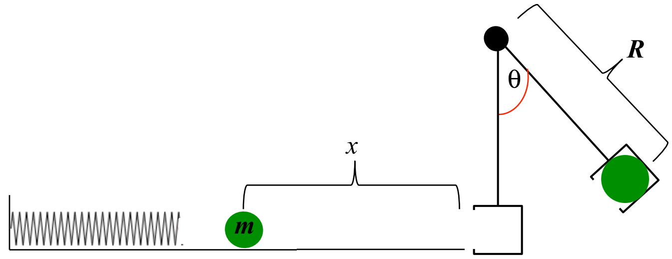
$$\Delta E_{system} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$

$$0 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 - \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 - \tfrac{1}{2}kx^2$$

$$0 = \left(m + \tfrac{2}{5}m\right)v^2 - kx^2 = \tfrac{7}{5}mv^2 - kx^2$$

$$v = \sqrt{\frac{5kx^2}{7m}} = \sqrt{\frac{5 \times 375\frac{N}{m} \times (0.15m)^2}{7 \times 0.050kg}} = 11\tfrac{m}{s}$$

   b. When the ball gets to the end of the horizontal surface, a distance $x = 0.5m$ of from the end of the spring, it strikes and becomes embedded in the stationary pendulum arm. Assume that the pendulum arm from pivot to cup has a length of $R = 31cm$ and that the pendulum arm has a mass $M = 0.250kg$. What is the *expression for the rotational speed of the* ***ball and pendulum arm*** *after the collision* if the pendulum arm rises through an angle of θ? Hint: After the collision the system does not translate, but rather only rotates. Also assume that there is no friction in the point about which the ball and pendulum swings. Your expression should not contain any numbers (except things like factors of 2) rather only symbols.



$$\Delta E_{system} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$

$$0 = -\tfrac{1}{2}I\omega^2 + mgh = -\tfrac{1}{2}I\omega^2 + m_{ball+pendulum}gR(1-\cos\theta)$$

$$\omega^2 = \frac{2gRm_{ball+pendulum}}{I}(1-\cos\theta)$$

$$\omega = \sqrt{\frac{2gRm_{ball+pendulum}}{I}(1-\cos\theta)}$$

c. After the collision between the ball and pendulum arm, the ball and pendulum arm swing through an angle of $\theta = 28^0$ and momentarily comes to rest. Using your expression for the rotational velocity from part b, what is the moment of inertia of the ball and pendulum arm? Hint: In the absence of any external torques the angular momentum is conserved and the initial angular momentum of the system is given by $L_i = rmv_i$.

$$L_i = L_f$$

$$Rm_{ball}v_i = I\omega = I\sqrt{\frac{2gRm_{ball+pendulum}}{I}(1-\cos\theta)}$$

$$R^2m_{ball}^2v_i^2 = 2gRIm_{ball+pendulum}(1-\cos\theta)$$

$$I = \frac{m_{ball}^2v_i^2R}{2gm_{ball+pendulum}(1-\cos\theta)} = \frac{(0.050kg)^2\times(11\tfrac{m}{s})^2\times 0.31m}{2\times 9.8\tfrac{m}{s^2}\times(0.050kg+0.250kg)\times(1-\cos 28)}$$

$$I = 0.141kgm^2$$

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^{\ 2} = v_{0r}^{\ 2} + 2a_r \Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	Triangles	Spheres
$C = 2\pi r$	$A = \tfrac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \tfrac{4}{3}\pi r^3$

$Quadratic\ equation: ax^2 + bx + c = 0,$

$whose\ solutions\ are\ given\ by: \ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$magnitude\ of\ a\,vector = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$$

$$direction\,of\ a\,vector \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}\!/\!_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/\!_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}\!/\!_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}\!/\!_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}\!/\!_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}\!/\!_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}[T_F - 32]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$$

$$PV = Nk_B T$$

$$\tfrac{3}{2}k_B T = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$$

$$a_r = r\omega^2$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho g d$$

$$F_B = \rho g V$$

$$A_1 v_1 = A_2 v_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho g h_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$$